Secondary One Mathematics: An Integrated Approach

Module 6

Congruence, Construction and Proof

By

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In partnership with the
Utah State Office of Education

Module 6 – Congruence, Construction and Proof

Classroom Task: Leaping Lizards! - A Develop Understanding Task
Developing the definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.1

Classroom Task: Is It Right? - A Solidify Understanding Task
Examining the slope of perpendicular lines (**G.CO.1, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.2

Classroom Task: Leap Frog– A Solidify Understanding Task
Determining which rigid-motion transformations carry one image onto another congruent image (**G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.3

Classroom Task: Leap Year – A Practice Understanding Task
Writing and applying formal definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.2, G.CO.4, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.4

Classroom Task: Symmetries of Quadrilaterals – A Develop Understanding Task
Finding rotational symmetry and lines of symmetry in special types of quadrilaterals (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.5

Classroom Task: Symmetries of Regular Polygons – A Solidify Understanding Task
Examining characteristics of regular polygons that emerge from rotational symmetry and lines of symmetry (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.6

Classroom Task: Quadrilaterals-Beyond Definition – A Practice Understanding Task
Making and justifying properties of quadrilaterals using symmetry transformations (**G.CO.3, G.CO.4, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.7

Classroom Task: Can You Get There From Here? – A Develop Understanding Task
Describing a sequence of transformations that will carry congruent images onto each other (**G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.8

Classroom Task: Congruent Triangles – A Solidify Understanding Task
Establishing the ASA, SAS and SSS criteria for congruent triangles (**G.CO.6, G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.9

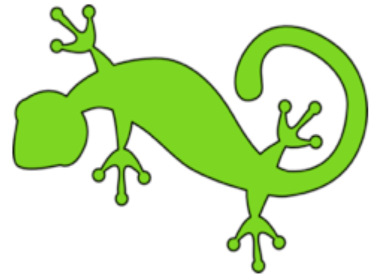
Classroom Task: Congruent Triangles to the Rescue – A Practice Understanding Task
Working with systems of linear equations, including inconsistent and dependent systems
(**G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.10

Classroom Task: Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct rhombuses and squares
(**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.11

Classroom Task: More Things Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct parallelograms, equilateral triangles and inscribed hexagons (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.12

Classroom Task: Justifying Constructions – A Solidify Understanding Task
Examining why compass and straightedge constructions produce the desired objects (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.13

Classroom Task: Construction Blueprints – A Practice Understanding Task
Writing procedures for compass and straightedge constructions (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.14



6.1 Leaping Lizards!

A Develop Understanding Task

Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns) or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:

{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the **pre-image** and **image** anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them

Lazy Lizard

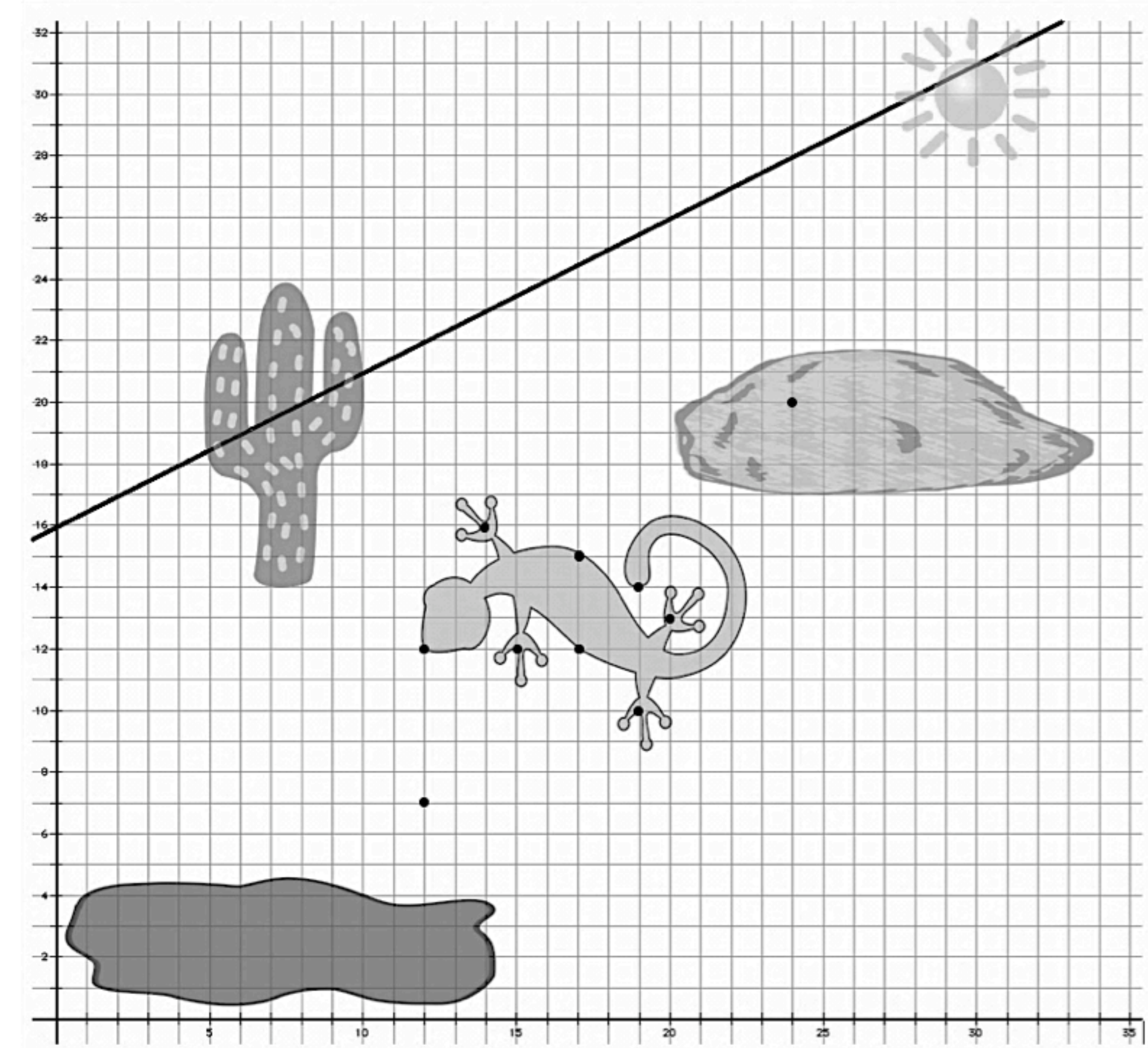
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

Lunging Lizard

Rotate the lizard 90° about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard

Reflect the lizard about given line $y = \frac{1}{2}x + 16$ so it looks like the lizard is doing a back flip over the cactus.



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6.1 Leaping Lizards! – Teacher Notes
A Develop Understanding Task

Purpose: This task provides an opportunity for formative assessment of what students already know about the three rigid-motion transformations: translations, reflections, and rotations. As students engage in the task they should recognize a need for precise definitions of each of these transformations so that the final image under each transformation is a unique figure, rather than an ill-defined sketch. The exploration and subsequent discussion described below should allow students to begin to identify the essential elements in a precise definition of the rigid-motion transformations, *e.g., translations move points a specified distance along a line parallel to a specified line; rotations move points along a circular arc with a specified center through a specified angle, and reflections move points across a specified line of reflection so that the line of reflection is the perpendicular bisector of each line segment connecting corresponding pre-image and image points.*

In addition to the work with the rigid-motion transformations, this task also surfaces thinking about the slope criteria for determining when lines are parallel or perpendicular. In a translation, the line segments connecting pre-image and image points are parallel, having the same slope. In a 90° rotation, the line segments connecting pre-image and image points are perpendicular, having opposite reciprocal slopes. Likewise, in a reflection, the line segments connecting pre-image and image points are perpendicular to the line of reflection.

Finally, this task reminds students that rigid-motion transformations preserves distance and angle measures—implying that the figures forming the pre-image and image are congruent. Students will be attending to two different categories of distances—the lengths of line segments that are used in the definitions of the transformations, and the lengths of the congruent line segments that are contained within the pre-image and image figures themselves. Students may determine that these lengths are preserved by counting units of "rise" and "run", or by using the Pythagorean Theorem. Ultimately, this work will lead to the development of the distance formula in future tasks.

Core Standards Focus:

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Related Standards: G.CO.2, G.CO.6, G.GPE.5

Teacher Note: Students' previous experiences with rigid motions may have surfaced intuitive ways of thinking about these transformations, but such informal definitions will not support students in proving geometric properties based on a transformational approach. Experiences with sliding, flipping and turning rigid objects will have provided experimental evidence that rigid-motion transformations preserve distance and angle, such that,

- Lines are taken to lines, and line segments to line segments of the same length.
- Angles are taken to angles of the same measure.
- Parallel lines are taken to parallel lines.

Students who have used technology to translate, rotate or reflect objects may not have attended to the essential features that define such transformations. For example, a student can mark a mirror line and click on a button to reflect an object across the mirror line without noting the relationship between the pre-image and image points relative to the line of reflection. Consequently, research shows that students harbor many misconceptions about the placement of an image after a transformation—erroneous assumptions such as:

- one of the sides of a reflected image must coincide with the line of reflection
- the center of a rotation must be located at a point on the pre-image (e.g., a vertex point) or at the origin
- a pre-image point and corresponding image point do not need to be the same distance away from the center of the rotation

Watch for these misconceptions as students engage in this task.

Launch (Whole Class):
Set the stage for the work of this learning cycle by discussing the ideas of computer animation as outlined in the first few paragraphs of this task. As part of the launch ask students why they think we need only keep track of a few anchor points, since the image of the lizard consists of infinitely many points, in addition to the eight points that are listed. The issue to be raised here is that rigid-motion transformations preserve distance and angle (properties that have been established in Math 8). Therefore a software animation program could draw features of the lizard, such as the toes on each of the feet, by starting at an anchor point and using predetermined angle and distance measures to locate other points on the toes. Make sure students pay attention to the order in which each of the anchor points should be listed after completing each of the transformations. This will help students pay attention to individual pairs of pre-image and image points.

Provide multiple tools for students to do this work, such as transparencies or tracing paper, protractors, rulers, and compasses. The coordinate grid on which the images are drawn is also a tool for doing this work, but initially students may not recognize the usefulness of the grid as a way of carrying out the transformations, but rather just as a way of designating the location of the points after the transformation is complete. Technology tools may obscure the ideas being surfaced in the task, so it is best to use the tools described, which will allow students to pay attention to the details of their work.

It is intended that students should work on the transformations in the order listed in the task.

Explore (Small Group):
This task provides a great opportunity to pre-assess what students know about each of the rigid-motion transformations, so don't worry if not all students are locating the final images correctly. Pay attention to the misconceptions that may arise (see teacher note).

If students use transparencies (or tracing paper) to copy the original lizard and then locate the image by sliding, turning or flipping the transparency, you will want to make sure they also think about these movements relative to the coordinate grid. Ask, "How could you have used the coordinate grid to locate this same set of points?" Focusing students' attention on the coordinate grid will facilitate connecting the details that need to be articulated in the definitions of the rigid-motion transformations to coordinate geometry ideas, such using slope to determine if lines are parallel or perpendicular. In this task, these ideas are surfaced and informally explored. In subsequent tasks these ideas are made more explicit and eventually justified.

Students should be fairly successful translating "Lazy Lizard", since the point at the tip of the nose moves up 8 units and right 14 units, every anchor point must move the same. Watch for two different strategies to emerge: some students may move each point up 8, right 14; others may move one point to the correct location, and then duplicate the relative positions of the points in the pre-image to locate points in the image—thereby preserving distance and angle between the points in the pre-image and those same points in the image.

To get started on "Lunging Lizard" you may want to direct students' attention to the point at the tip of the lizard's nose, which lies on a vertical line, 5 units above the center of rotation. Ask students where this point would end up after rotating 90° counterclockwise. Watch for students who are attending to the 90° angle of rotation by drawing line segments from the center of rotation to the image and corresponding pre-image points. Also watch for how students determine that an image points is the same distance away from the center of rotation as its corresponding pre-image point: do they measure with a ruler, do they draw concentric circles centered at (12, 7), do they count the rise and run from (12, 7) to a point on the lizard and then use a related way of counting rise and run to locate the image point—intuitively using the Pythagorean Theorem to keep the same distance, or do they ignore distance altogether?

For "Leaping Lizard" watch for students who may have noticed that an image point and its corresponding pre-image point are equidistant from the line of reflection. Listen for how they justify that these distances are the same: do they measure with a ruler, do they fold the paper along the line of reflection, do they count the rise and run from the pre-image to the line of reflection and then from the line of reflection to the image point—intuitively using the Pythagorean Theorem to keep the same distance. Also watch for students who notice that the line segments connecting the image points to their corresponding pre-image points are all parallel to each other—perhaps even noticing that all of these line segments have a slope of -2.

Discuss (Whole Class):
If students have not all located the same set of points for the images of the transformations, have students discuss whether this is reasonable or not. Inform students, "That transformations are like functions—any set of points that form a pre-image should have a unique set of points that form the image that is the result of the transformation. If we have not obtained unique images, then we have not recognized the precise nature of these transformations. That is the goal of our work today, to notice what is important about each transformation so the images produced by the transformation are precisely defined."

Discuss strategies for locating the images of the anchor points for each transformation. Here is a suggested list of a sequence of ideas to be presented, if available. While we will not be writing precise definitions for the transformations until the task *Leap Year*, it is important that the ideas of distance and direction (e.g., along a parallel line, perpendicular to a line, or along a circle) emerge during this discussion. If not all of the suggested strategies are available in the student work, at least make sure the debrief of each transformation does focus on both distance and direction. If either idea is missing, ask additional questions to prompt for it. For example, "How did you know how far away from the center point (or the reflecting line) this image point should be?" Also, be aware of the tasks that follow in this learning cycle—not everything needs to be neatly wrapped up in this discussion.

Debriefing the translation:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who moved each anchor point up 8, right 14 units.
- Finally, have a student present who moved one anchor point up 8, right 14 units and then used the relative positions of the points in the original figure to locate related points in the image figure. Discuss that this is possible because translations preserve distance, angle and parallelism.

Debriefing the rotation:

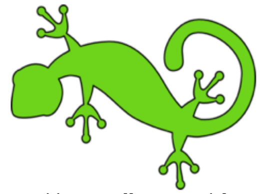
- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who used a protractor to measure 90° and a ruler to measure distances from the center of rotation. Draw in the line segments between (12, 7) and the corresponding image and pre-image points, using a different color for each image/pre-image pair. This will highlight the 90° angle of rotation, centered at (12, 7).
- Next, have a student present who drew concentric circles (or arcs) to show that pairs of image/pre-image points are the same distance from (12, 7) because they lie on the same circle.
- Finally, have a student present who showed that image/pre-image points are the same distance from (12, 7) by using the Pythagorean Theorem, or some strategy that is intuitively equivalent.

Debriefing the reflection:

- Have a student present who used a transparency or tracing paper to get a set of image points that the whole class can agree upon.
- Next, have a student present who used a ruler to measure distances from the line of reflection.
- If available, have a student describe how they determined these distances from the line of reflection using the Pythagorean Theorem, or some strategy that is intuitively equivalent.
- Next, have a student present who noticed that the segments connecting pairs of image/pre-image points are parallel, perhaps by pointing out that they have the same slope.
- Finally, have a student present who might argue that the segments connecting pairs of image/pre-image points are perpendicular to the line of reflection.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.1

Ready, Set, Go!

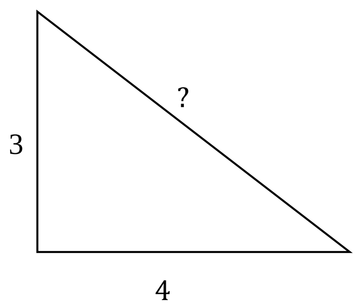


Ready

Topic: Pythagorean Theorem

For each of the following right triangles determine the number units measure for the missing side.

1.



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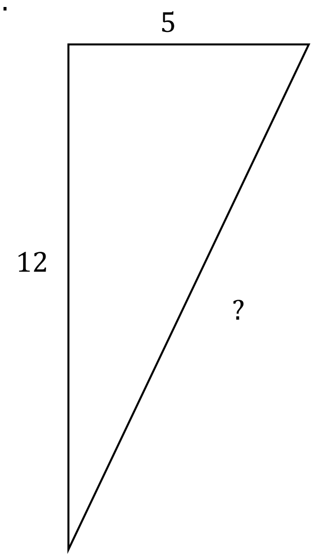

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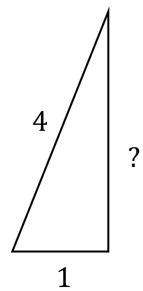

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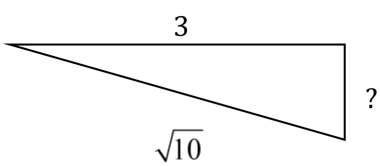
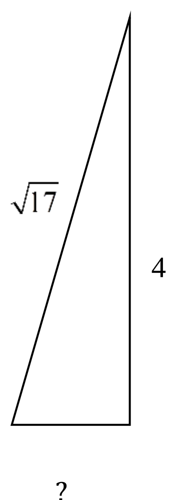
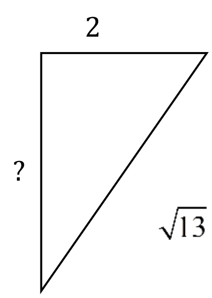


5.



6.



Set

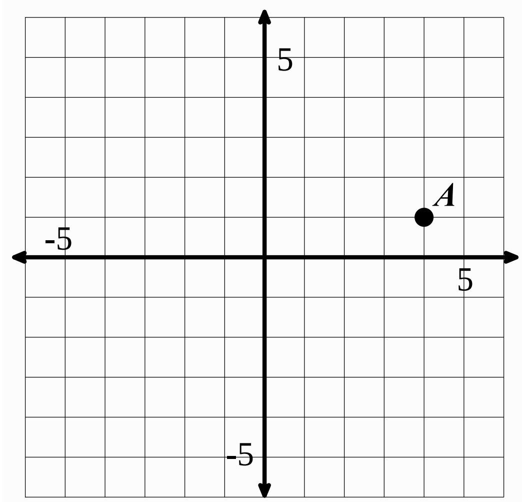
Topic: Transformations

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A'

b. Reflect point A over x-axis, label as A''

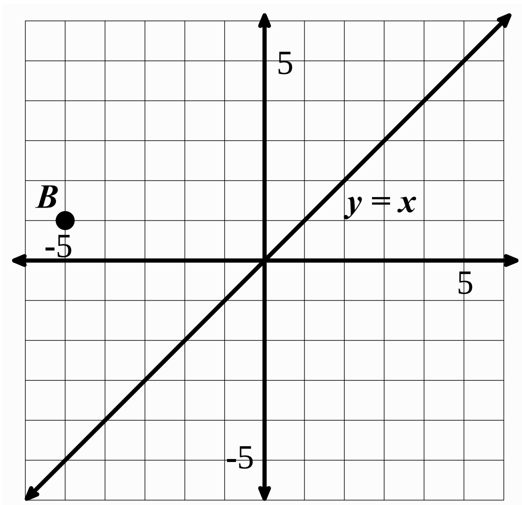
c. Apply the rule $(x-2, y-5)$, to point A and label A'''



8a. Reflect point B over the line $y = x$, label as B'

b. Rotate point B 180° about the origin, label as B''

c. Translate point B the point up 3 and right 7 units, label as B'''

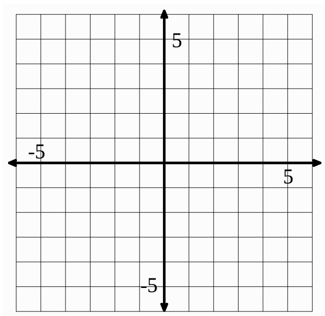


Go

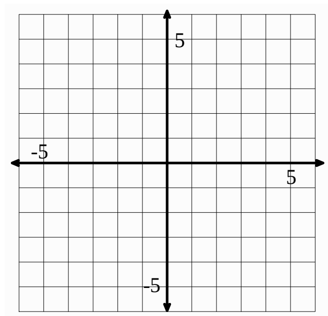
Topic: Graphing linear equations

Graph each equation on the coordinate grid provided. Extend the line as far as the grid will allow.

9. $y = 2x - 3$

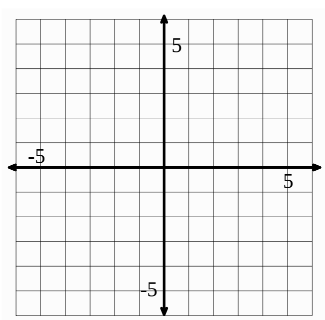


10. $y = -2x - 3$

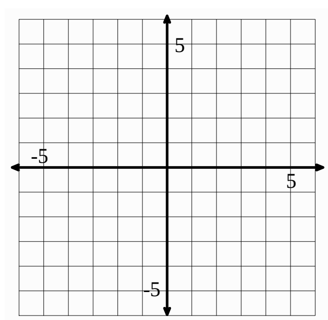


11. What similarities and differences are there between the equations in number 13 and 14?

12. $y = \frac{2}{3}x + 1$

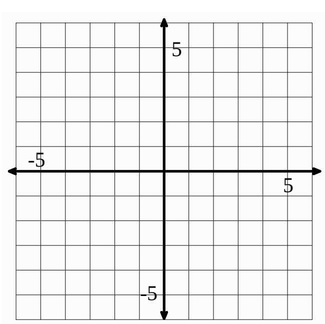


13. $y = -\frac{3}{2}x + 1$

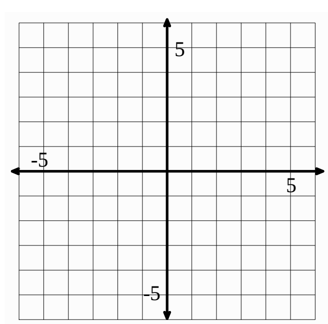


14. What similarities and differences are there between the equations in number 15 and 16?

15. $y = x + 1$



16. $y = x - 3$



17. What similarities and differences are there between the equations in number 15 and 16?

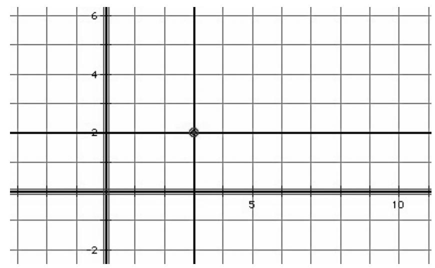
6.2 Is It Right?

A Solidify Understanding Task

In *Leaping Lizards* you probably thought a lot about perpendicular lines, particularly when rotating the lizard about a 90° angle or reflecting the lizard across a line.

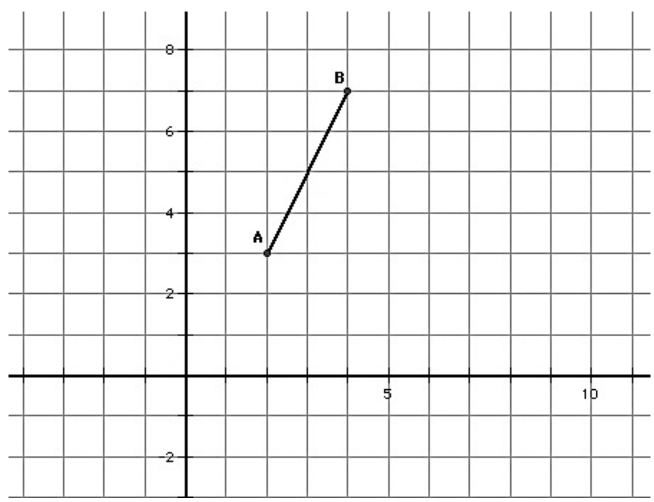
In previous tasks, we have made the observation that *parallel lines have the same slope*. In this task we will make observations about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: *Horizontal and vertical lines are perpendicular.* For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.



But what if a line or line segment is not horizontal or vertical?
How do we determine the slope of a line or line segment that will be perpendicular to it?

<u>Experiment 1</u>



1. Consider the points A (2, 3) and B (4, 7) and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

2. Locate a third point C (x, y) on the coordinate grid, so the points A (2, 3), B (4, 7) and C (x, y) form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

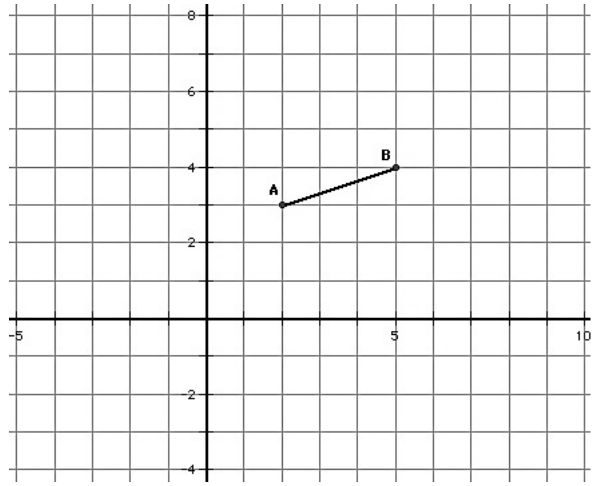
3. Explain how you know that the triangle you formed contains a right angle?

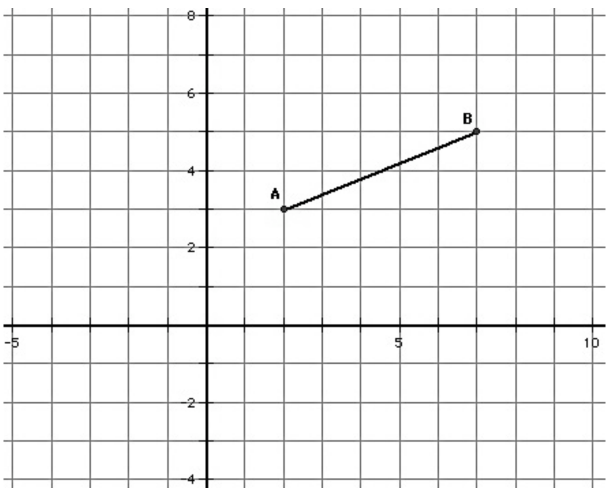
4. Now rotate this right triangle 90° about the vertex point (2, 3). Explain how you know that you have rotated the triangle 90°.

5. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

Experiment 2

Repeat steps 1-5 above for the points A (2, 3) and B (5, 4).



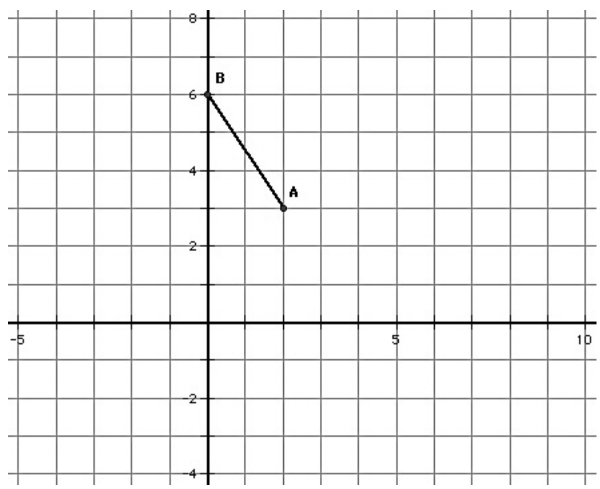


Experiment 3

Repeat steps 1-5 above for the points A (2, 3) and B (7, 5).

Experiment 4

Repeat steps 1-5 above for the points A (2, 3) and B (0, 6).



Based on experiments 1-4, state an observation about the slopes of perpendicular lines.

While this observation is based on a few specific examples, can you create an argument or justification for why this is always true? (Note: You will examine a formal proof of this observation in the next module.)

6.2 Is It Right?

A Solidify Understanding Task

Purpose: In this task students make a conjecture about the slopes of perpendicular lines. This observation will be formally proved in a later task, but the representation-based argument presented in this task suggests that this is a reasonable generalization across all cases. In the previous task, students encountered the idea of perpendicularity when they were asked to rotate the lizard 90°, and they may also have noticed the idea when they considered the relationship between the line of reflection and the points on the reflected image of the lizard. Students might have used the square corner of a piece of paper or a protractor to measure these right angles. In this task they consider how the coordinate grid can be used to determine if two lines are perpendicular. This is one example of the connections that can be made between coordinate geometry and transformations. It is powerful for students to be able to draw upon two different representational systems to think about the same ideas.
In this task, students also consider a definition of perpendicular that is related to reflections: *Two lines are perpendicular if they meet to form congruent adjacent angles*.

Core Standards Focus:

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Related Standards: G.GPE.4

Launch (Whole Class):
Students already have a sense of what it means for two lines to be perpendicular. Have them state their "definitions" and list their ideas on the board (e.g., they meet at right angles, they form 90° angles, if one is horizontal then the other is vertical, etc.) Then propose the following definition of perpendicular lines: *Lines are perpendicular if they meet to form congruent adjacent angles*. Ask students how this definition connects to the ideas they have recorded on the board about perpendicular lines (e.g., since the two adjacent angles form a straight angle measuring 180°, if the adjacent angles are also congruent they must each measure 90°; by definition, a right angle measures 90°). Ask students how they might justify this definition of perpendicular lines using transformations. Students might suggest folding or reflecting half of the line onto the other half, creasing the fold along the perpendicular line or using the perpendicular line as the line of reflection. Since reflections preserve angle measure, the image angle and its adjacent pre-image angle are congruent.

With these ideas about perpendicular lines activated for students, turn their attention to the experiments outlined in the task. Inform students that they are going to make a conjecture about lines that are perpendicular on a coordinate grid. For the sake of these experiments, we will agree that horizontal and vertical lines are perpendicular.

As part of the launch, make sure that students can accurately plot point *C* to form the third vertex of a right triangle *ABC* with segment *AB* as the hypotenuse. There are two positions where *C* can be located so that the legs of the right triangle lie along horizontal and vertical lines, thus guaranteeing that we have a right angle at *C.*

Explore (Small Group):
The exploration asks students to rotate a right triangle 90° around a vertex located at one of the acute angles. Since the legs of the right triangle are oriented along horizontal and vertical lines, students will know that they have rotated the triangle 90° when the leg adjacent to point *A* in the pre-image right triangle is oriented in the other direction—horizontal or vertical—in the resulting image right triangle. (See diagrams below)

Watch for students to plot point *C* in a correct location, and listen for their arguments as to how they know they have rotated the right triangle 90°. Ask students how they know that the hypotenuse of the rotated right triangle is perpendicular to the hypotenuse of the original right triangle as a result of this rotation. It may be helpful to have students use a compass to connect points *B* and *C* with their image points along circles, centered at *A*—particularly if this was a strategy used to analyze the rotation in the previous task.

Make sure that students are recording the slopes of the hypotenuse of the pre-image right triangle and its resulting image right triangle correctly.

Discuss (Whole Class):
Post samples of student work that illustrate what happens when *C* is located below *B* and to the right of *A*, as well as what happens when *C* is located above *A* and to the left of *B*, as in the following diagram:

Have students articulate how they know when they have rotated the triangle 90° about point A by having them identify the horizontal/vertical relationship between leg AC and its image. Have them justify why this means that the hypotenuse of the rotated right triangle is also perpendicular to the hypotenuse of the original right triangle. Then have them state a conjecture about the slope of perpendicular lines based on these four examples. Remind students that four examples do not prove a conjecture, and have them suggest how they might generalize this work—that is, does the visual representation of the perpendicular hypotenuses hold, regardless of the size of the right triangle or the measures of the acute angles in the right triangle? This is an informal argument for an idea that will be formally proven in a task in the next module.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.2

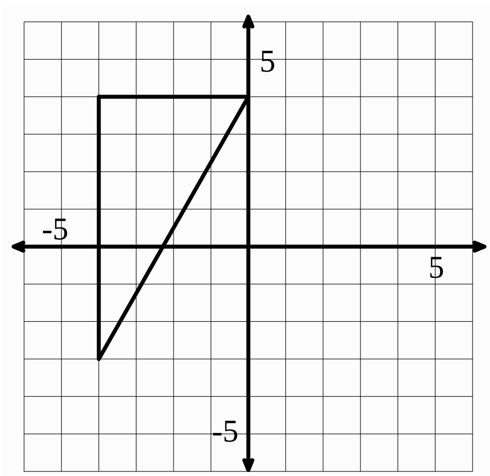
Ready, Set, Go!

Ready

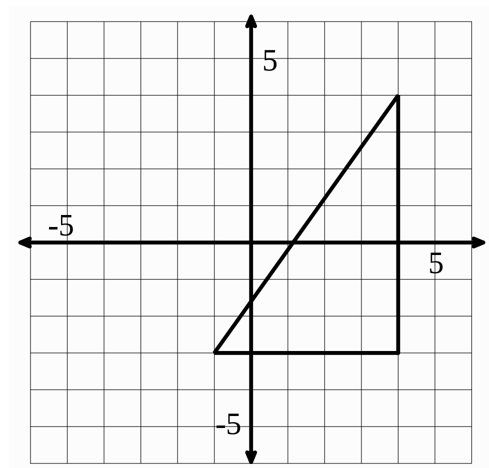
Topic: Finding Distance using Pythagorean Theorem

Use the coordinate grid to find the length of each side of the triangles provided.

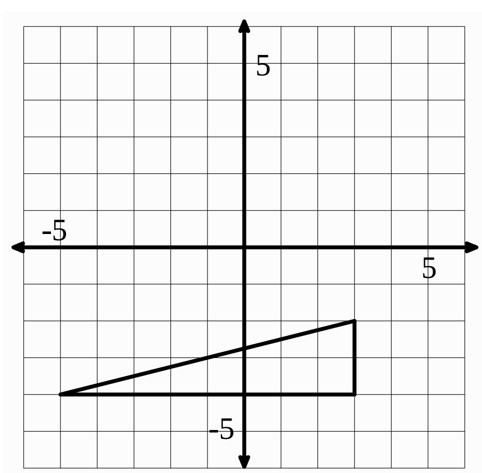
1.



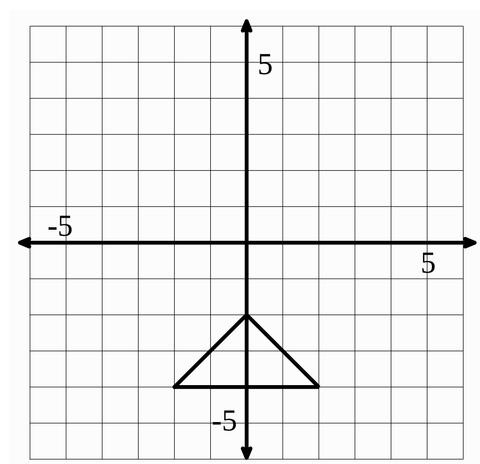
2.



3.



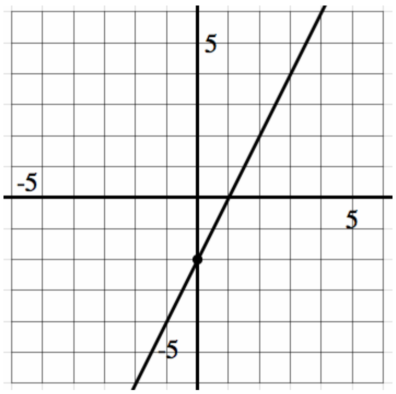
4.



Set

Topic: Slopes of parallel and perpendicular lines.

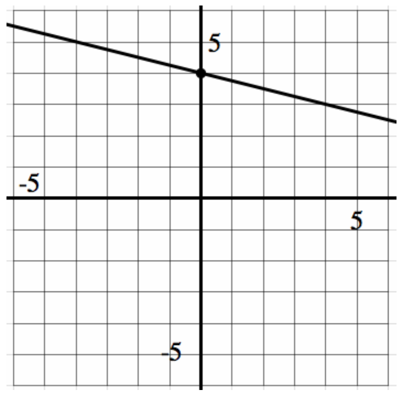
5. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

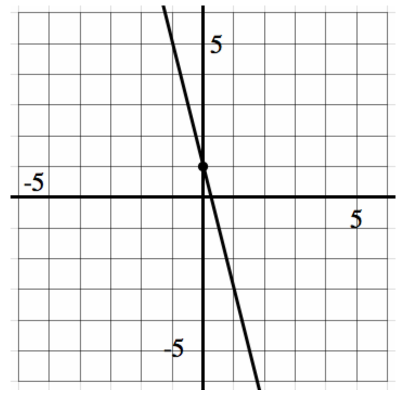
6. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

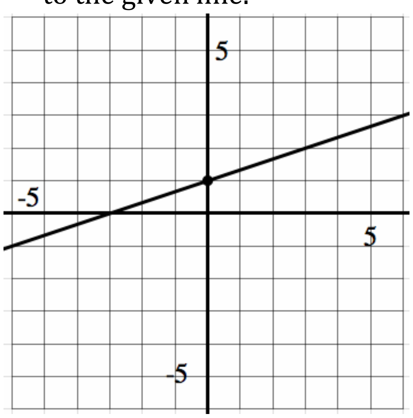
7. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

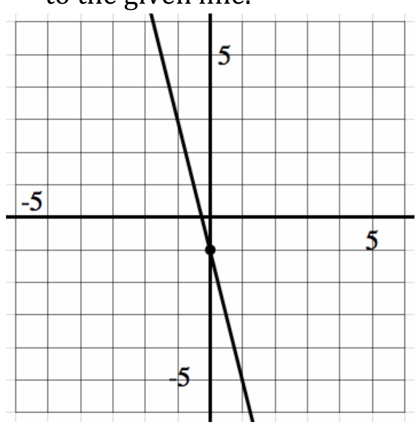
8. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

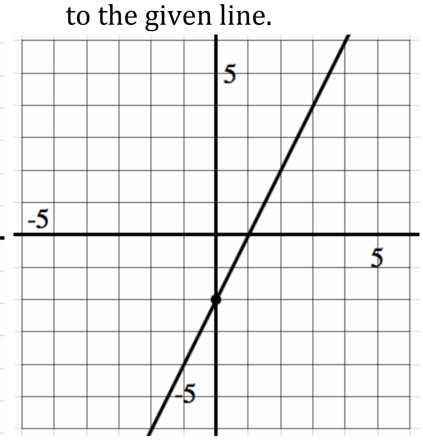
9. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

10. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

Go

Topic: Solve the following equations.

Solve each equation for the indicated variable.

11. $3(x-2)=5x+8$; Solve for *x*.

12. $-3+n=6n+22$; Solve for *n*.

13. $y-5=m(x-2)$; Solve for *x*.

14. $Ax+By=C$; Solve for *y*.

6.3 Leap Frog

A Solidify Understanding Task

Josh is animating a scene where a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent". In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.
- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.
- If you decide the transformation is a translation you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.
- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	

images this page:
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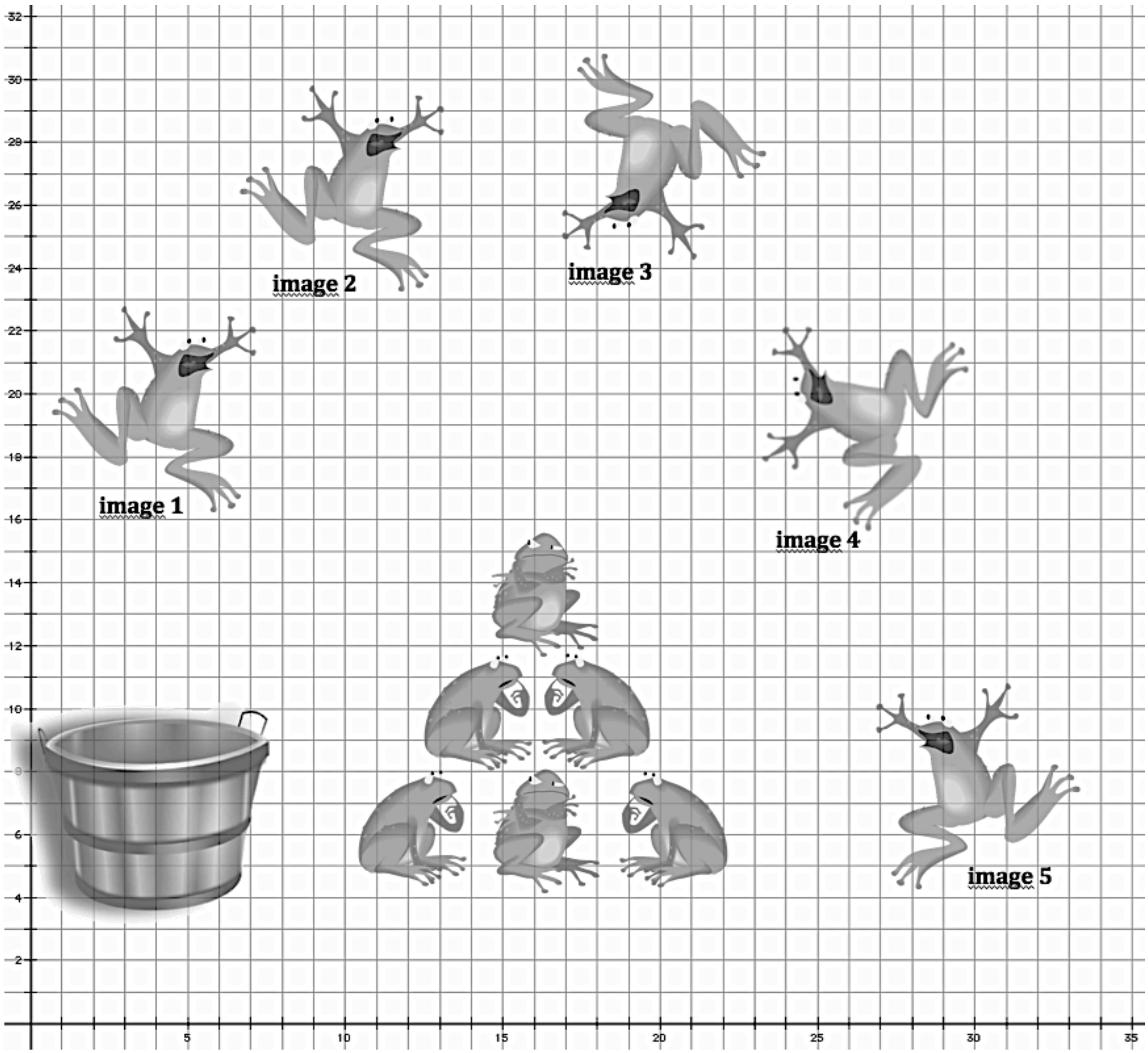
©2012 http://openclipart.org/detail/33985/architetto
©2012 http://openclipart.org/detail/170806/hatalar205



6.3 Leap Frog – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to continue solidifying the definitions of the rigid-motion transformations. Building on the work started in *Leaping Lizards*, students describe translations, rotations and reflections in terms of the important features that define these transformations, such as the distance from the center of rotation or from the line of reflection. Students will also surface the idea that it may take a combination of transformations to move from one image to another.

Core Standards Focus:

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Specify a sequence of transformations that will carry a given figure onto another.

Related Standards: G.CO.1, G.CO.2, G.CO.6

Launch (Whole Class):

Remind students that in *Leaping Lizards* they were trying to find where an image would end up after undergoing a specified transformation. In that task the transformation was described and they had to locate the image. In this task the work is reversed: the image and its pre-image are specified, and they have to describe the transformation that carries one onto the other. Review with students the bulleted list of features they should include in a description of each type of transformation. Also point out the possibility that it may take more than one transformation to get from one figure to another. In such a case, they are to describe each transformation in the sequence of transformations they would perform.

Explore (Small Group):

Since no anchor points are specified, students will need to select a few points to pay attention to, however they should intuitively recognize that all of the points on the pre-image get transformed in the same way. (Technically every point in the plane moves to a new location except for the points on a line of reflection or the point at the center of a rotation. Students may not recognize that the white points move in the same way as the colored points do.) Students might focus on points that lie on lattice points of the grid, such as the center of the frog's tongue or the end of the middle toe on the left foot.

Listen for ways that students are describing the transformations, and that they are noting the important characteristics of each transformation, such as finding both the center and the angle of rotation for figures that have been rotated. The last two pre-image/image pairs will require more than one transformation—a reflection and translation in the 4th pair, and a translation and rotation in the 5th pair—however, there are many possible combinations of transformations that will work.

These last two problems are setting up the work of a later task, *Can You Get There From Here*, and it is not necessary at this time that all students successfully finish these last two problems.

Discuss (Whole Class):
Select students to share their descriptions on each of the five pre-image/image pairs. Emphasize the essential features of each transformation in these discussions: every point in a translation moves the same distance along parallel lines; every point on a rotation moves through a specified angle along circular arcs that all have the same center point; and, the line of reflection is the perpendicular bisector of the segments connecting image and pre-image points in a reflection. Students will write formal definitions of the three rigid-motion transformations in the next task, so in this discussion it is sufficient for students to be noticing these details and supporting their claims as they analyze the transformations that carry one figure onto another.

There are multiple sequences of transformations that work for the 4th and 5th pre-image/image pairs. If available, share a couple of descriptions for each. For example, to get from image 1 to image 5 you might reflect image 1 about some vertical line, then translate the reflected image until it coincides with image 5; or, you might translate image 1 until you can identify a mirror line in which to reflect the image to get it to coincide with image 5.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.3

Ready, Set, Go!

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Ready

Topic: Basic Rotations and Reflections of objects

In each problem there will be a preimage and several images based on the give preimage. Determine which of the images are rotations of the given preimage and which of them are reflections of the preimage. If an image appears to be created as the result of a rotation and a reflection then state both.

1.

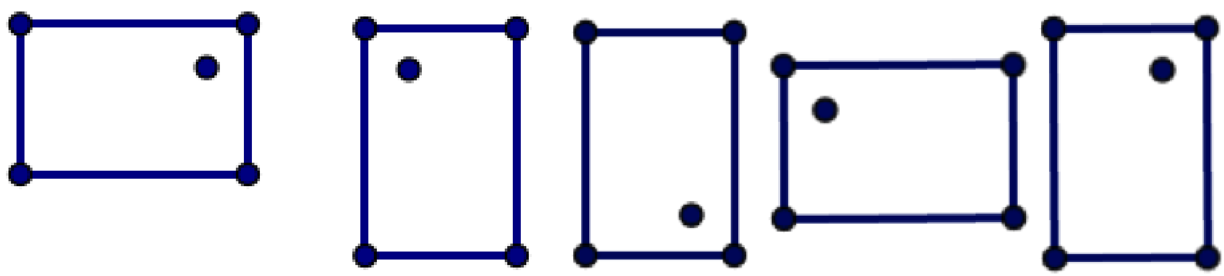
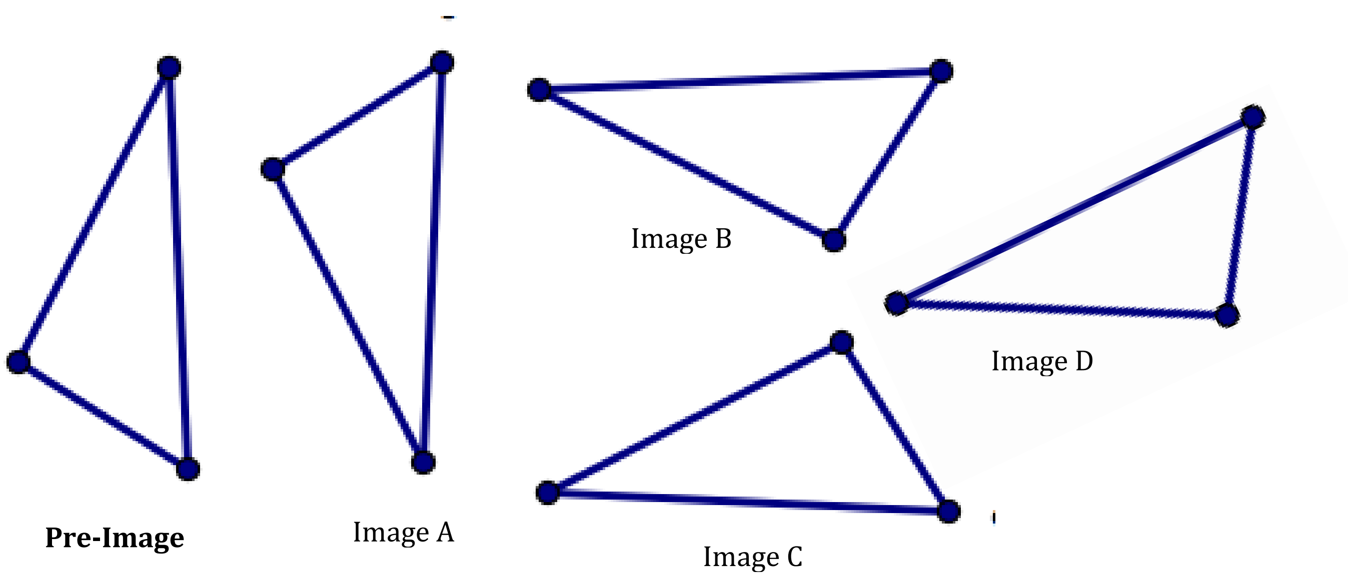


Image A Image B Image C Image D

Pre-Image

2.

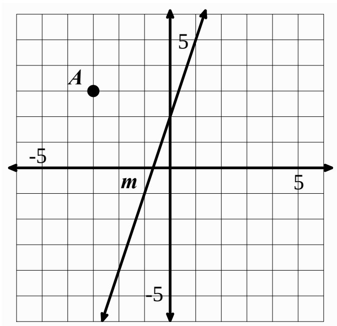


Set

Topic: Reflecting and Rotating points

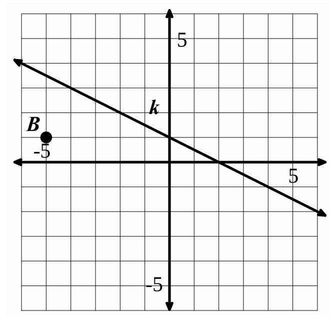
On each of the coordinate grids there is a labeled point and line. Use the line as a line of reflection to reflect the given point and create its reflected image over the line of reflection.

3.



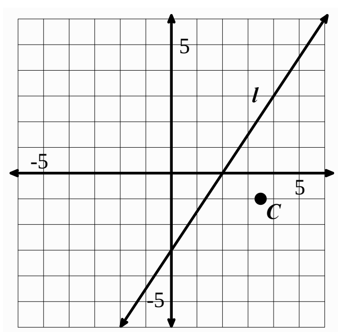
Reflect point ***A*** over line ***m*** and label the image ***A'***

4.



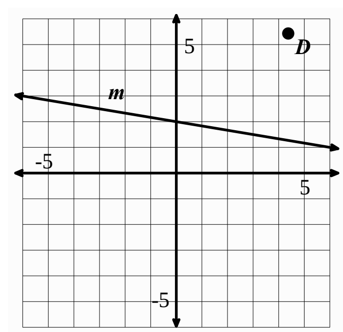
Reflect point ***B*** over line ***k*** and label the image ***B'***

5.



Reflect point ***C*** over line ***l*** and label the image ***C'***

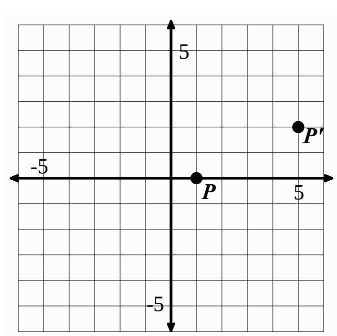
6.



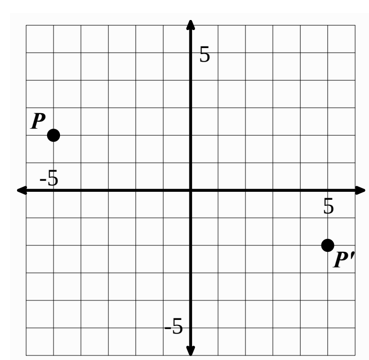
Reflect point ***D*** over line ***m*** and label the image ***D'***

For each pair of point, *P* and *P'* draw in the line of reflection that would need to be used to reflect *P* onto *P'*. Then find the equation of the line of reflection.

7.

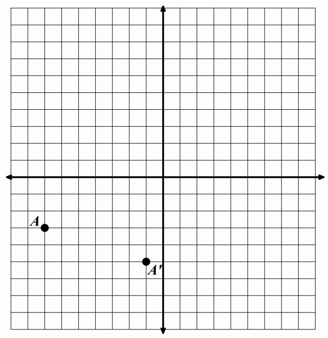


8.

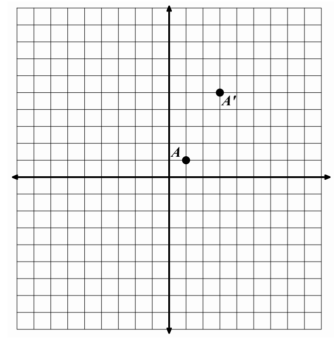


For each pair of point, *A* and *A'* draw in the line of reflection that would need to be used to reflect *A* onto *A'*. Then find the equation of the line of reflection. Also, draw a line connecting *A* to *A'* and find the equation of this line. Compare the slopes of the lines of reflection containing *A* and *A'*.

9.



10.



Go

Topic: Slopes of parallel and perpendicular lines and finding both distance and slope between two points.

For each linear equation write the slope of a line parallel to the given line.

11. $y = -3x + 5$

12. $y = 7x - 3$

13. $3x - 2y = 8$

For each linear equation write the slope of a line perpendicular to the given line.

14. $y = -\frac{2}{7}x + 5$

15. $y = \frac{1}{5}x - 4$

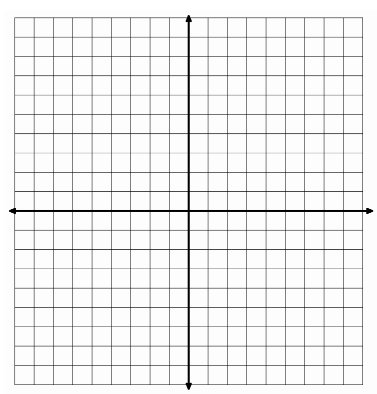
16. $3x + 5y = -15$

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. You may use the graph to help you as needed.

17. (-2, -3) (1, 1)
a. Slope: b. Distance:

18. (-7, 5) (-2, -7)
a. Slope: b. Distance:

19. (2, -4) (3, 0)
a. Slope: b. Distance:



6.4 Leap Year

A Practice Understanding Task

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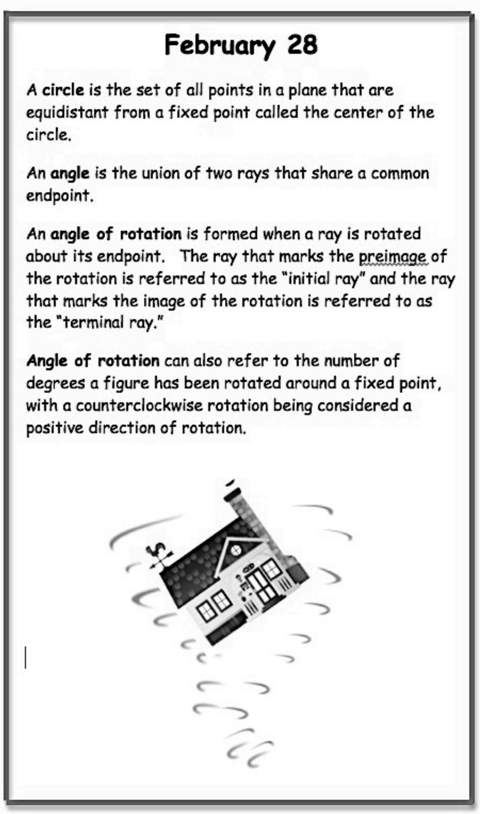
Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining since images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational since each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," she explains, "you need 366 pages."

So now Carlos and Clarita have the dilemma of having to create an extra page to insert between February 28 and March 1. Here are the planner pages they have already designed.



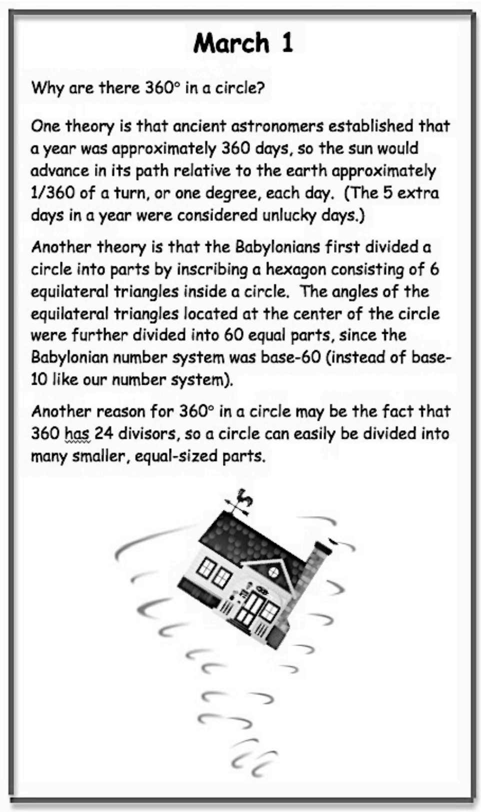
February 28

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

An **angle** is the union of two rays that share a common endpoint.

An **angle of rotation** is formed when a ray is rotated about its endpoint. The ray that marks the preimage of the rotation is referred to as the "initial ray" and the ray that marks the image of the rotation is referred to as the "terminal ray."

Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.



March 1

Why are there 360° in a circle?

One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance in its path relative to the earth approximately 1/360 of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.)

Another theory is that the Babylonians first divided a circle into parts by inscribing a hexagon consisting of 6 equilateral triangles inside a circle. The angles of the equilateral triangles located at the center of the circle were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10 like our number system).

Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts.

Part 1

Since the theme for the facts for February is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

1. A translation of a set of points in a plane . . .

2. A rotation of a set of points in a plane . . .

3. A reflection of a set of points in a plane . . .

4. Translations, rotations and reflections are rigid motion transformations because . . .

Carlos and Clarita used these words and phrases in their definitions: perpendicular bisector, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, preserves distance and angle measures.

Revise your definitions so they also use these words or phrases.

Part 2

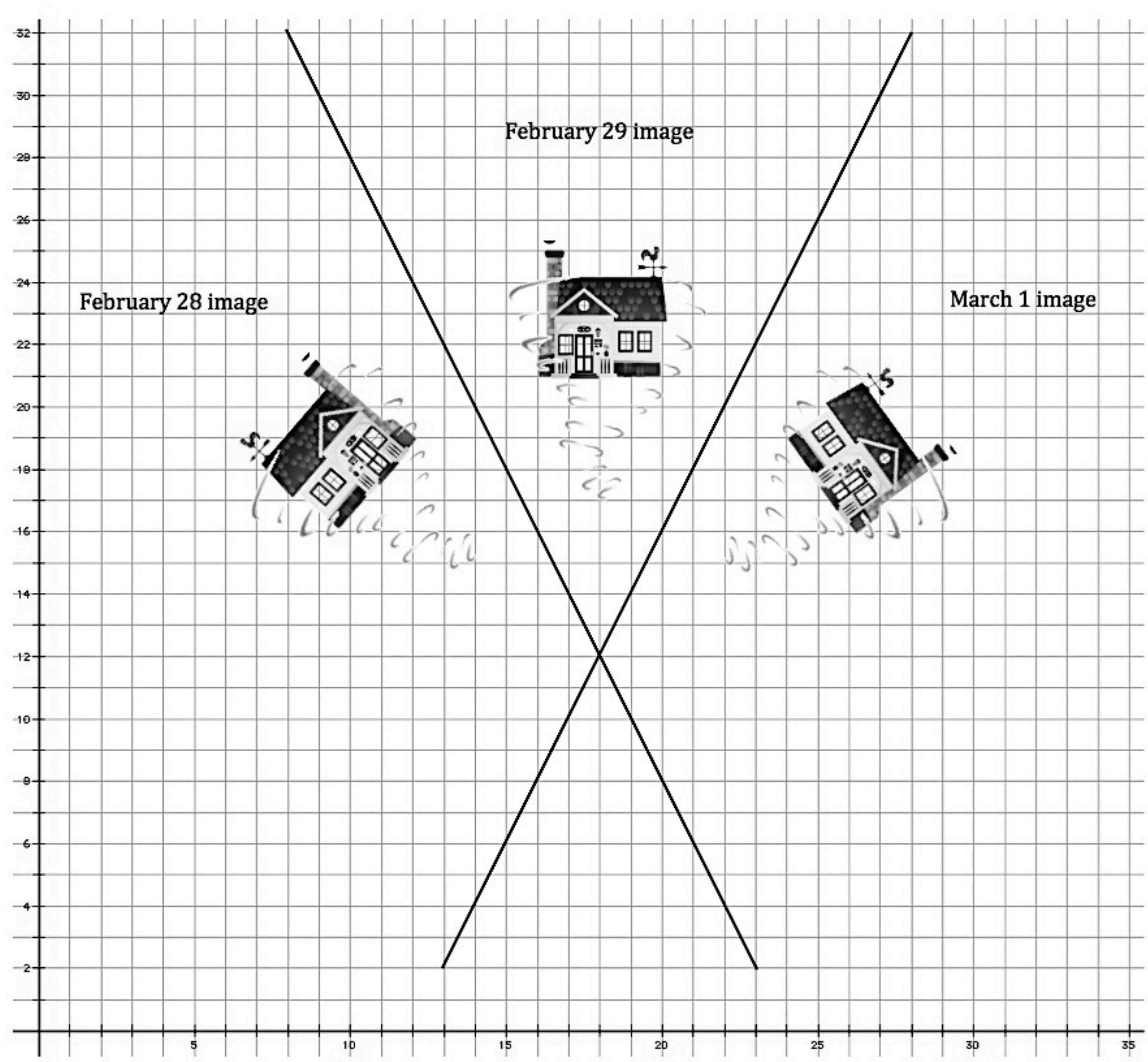
In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from the Wizard of Oz as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that he can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.

Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as he intended. For example,

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?



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www.clker.com/clipart.tornado-gray

6.4 Leap Year – Teacher Notes
A Practice Understanding Task

Purpose: In this task students will write precise definitions for the three rigid-motion transformations, based on the observations they have made in the previous tasks in this learning cycle. To prepare students for writing their own definitions, they will study the language used in the definitions given for *circle, angle,* and *angle of rotation.* They will also write a definition for the word *degree* based on the information given. In part 2 of this task students will use their definitions to justify that multiple images have been correctly drawn based on specified transformations. As part of this task students will also explore the idea that two consecutive reflections produce a rotation when the lines of reflection are not parallel.

Core Standards Focus:

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Related Standards: G.CO.5, G.CO.6

Launch (Whole Class):
Read and clarify the definition of *circle* given on the February 28 calendar page. Ask how the words used identify only the points on a circle and omit every other point in the plane from being identified as points belonging to the circle. Ask what would happen if the words "in a plane" were removed from the definition.

Ask students about the images formed in their minds from the two definitions for *angle* and *angle of rotation*. Make sure they can use either of these definitions to describe an angle. Then have students read the historical notes considering why there are 360° in a circle. (What does this really mean—where are each of the 360 degrees located "in the circle"?) Based on this discussion, ask students to write a definition for the word degree. Press for something like, "A degree is the measure of an angle of rotation that is equal to 1/360 of a complete rotation around a fixed point."

After emphasizing the precision of language in a formal definition, have partners write definitions for each of the three rigid-motion transformations. Let students know that you will be formalizing these definitions as a class before working on part 2 of the task.

Explore (Small Group), part 1:
Give students time to consider all three definitions. Listen for language about distance and direction in each definition. Students who finish their three definitions can also work on writing a definition for *rigid-motion transformation.*

Discuss (Whole Class), part 1:
As a whole class, write definitions that students agree will define each transformation with precision. The essential elements of each definition are as follows:
Translation: *translations move points the same distance along lines that are parallel to each other*
Rotation: *rotations move points along concentric circles and through the same angle of rotation*
Reflection: *reflections move points across a specified line of reflection so that the line of reflection is the perpendicular bisector of each line segment connecting corresponding pre-image and image points*

Explore (Small Group), part 2:
In part 2, students should justify their answers to the three questions by using the definitions they wrote for the three rigid-motion transformations. Here are some additional prompting questions, if students are not attending to the definitions:

- What evidence can you provide that the first given line is a line of reflection? How can we convince ourselves that the line is the perpendicular bisector of the line segments connecting pre-image and image points of the first two drawings of Dorothy's house? (Hint: You may want to use the Pythagorean theorem and also think about slopes.)
- What evidence can you provide that the second line is the perpendicular bisector of the line segments connecting pre-image and image points of the last two drawings of Dorothy's house?
- Where is the center of this rotation located? What evidence can you provide that pre-image and image points are equidistant from the center of rotation? What evidence can you provide that there is a 90° rotation between the February 28 and March 1 images?

Discuss (Whole Class), part 2:
The focus of this discussion is on using the definitions to justify the transformations that Carlos claims to have used. Therefore, students will need to verify that corresponding vertex points on the image and pre-image satisfy the conditions defining the particular transformation. For example, is the line of reflection the perpendicular bisector of each line segment joining a vertex point on the image with its corresponding vertex point on the pre-image? How did students verify this? (Note: the question of determining if a point is a midpoint of a line segment may come up in this

discussion.   If so, allow students to discuss how they think they might find the coordinates of the midpoint of a segment when they know the coordinates of the endpoints.)

Point out to students that these two reflections produced a rotation. You might ask them to consider if this would always be the case, and what makes them think this might be so, or under what conditions it might not be so.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.4

Ready, Set, Go!

Ready

Topic: Defining geometric shapes and components

For each of the geometric words below write a definition of the object that addresses the essential elements. Also, list necessary attributes and characteristics.

1. Quadrilateral:

2. Parallelogram:

3. Rectangle:

4. Square:

5. Rhombus:

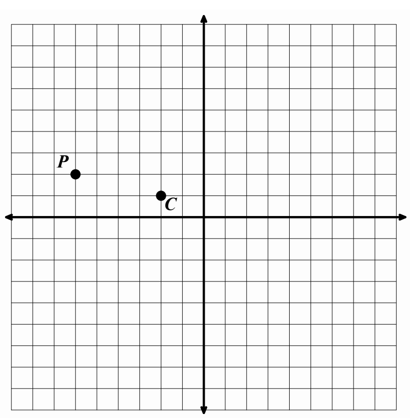
6. Trapezoid:

Set

Topic: Reflections and Rotations, composing reflections to create a rotation

Perform the indicated rotations.

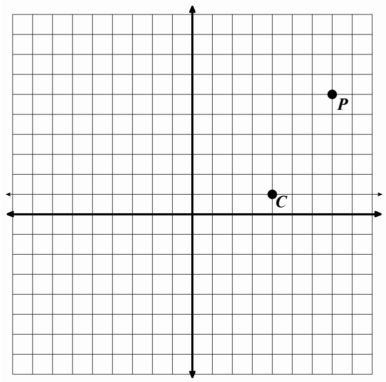
7.



Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90°. Label the image ***P***'.

With point ***C*** as a center of rotation also rotate point ***P*** 180°. Label this image ***P***''.

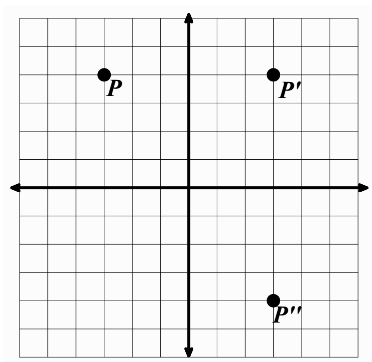
8.



Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P'***.

With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P''***.

9.

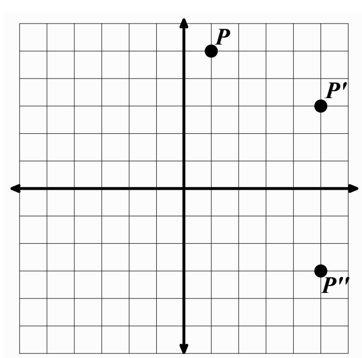


a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

10.

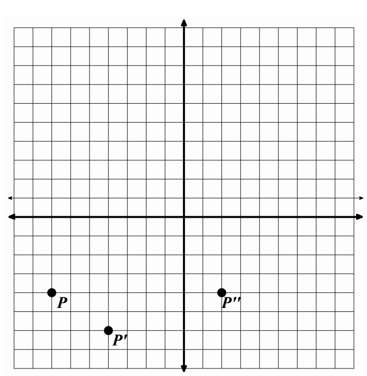


a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

11.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

Go

Topic: Rotations about the origin

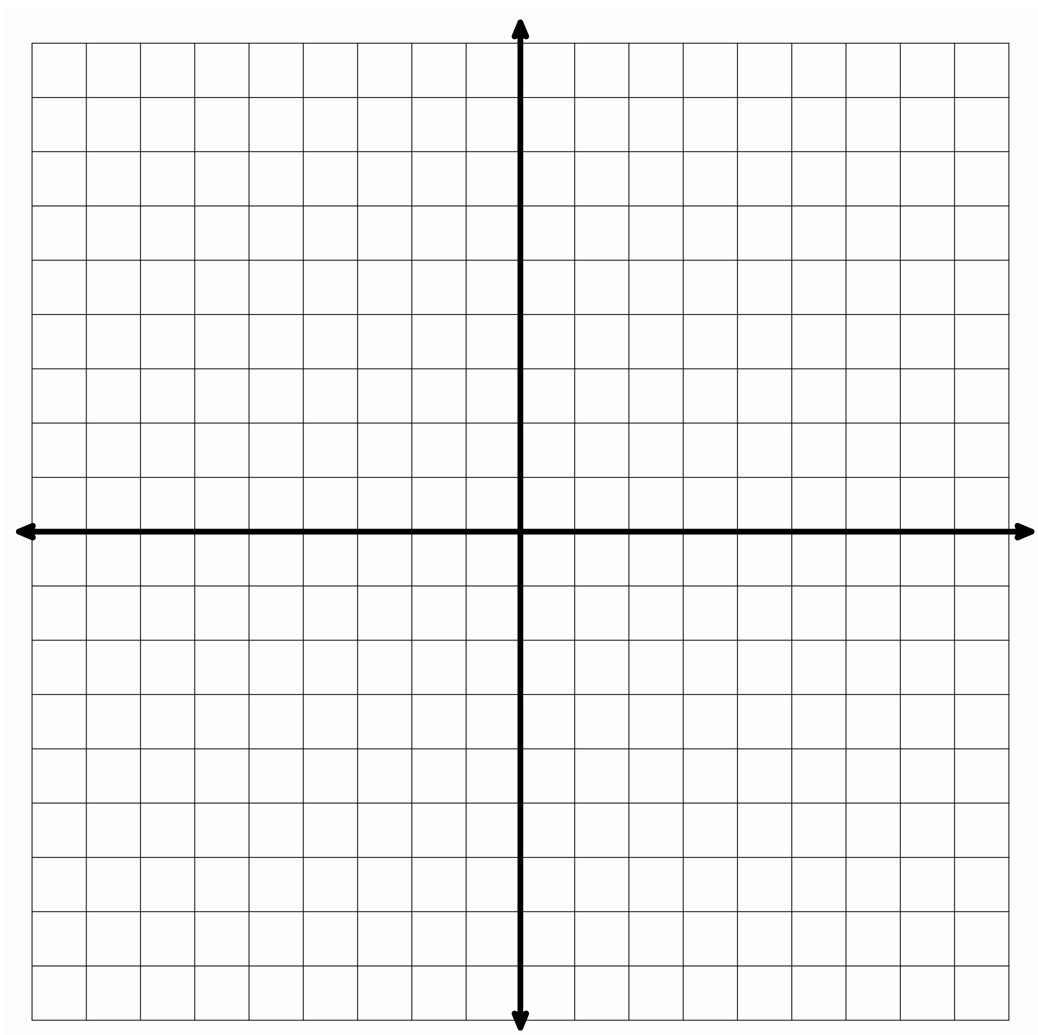
Plot the given coordinate and then perform the indicated rotation in a clockwise direction around the origin, the point (0 , 0), and plot the image created. State the coordinates of the image.

12. Point ***A*** (4, 2) rotate 180^0
    Coordinates for Point ***A***' (___ , ___)

13. Point ***B*** (-5, -3) rotate 90^0 clockwise
    Coordinates for Point ***B***' (___ , ___)

14. Point ***C*** (-7, 3) rotate 180^0
    Coordinates for Point ***C***' (___ , ___)

15. Point ***D*** (1, -6) rotate 90^0 clockwise
    Coordinates for Point ***D***' (___ , ___)



6.5 Symmetries of Quadrilaterals

A Develop Understanding Task

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A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

1. A **rectangle** is a quadrilateral that contains four right angles. Is it possible to reflect or rotate a rectangle onto itself?

For the rectangle shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rectangle onto itself.

Describe the rotations and/or reflections that carry a rectangle onto itself. (Be as specific as possible in your descriptions.)

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel. Is it possible to reflect or rotate a parallelogram onto itself?

For the parallelogram shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the parallelogram onto itself.

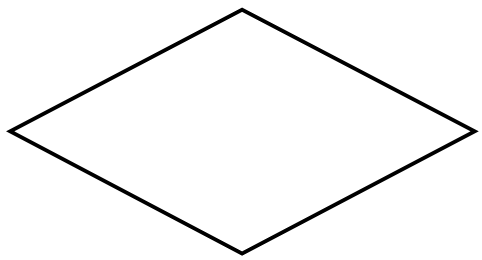
Describe the rotations and/or reflections that carry a parallelogram onto itself. (Be as specific as possible in your descriptions.)

3. A **rhombus** is a quadrilateral in which all sides are congruent. Is it possible to reflect or rotate a rhombus onto itself?

For the rhombus shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rhombus onto itself.



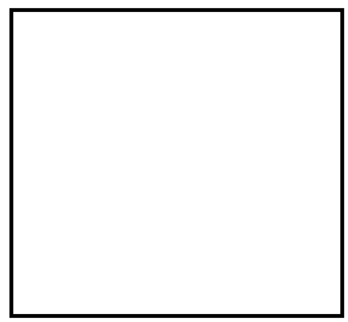
Describe the rotations and/or reflections that carry a rhombus onto itself. (Be as specific as possible in your descriptions.)

4. A **square** is both a rectangle and a rhombus. Is it possible to reflect or rotate a square onto itself?

For the square shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the square onto itself.



Describe the rotations and/or reflections that carry a square onto itself. (Be as specific as possible in your descriptions.)

5. A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find

- any lines of symmetry, or
- any centers of rotational symmetry

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

6.5 Symmetries of Quadrilaterals – Teacher Notes

A Develop Understanding Task

Purpose: In this learning cycle, students focus on classes of geometric figures that can be carried onto themselves by a transformation—figures that possess a line of symmetry or rotational symmetry. In this task the idea of "symmetry" is surfaced relative to finding lines that reflect a figure onto itself, or determining if a figure has rotational symmetry by finding a center of rotation about which a figure can be rotated onto itself. This work is intended to be experimental (e.g., folding paper, using transparencies, using technology, measuring with ruler and protractor, etc.), with the definitions of reflection and rotation being called upon to support students' claims that a figure possesses some type of symmetry. The particular classes of geometric figures considered in this task are various types of quadrilaterals.

Core Standards Focus:

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.4, G.CO.5

Launch (Whole Class):
Discuss the concept of symmetry in terms of finding a rigid-motion transformation that carries a geometric figure onto itself. Help students recognize that two such types of symmetries exist: a line of symmetry might exist that reflects a figure onto itself, or a center of rotation might exist about which a figure might be rotated onto itself. Also remind students of the definitions of "quadrilateral" and "diagonal" giving in the introduction of the task.

Students are to experiment with the various types of quadrilaterals listed in the task to determine if they can find any lines of symmetry or centers and angles of rotation that will carry the given quadrilateral onto itself. You will need to decide what tools to make available for this investigation. For example, you could provide cut-outs of each of the figures which would allow students to find lines of symmetry by folding the figures onto themselves (note that a handout of the set of figures is provided at the end of the teacher notes). This would also be a good task to support using dynamic geometry software programs, such as Geometer's Sketchpad or Geogebra. If you use technology, students will need to be provided with a set of well-constructed quadrilaterals, so they can focus on searching for lines of symmetry and centers of rotation, rather than on the construction of the geometric figures themselves.

Explore (Small Group):
Since students are dealing with classes of quadrilaterals, rather than individual quadrilaterals, in addition to finding the line of symmetry or the center and angle of a rotation, they should also provide some type of justification as to how they know that this symmetry exists for all members of the class. The given definitions for each quadrilateral should support making such an argument.

For example, if students say that the diagonal of a square is a line of symmetry, they might note that distance and angle are preserved by this reflection since adjacent sides of a square are congruent and opposite angles of a square are both right angles. Try to press students to move away from basing their decisions about lines of symmetry or centers of rotation simply on intuition and "it looks like it works" type of justification, and towards arguments based on the definitions of the rigid-motion transformations and the defining properties of the geometric figures.

Look for students who find all of the lines of symmetry or describe all of the possible rotations that might exist for each type of quadrilateral. For example, a square has two different types of lines of symmetry: the diagonals, and the lines passing through the midpoints of opposite sides. Hence, there are four lines of symmetry in a square. The point where the two diagonals of a square intersect locates the center of rotation for describing the rotational symmetry of a square. A square can be rotated 90°, 180°, 270° or 360° about this center of rotation. Each reflection or rotation carries a segment onto another segment of the same length, or a right angle onto another right angle, due to the defining properties of a square.

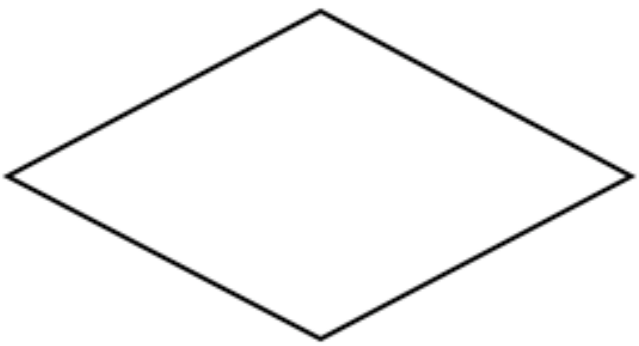
Watch for misconceptions that might arise, such as the diagonals of a parallelogram being identified as lines of reflection. Experimentation with technology or paper folding will disprove this conjecture, but it is important to have students describe why they initially thought it was true, and how they might convince themselves that this conjecture isn't true based on the definition of a reflection.

Discuss (Whole Class):
Start the discussion by asking if the diagonals of a parallelogram are lines of symmetry for the parallelogram. Ask students how they know a diagonal is not a line of symmetry. If their only arguments are experimental in nature (e.g., "if you fold it on the diagonal opposite vertices don't match up" or "when I used the diagonal as a mirror line in GSP it didn't work"), press for an explanation based on the essential ideas of a reflection (e.g., "since adjacent sides of a parallelogram aren't necessarily congruent, we can't find a line of reflection that will reflect a side of a parallelogram onto an adjacent side so that distance is preserved").

Ask students how they determined where the center of rotation is located in various classes of quadrilaterals. This should lead to a discussion about the point of intersection of the diagonals.

Ask students which type of quadrilateral has the most types of symmetry, and why this might be so.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.5



Ready, Set, Go!

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Ready

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

2. Fill in the names of each polygon based on the number of sides the polygon has.

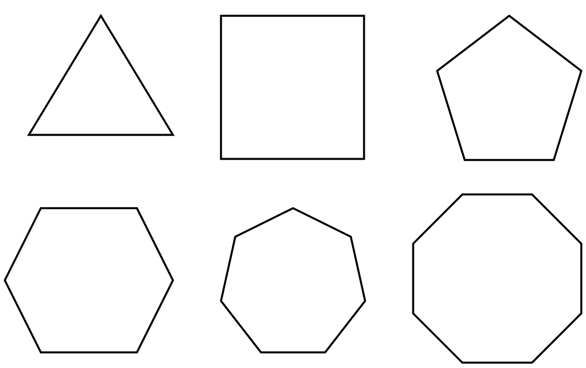
Number of Sides	Name of Polygon
3	
4	
5	
6	
7	
8	
9	
10	

Set

Topic: Lines of symmetry and diagonals

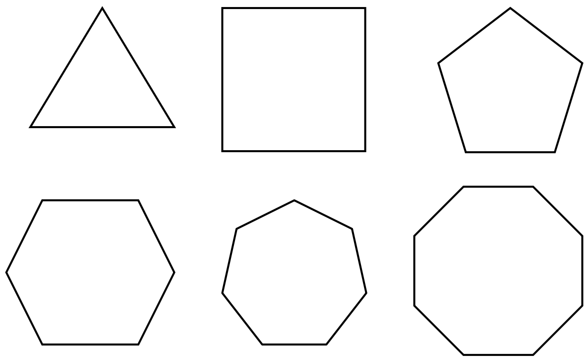
3. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a ***n***-sided polygon.

Number of Sides	Number of lines of symmetry
3	
4	
5	
6	
7	
8	
n	



4. Find

all of the diagonals in each regular polygon. Fill in the table including an expression for the number of diagonals in a ***n***-sided polygon.



Number of Sides	Number of diagonals
3	
4	
5	
6	
7	
8	
n	

5. Are all lines of symmetry also diagonals? Explain.

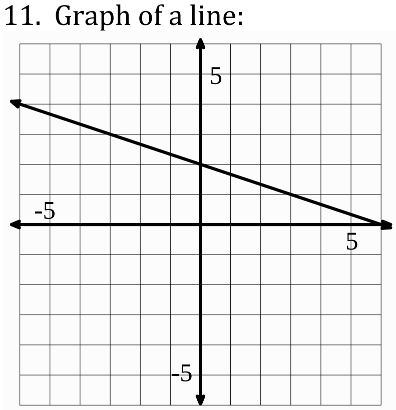
6. Are all diagonals also lines of symmetry? Explain.

7. What shapes will have diagonals that are not lines of symmetry? Name some and draw them.

8. Will all parallelograms have diagonals that are lines of symmetry? If so, draw and explain. If not draw and explain.

Go

Topic: Equations for parallel and perpendicular lines.

<table>
<tr><th></th><th>Find the equation of a line PARALLEL to the given info and through the indicated point.</th><th>Find the equation of a line PERPENDICULAR to the given line and through the indicated point.</th></tr>
<tr><td>9. Equation of a line:
$y = 4x + 1$.</td><td>a. Parallel line through point (-1 , -7):</td><td>b. Perpendicular to the line line through point (-1 , -7):</td></tr>
<tr><td>10. Table of a line:
<table><tr><th>x</th><th>y</th></tr><tr><td>3</td><td>-8</td></tr><tr><td>4</td><td>-10</td></tr><tr><td>5</td><td>-12</td></tr><tr><td>6</td><td>-14</td></tr></table></td><td>a. Parallel line through point (3 , 8):</td><td>b. Perpendicular to the line through point (3 , 8):</td></tr>
<tr><td>11. Graph of a line:

</td><td>a. Parallel line through point (2 , -9):</td><td>b. Perpendicular to the line through point (2 , -9):</td></tr>
</table>

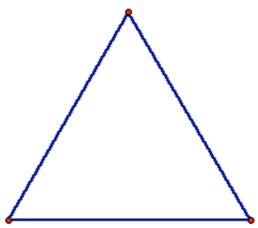
6.6 Symmetries of Regular Polygons

A Solidify Understanding Task

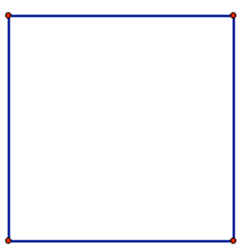
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

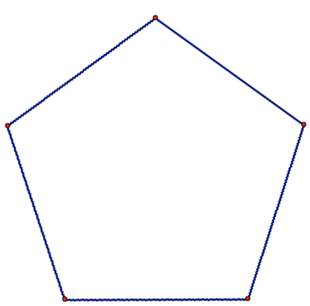
1. An equilateral triangle



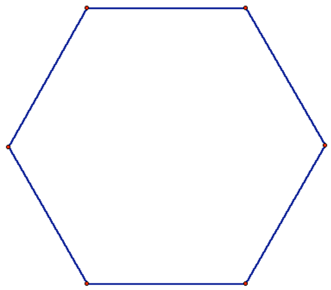
2. A square



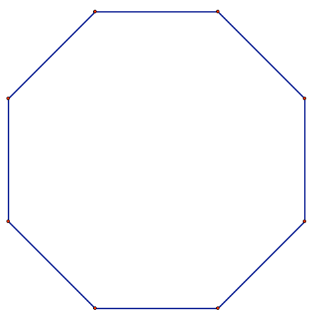
3. A regular pentagon



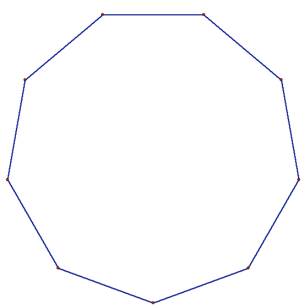
4. A regular hexagon



5. A regular octagon



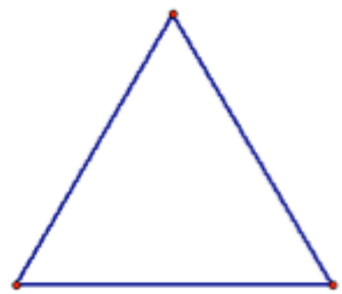
6. A regular nonagon



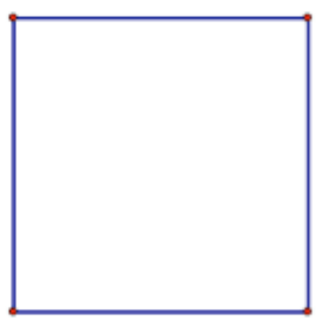
What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

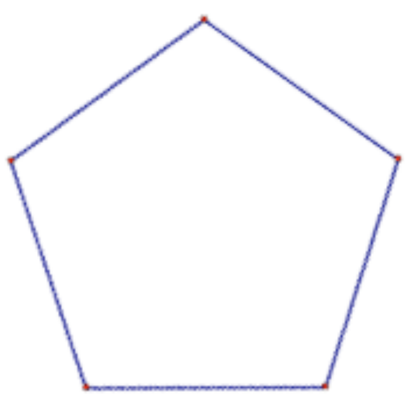
1. An equilateral triangle



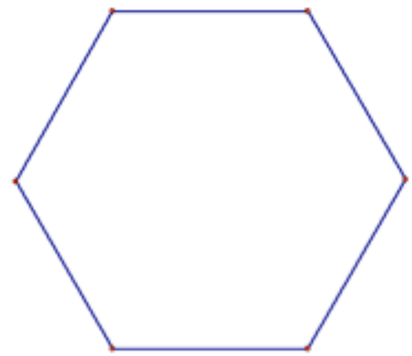
2. A square



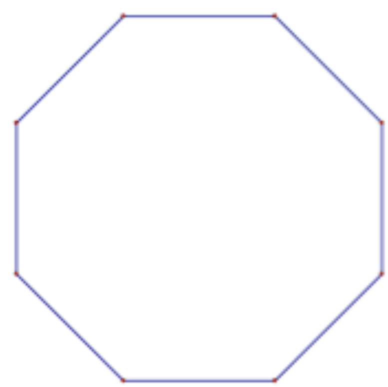
3. A regular pentagon



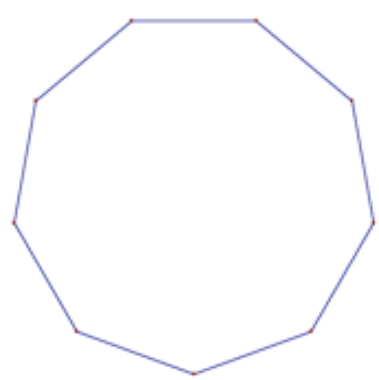
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



6.6 Symmetries of Regular Polygons – Teacher Notes

A Solidify Understanding Task

Purpose: In this task, students continue to focus on classes of geometric figures that can be carried onto themselves by a transformation—figures that possess a line of symmetry or rotational symmetry. Students solidify the idea of "symmetry" relative to finding lines that reflect a figure onto itself, or determining if a figure has rotational symmetry by finding a center of rotation about which a figure can be rotated onto itself. They also look for and describe the structure that determines if a figure possesses some type of symmetry. This work can be experimental (e.g., folding paper, using transparencies, using technology, measuring with ruler and protractor, etc.), or theoretical, with the definitions of reflection and rotation being called upon to support students' claims that a figure possesses some type of symmetry. The particular classes of geometric figures considered in this task are various types of regular polygons, and students will look for patterns in the types of lines of symmetry a regular polygon with an odd number of sides possesses, versus those with an even number of sides. They should also note a pattern between the smallest angle of rotation that carries a regular polygon onto itself and the number of sides of the polygon.

Core Standards Focus:

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.4, G.CO.5

Launch (Whole Class):

Based on the level of student thinking that exists in your class, decide if you want the work of this task to be experimental (e.g., using tools, cut-outs and/or dynamic geometric software), or theoretical, making decisions based on the definitions of reflections and rotations. Introduce the task as being similar in nature to the previous task. If you have decided to provide experimental tools, outline what is available for students (note that a handout of the figures is provided at the end of the teacher notes, if needed.) You might also choose to just hand out the task and let students decide if they want to draw upon reasoning or tools to support their work. Or, you might want to press students to analyze the figures using reasoning based on the definitions of reflection and rotation.

Before having students start on the task, read the last two questions together, and point out that this is the goal of the task: to look for patterns in the number and characteristics of the lines of symmetry in a regular *n*-gon, and to look for patterns that describe the nature of the rotational symmetry in a regular *n*-gon. Encourage students to keep these goals in mind as they explore.

Explore (Small Group):
Listen for how students are determining the types of symmetry that exists for each regular polygon, and make sure they are identifying both types of symmetry—lines of symmetry and rotational symmetry. For rotational symmetry, make sure they are identifying all possible angles of rotation. As you observe students work, you may want to suggest additional tools for exploration, or set tools aside if you feel students are capable of noting the symmetries by reasoning with the definitions of rotation and reflection and the properties of regular polygons.

It is important for students to examine the last two questions in small groups, before moving to a whole class discussion. Keep reminding students of these goals, even before they have examined all of the listed regular polygons. That way, students will be able to attend to the presentations during the discussion, even if they haven't examined the complete set of polygons.

Discuss (Whole Class):
The discussion should focus on the last two questions, and students might draw upon specific examples of regular polygons to support their conjectures.

Ask students to state a conjecture as to the number of lines of symmetry in a regular n-gon, and to provide some justification of their conjecture. If students have not noticed any patterns in the number of lines of symmetry, make a table on the board consisting of "number of sides" as the input and "number of lines of symmetry" as the output. Ask what they may have noticed about the types of lines of symmetry in a regular polygon with an even number of sides versus an odd number of sides. What accounts for the differences in the ways they located the lines of symmetry?

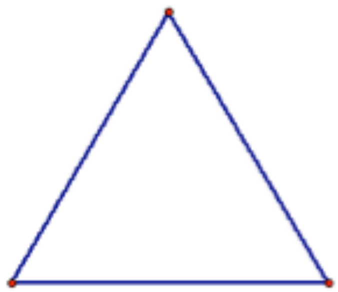
Students should notice that in a regular polygon with an odd number of sides they can only draw lines of symmetry (or locate a crease line that folds the polygon onto itself) by using the lines that pass through a vertex point and the midpoint of the opposite side. Since such a line can be drawn through each vertex, a regular n-gon will possess n lines of symmetry.

In a regular polygon with an even number of sides you can draw (or fold) a line of reflection through opposite vertices. Since only one line of symmetry exists for each pair of opposite vertices, there are $n/2$ such lines of symmetry. You can also draw lines of symmetry through the midpoints of opposite sides of the polygon—the sides that are parallel to each other. Since only one line of symmetry exists for each pair of opposite sides, there are also $n/2$ such lines of symmetry. Consequently, a regular polygon with an even number of sides also has $n/2 + n/2 = n$ lines of symmetry, but for different reasons than in the case of regular polygons with an odd number of sides. Make sure that the arguments for n lines of symmetry in any regular polygon are based on the structure of the geometric figures themselves, and not just on the pattern observed in the table.

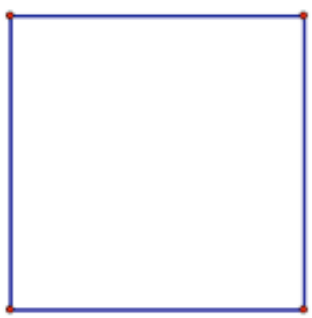
Turn the focus of the discussion to the second question: What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon? Again it may be helpful to create a table with the input representing “number of sides” and output representing “the smallest angle of rotation”. Point out that every regular polygon can be rotated onto itself by rotating 360° about the point of intersection of the diagonals of the polygon. How might the smallest angle of rotation be related to this 360° rotation? You might draw the line segments between a pair of consecutive vertices and the center of rotation and ask, “What is the measure of this angle, and how do you know?” Students should notice that the smallest angle of rotation in a regular n-gon is $360°/n$ and they should be able to justify why this is so. They should also note that any whole-number multiple of this smallest angle of rotation is also an angle of rotation for the polygon.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.6

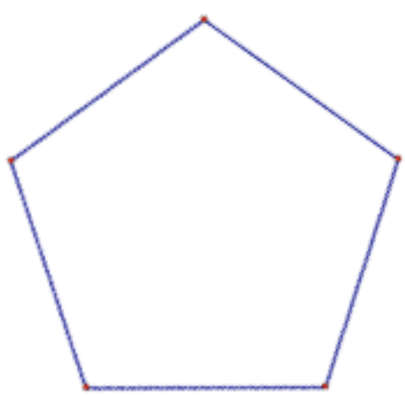
1. An equilateral triangle



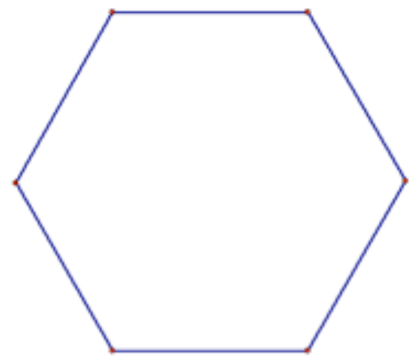
2. A square



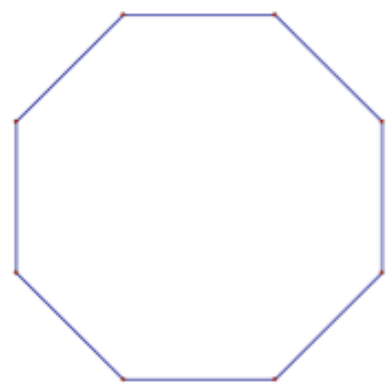
3. A regular pentagon



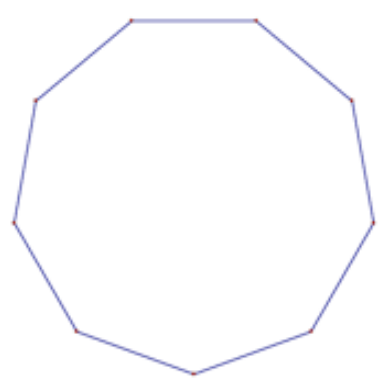
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



Ready, Set, Go!

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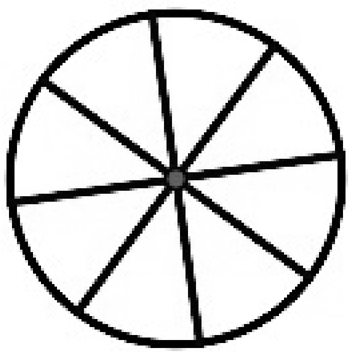
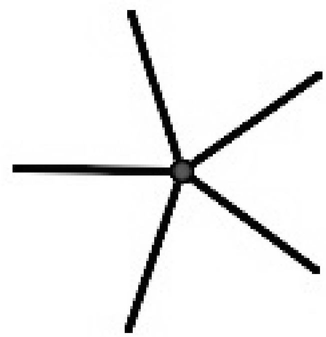
Ready

Topic: Rotation as a transformation, what does it mean?

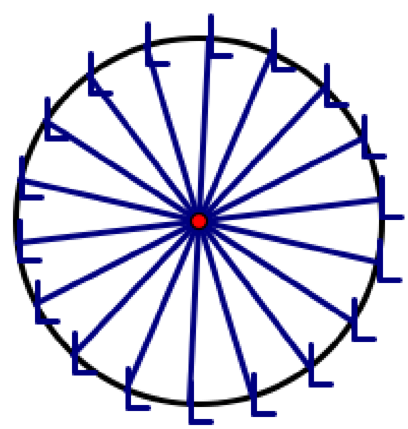
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



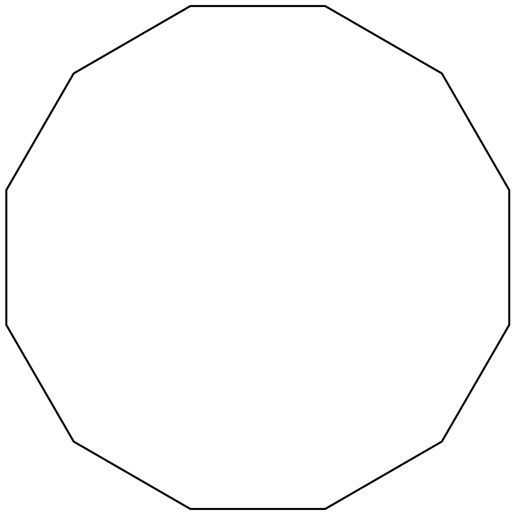
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



Set

Topic: Finding angels of rotation for regular polygons.

4. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



5. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

6. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

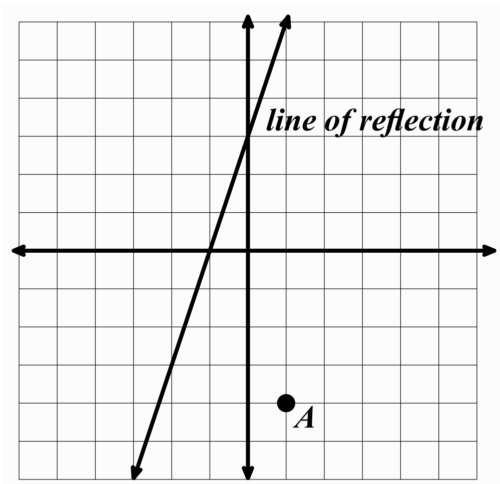
7. How many sides does a regular polygon have that has an angle of rotation equal to 18^0? Explain.

8. How many sides does a regular polygon have that has an angle of rotation equal to 20^0? How many lines of symmetry will it have?

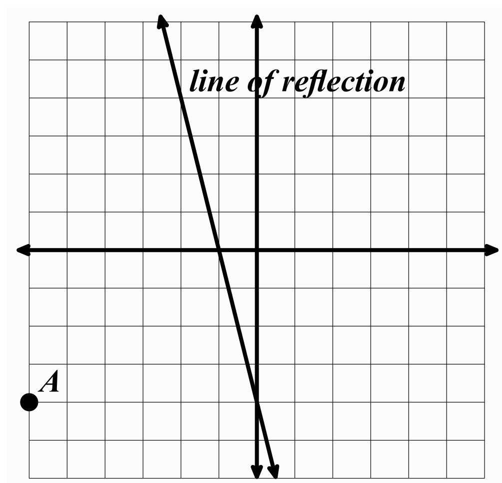
Go

Topic: Reflecting and Rotating points on the coordinate plane.

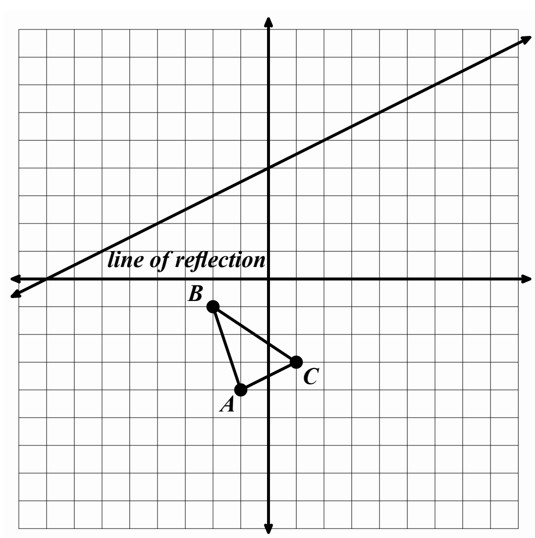
9. Reflect point ***A*** over the line of reflection and label the image ***A'***.



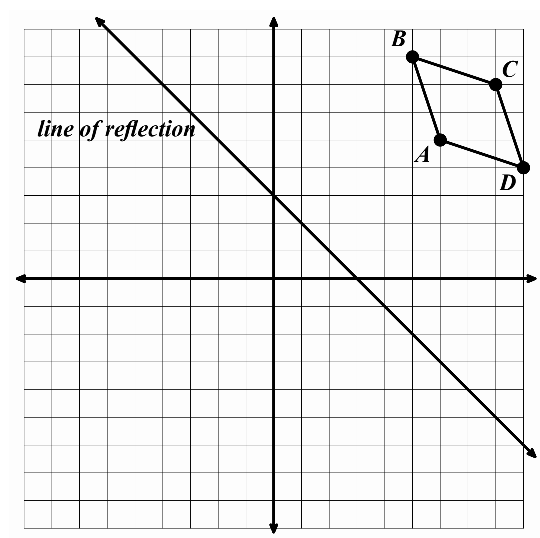
10. Reflect point ***A*** over the line of reflection and label the image ***A'***.



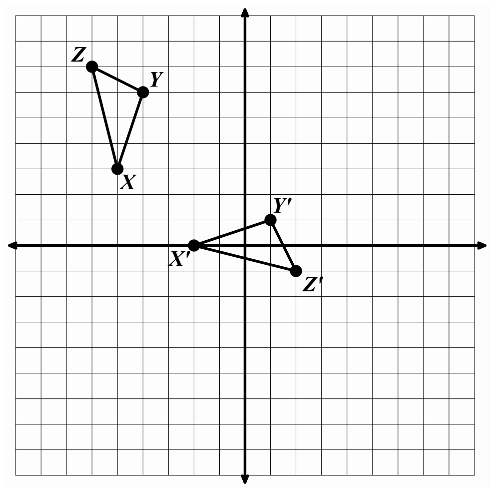
11. Reflect triangle ***ABC*** over the line of reflection and label the image ***A'B'C'***.



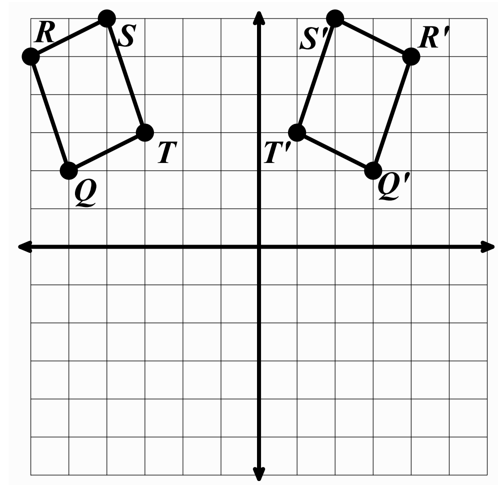
12. Reflect parallelogram ***ABCD*** over the line of reflection and label the image ***A'B'C'D'.***



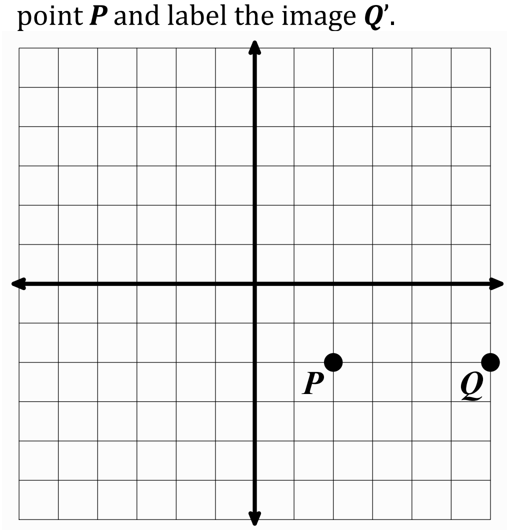
13. Given triangle ***XYZ*** and its image ***X'Y'Z'*** draw the line of reflection that was used.



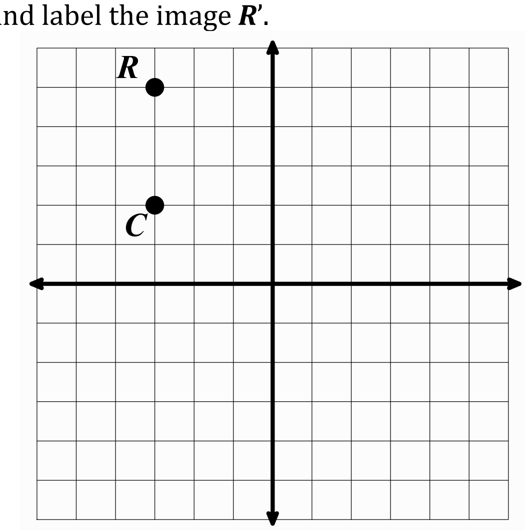
14 Given parallelogram ***QRST*** and its image ***Q'R'S'T'*** draw the line of reflection that was used.



15. Using point ***P*** as a center of rotation. Rotate point ***Q*** 120^0 clockwise about point ***P*** and label the image ***Q'***.



16. Using point ***C*** as the center or rotation. Rotate point ***R*** 270^0 counter-clockwise about point ***C*** and label the image ***R'***.



6.7 Quadrilaterals—Beyond Definition
A Practice Understanding Task

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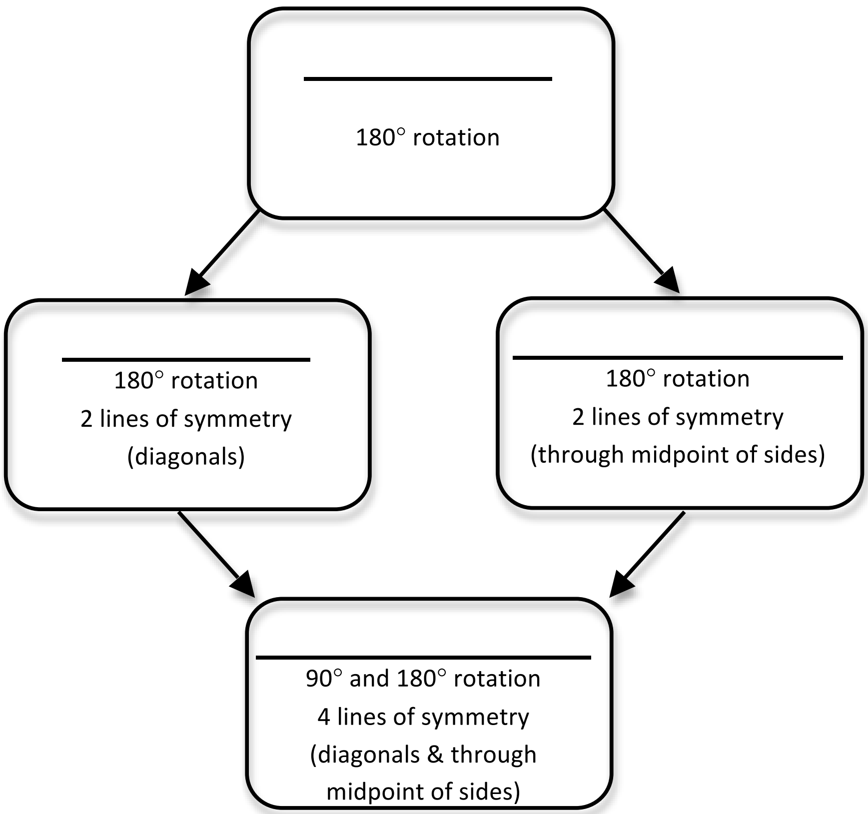
We have found that many different quadrilaterals possess line and/or rotational symmetry.

In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.

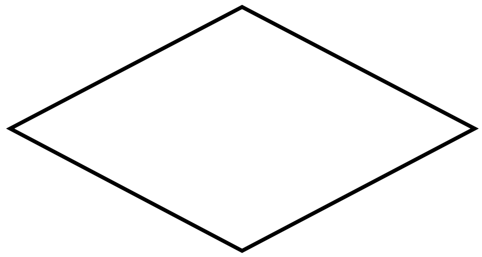
1. A **rectangle** is a quadrilateral that contains four right angles.

Based on what you know about transformations, what else can we say about rectangles besides the defining property that all four angles are right angles? Make a list of additional properties of rectangles that seem to be true based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals.

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

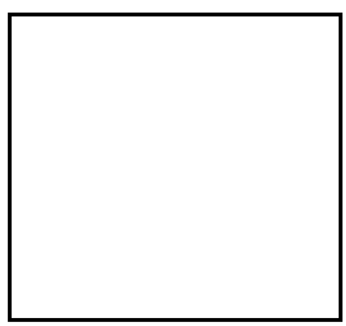
Based on what you know about transformations, what else can we say about parallelograms besides the defining property that opposite sides of a parallelogram are parallel? Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles and the diagonals.

3. A **rhombus** is a quadrilateral in which all four sides are congruent.



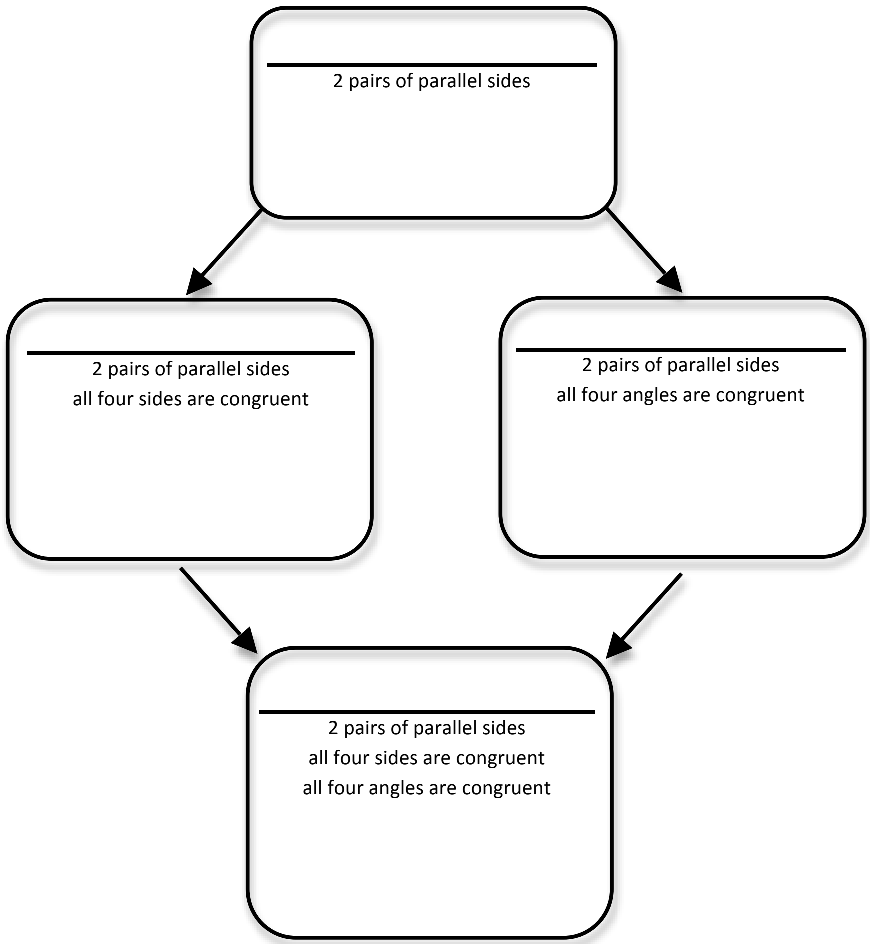
Based on what you know about transformations, what else can we say about a rhombus besides the defining property that all sides are congruent? Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles and the diagonals.

4. A **square** is both a rectangle and a rhombus.



Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles and the diagonals.

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and highlighted in the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

6.7 Quadrilaterals—Beyond Definition – Teacher Notes

A Practice Understanding Task

Purpose: This task allows students to extend their work with symmetries of quadrilaterals and practice making conjectures about geometric figures that are based on reasoning with the definitions of reflection and rotation. The work of this task will be revisited in Mathematics II, where students will be asked to create formal proofs for the conjectures they are making in this task about the properties of different types of quadrilaterals. Therefore, while this is classified as a practice understanding task, the mathematics students should be practicing is making and justifying conjectures about geometric figures based on the definitions of rigid-motion transformations, rather than practicing knowledge about the specific properties of different types of quadrilaterals. Whatever properties about sides, angles and diagonals of quadrilaterals students surface is sufficient for this task.

Core Standards Focus:

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.11

Launch (Whole Class):
Give students a few minutes to examine the chart on the first page of the task. They should summarize their work with symmetries of quadrilaterals by identifying the types of quadrilaterals that possess the symmetries being described in each portion of the chart. Discuss the question: What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart? Help students notice that special quadrilaterals inherit the symmetries of all categories of quadrilaterals to which they belong.

Remind students that in the previous task they were able to make some conjectures about properties of regular polygons based on features that revealed themselves when they looked at the symmetries of the polygons. In this task they will return to quadrilaterals and see what conjectures they might make about relationships between sides, angles and diagonals of different types of quadrilaterals based on the symmetries of the quadrilateral. Have students practice making

conjectures by working on problem 1: what else might be true about a rectangle—in addition to being a quadrilateral with four right angles—and how can they justify these observations based on the definitions of reflection and rotation.
After a few minutes summarize what groups have observed and share the arguments they might make to support their claims. Don't bring up properties that students have not observed on their own, but you might prompt further discussion by asking if students noticed anything about the diagonals of a rectangle that could be justified based on lines of symmetry or rotational symmetry. If they haven't already noticed anything about the diagonals suggest that they could add thinking about the diagonals to their list of things to pay attention to during their exploration.

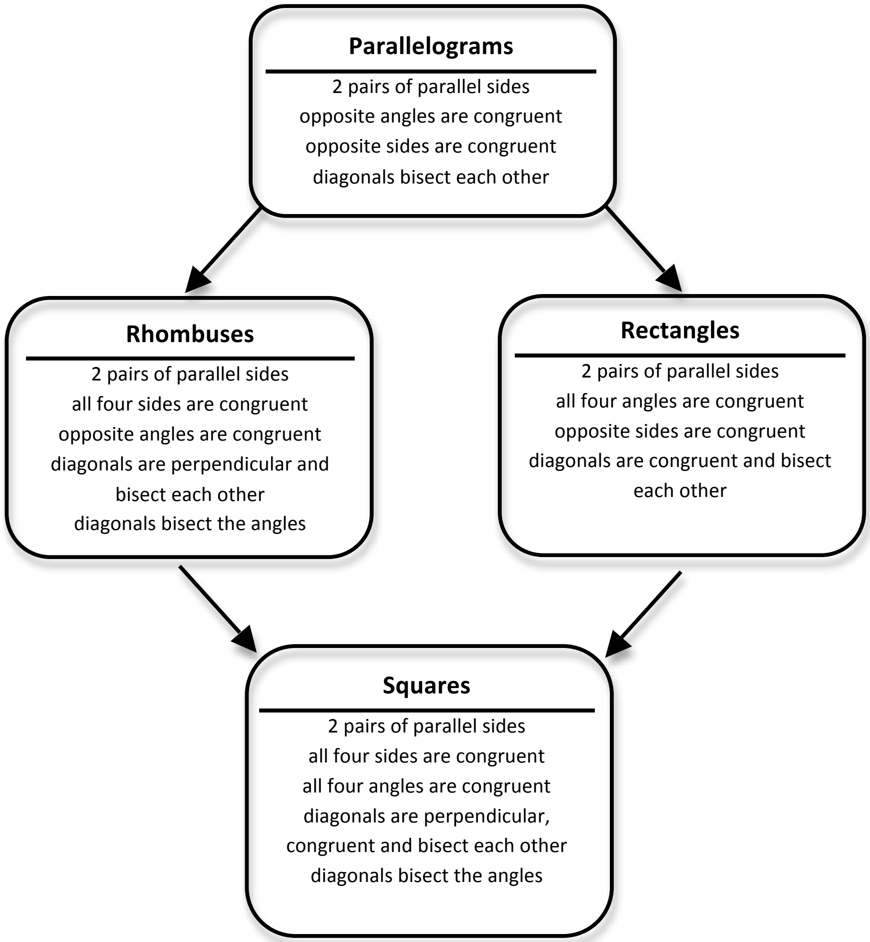
Now that students have a sense of the work that is expected of them on this task, assign them to work on making conjectures about the remaining quadrilaterals.

Explore (Small Group):
Listen for the types of conjectures students are making about each quadrilateral. Press them to look for more conjectures by asking questions like, "Is there anything you can say about opposite sides? Opposite angles? Adjacent sides? Adjacent angles? The diagonals and the way they interact with the sides and angles and with each other?"

Whenever students state a conjecture ask them why they think that conjecture is true—is it based on intuitive guessing, experimentation with tools, or on reasoning with the definitions of reflection and rotation? Press for justifications that are based on reasoning with transformations.

Discuss (Whole Class):
Since the purpose of this task is to practice making conjectures based on symmetry, there are no specific conjectures that need to be highlighted. Select students to share conjectures for which they have some justification based on transformations. As conjectures are shared and discussed, have students list those conjectures in the appropriate places on the chart at the end of the task. (They will first need to label the portions of the chart based on the defining properties of the different types of quadrilaterals. As the chart evolves, you will want to relate the chart at the beginning of this task to the chart at the end as a way of acknowledging the role that symmetry plays in the inherited properties that quadrilaterals possess based on the different categories of quadrilaterals to which they belong.)

Possible lists of properties of quadrilaterals that may surface are summarized in the following chart. However, not all of these properties need to be discussed.



Aligned Ready, Set, Go: Congruence, Construction and Proof 6.7

Ready, Set, Go!

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Ready

Topic: Defining Congruence and Similarity.

1. What do you know about two figures if they are congruent?

2. What do you need to know about two figures to be convinced that the two figures are congruent?

3. What do you know about two figures if they are similar?

4. What do you need to know about two figures to be convinced that the two figures are similar?

Set

Topic: Classifying quadrilaterals based on their properties.

Using the information given determine the most accurate classification of the quadrilateral.

5. Has 180^0 rotational symmetry.

6. Has 90^0 rotational symmetry.

7. Has two lines of symmetry that are diagonals.

8. Has two lines of symmetry that are not diagonals.

9. Has congruent diagonals.

10. Has diagonals that bisect each other.

11. Has diagonals that are perpendicular.

12. Has congruent angles.

Go

Topic: Slope and distance

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points.

13. (-3 , -2) , (0 , 0)

a. Slope: b. Distance:

14. (7 , -1) , (11 , 7)

a. Slope: b. Distance:

15. (-10 , 13) , (-5 , 1)

a. Slope: b. Distance:

16. (-6 , -3) , (3 , 1)

a. Slope: b. Distance:

17. (5 , 22) , (17 , 28)

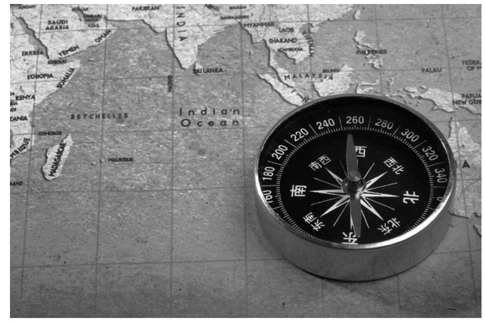
a. Slope: b. Distance:

18. (1 , -7) , (6 , 5)

a. Slope: b. Distance:

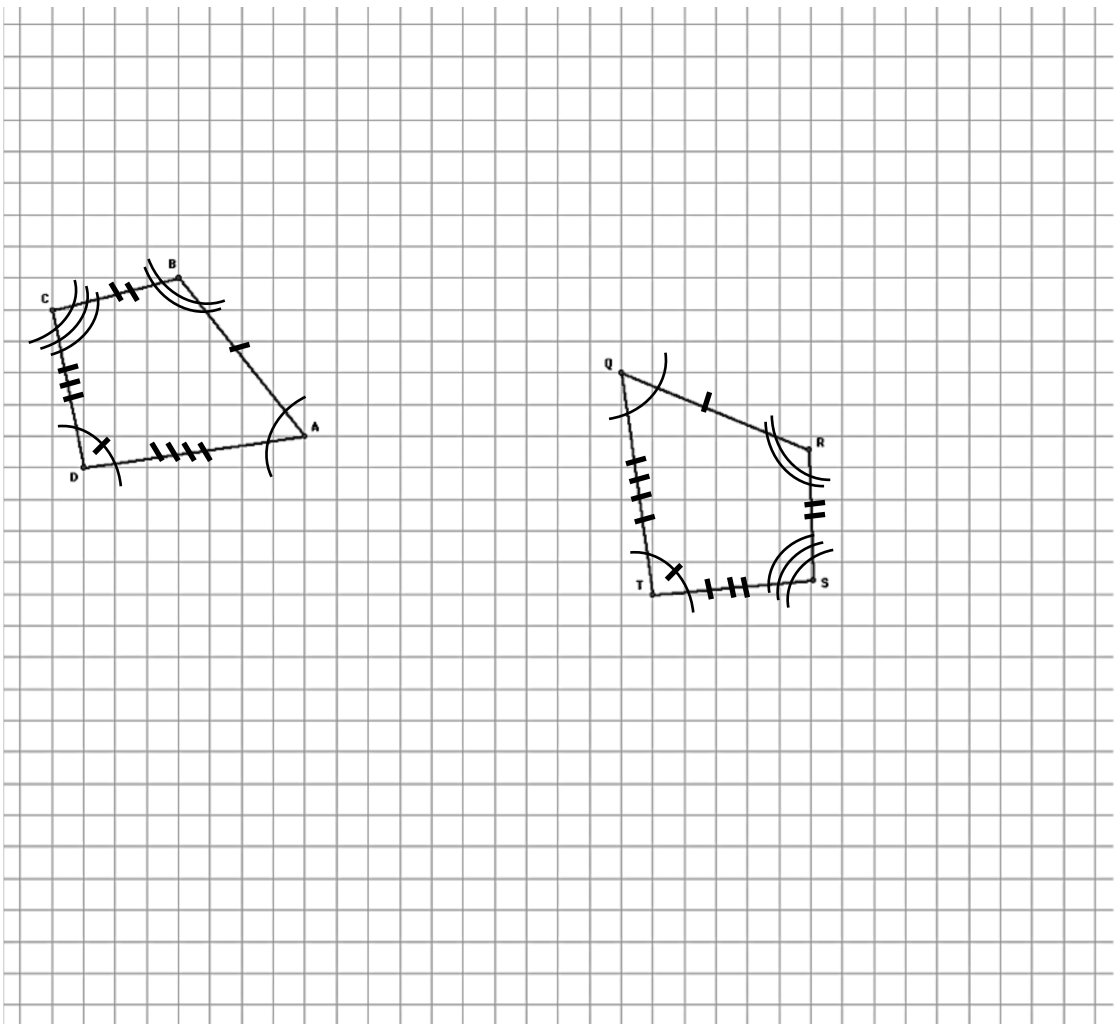
6. 8 Can You Get There From Here?

A Develop Understanding Task



The two quadrilaterals shown below, quadrilateral *ABCD* and quadrilateral *QRST* are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



Can You Get There From Here? – Teacher Notes

A Develop Understanding Task

Purpose: In Math 8 students came to understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In this task students are given two congruent figures and asked to describe a sequence of transformations that exhibits the congruence between them. While exploring potential sequences of transformations, students will notice how corresponding parts of congruent figures have to be carried onto one another, and they may look for ways that this can be accomplished in the fewest possible steps.

Core Standards Focus:

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Related Standards: G.CO.6

Launch (Whole Class):

Give students the handout and discuss the markings used to indicate the corresponding parts of the quadrilaterals that are congruent. For example, ask what angle in quadrilateral *QRST* is congruent to angle *B* in quadrilateral *ABCD* or what segment in quadrilateral *QRST* is congruent to $\overline{BC}$. Point out the convention for naming congruent figures so that corresponding angles and sides can be determined from the names of the figures without having to have a marked diagram. You may want to spend some time practicing this convention by naming two triangles and asking students to mark congruent angles and sides based on the naming convention.

Provide transparencies or tracing paper for students to experiment with a sequence of transformations that will carry parts of one quadrilateral onto the corresponding congruent parts of the other.

Explore (Small Group):

Allow students time to experiment with potential sequences of transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Watch for the strategies they use to get congruent parts to match. As students work, encourage them to look for ways that this can be accomplished in the fewest possible steps. Challenge them to develop a consistent and efficient strategy for determining a sequence of transformations that will always work to carry a given figure onto another congruent figure.

Students who finish faster than other students should be challenged to make their list of transformations as short as possible, helping them to recognize that a translation, a rotation and a reflection are all necessary to complete the sequence. Once they are convinced they have the shortest list of transformations possible to carry quadrilateral *ABCD* onto quadrilateral *QRST*, challenge them to find a sequence of transformations that will carry quadrilateral *QRST* onto quadrilateral *ABCD.* Encourage students to look for how these two sequences of transformations are similar.

Discuss (Whole Class):
Select and sequence student presentations so that each strategy involves fewer transformations, and is therefore more consistent (e.g., less haphazard) and efficient. Work towards a strategy that translates a vertex on quadrilateral *ABCD* to the corresponding vertex on quadrilateral *QRST*, then rotates quadrilateral *ABCD* about that vertex until a pair of corresponding congruent sides match up, and then reflects quadrilateral *ABCD* over the sides that has been superimposed so that quadrilateral *ABCD* coincides with quadrilateral *QRST*.

Have students check out a "translation-rotation-reflection" sequence to see if it can be applied to carrying quadrilateral *QRST* onto quadrilateral *ABCD.*

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.8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Ready, Set, Go!

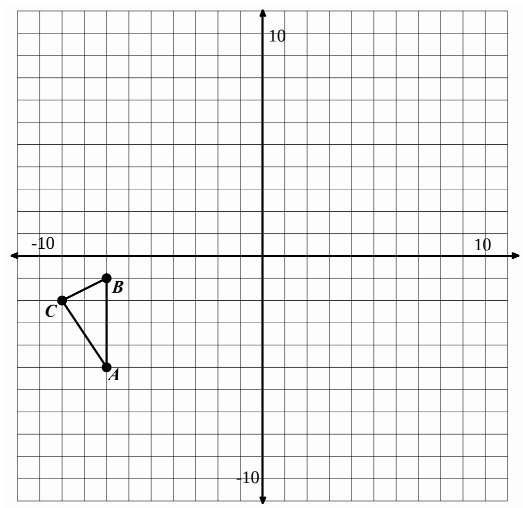
Ready

Topic: Performing a sequence of transformations.

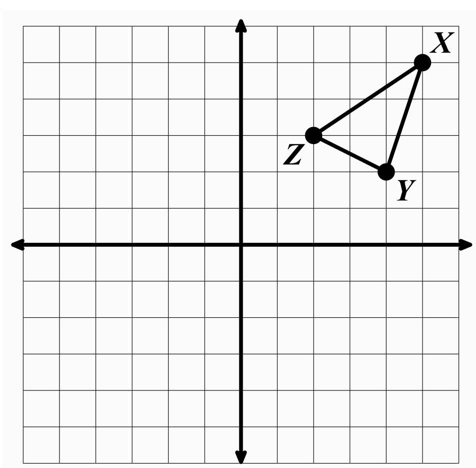
The given figures are to be used as pre-images. Perform the stated transformations to obtain an image. Label the corresponding parts of the image in accordance with the pre-image.

1. Reflect triangle ***ABC*** over the line $y = x$ and label the image ***A'B'C'***.

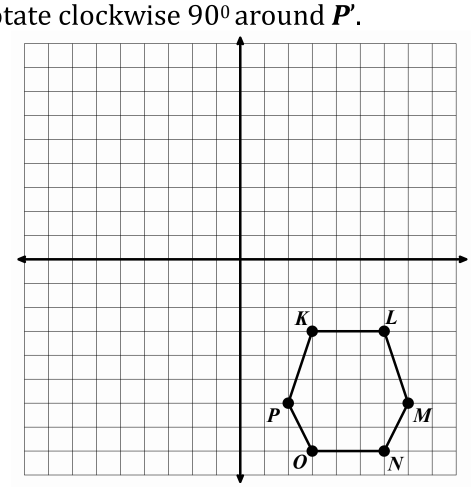
   Rotate triangle ***A'B'C'*** 180^0 counter clockwise around the origin and label the image ***A''B''C''***.



2. Reflect over the line $y = -x$.

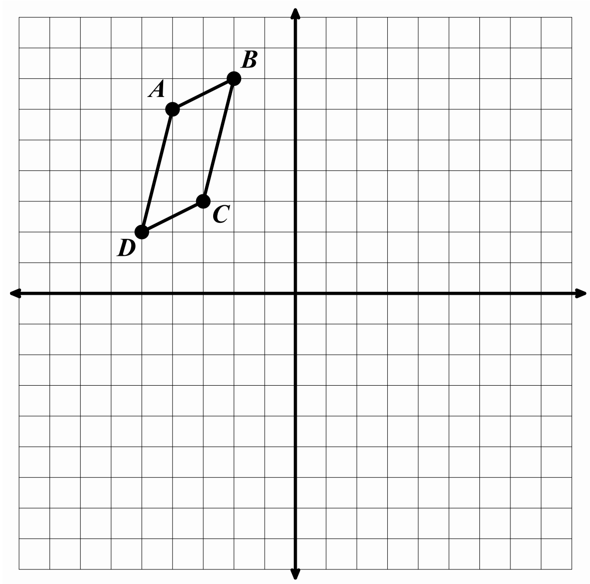


3. Reflect over y-axis and then Rotate clockwise 90^0 around ***P'***.



4. Reflect quadrilateral ABCD over the line $y = 2 + x$ and label the image A'B'C'D'.

Rotate quadrilateral A'B'C'D' counter-clockwise 90^0 around (-2, -3) as the center of rotation label the image A''B''C''C''.

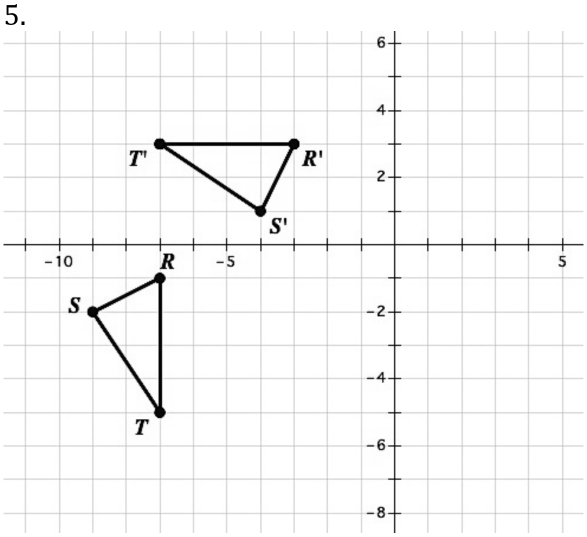


Set

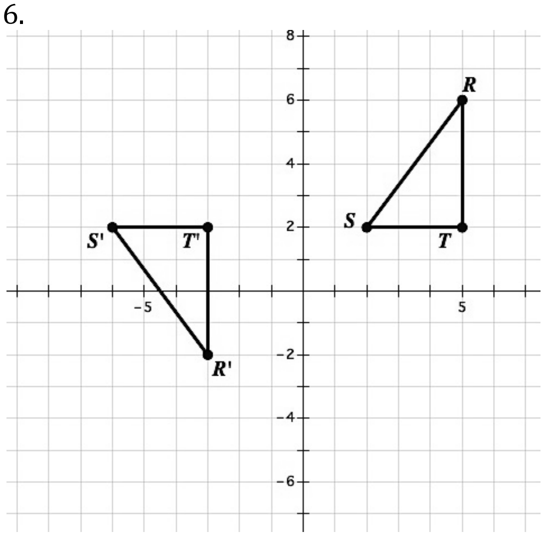
Topic: Find the sequence of transformations.

Find the sequence of transformations that will carry triangle ***RST*** onto triangle ***R'S'T'***. Clearly describe the sequence of transformations below each grid.

5.



6.



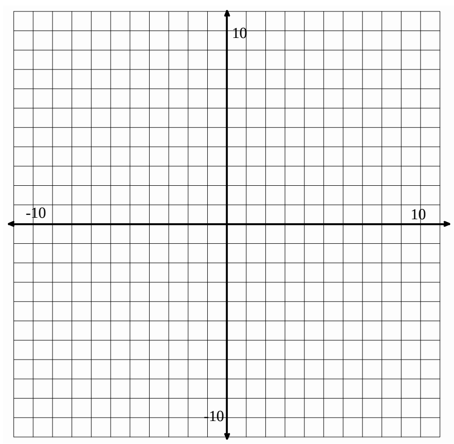
Go

Topic: Graphing functions and making comparisons.

Graph each pair of functions and make an observation about how the functions compare to one another.

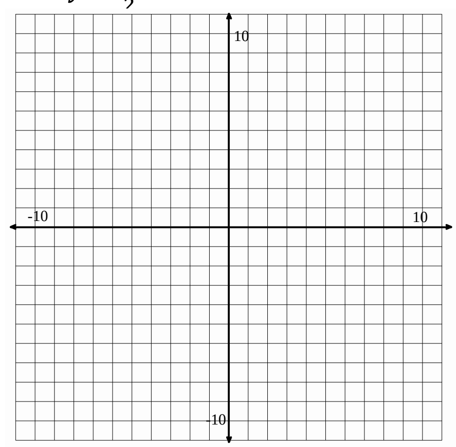
7. $y = \frac{1}{3}x - 1$

   $y = -3x - 1$



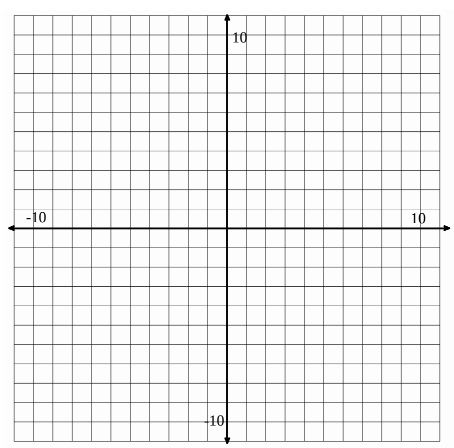
8. $y = -\frac{2}{3}x + 5$

   $y = \frac{3}{2}x + 5$



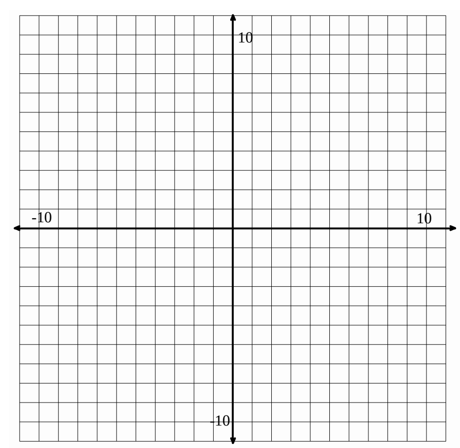
9. $y = \frac{1}{4}x + 2$

   $y = -\frac{1}{4} + 2$



10. $y = 2^x$

    $y = -2^x$



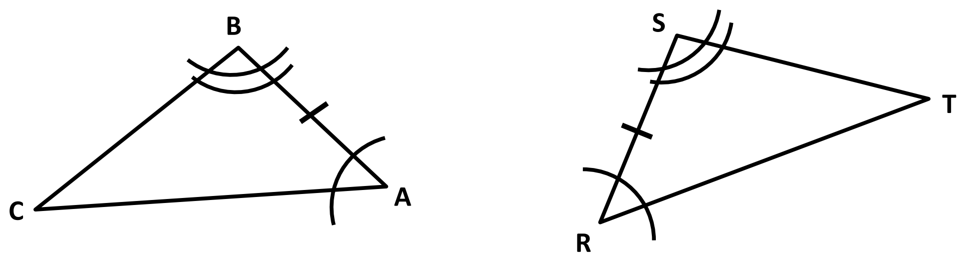
6.9 Congruent Triangles

A Solidify Understanding Task

Zac and Sione are trying to decide how much information they need to know about two triangles before they can convince themselves that the two triangles are congruent.

They are wondering if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria their teacher refers to as ASA—is enough to know that the two triangles are congruent. They are trying to justify that this would be so.

To start reasoning about the congruence of the two triangles, Zac and Sione have created the following diagram in which they have marked an ASA relationship between the triangles.



1. Based on the diagram, which angles have Zac and Sione indicated are congruent? Which sides?

2. To convince themselves that the two triangles are congruent, what else would Zac and Sione need to know?

<u>Zac's Argument</u>

"I know what to do," said Zac. "We can translate point A until it coincides with point R, then rotate $\overline{AB}$ about point R until it coincides with $\overline{RS}$. Finally, we can reflect ΔABC across $\overleftrightarrow{RS}$ and then everything coincides so the triangles are congruent." [Zac and Sione's teacher has suggested they use the word "coincides" when they want to say that two points or line segments occupy the same position on the plane. They like the word, so they plan to use it a lot.]

What do you think about Zac's argument? Does it convince you that the two triangles are congruent? Does it leave out any essential ideas that you think need to be included?

3. Write a paragraph explaining your reaction to Zac's argument:

Sione isn't sure that Zac's argument is really convincing. He asks Zac, "How do you know point *C* coincides with point *T* after you reflect the triangle?"

4. How do you think Zac might answer Sione's question?

While Zac is trying to think of an answer to Sione's question he adds this comment, "And you really didn't use all of the information about the corresponding congruent parts of the two triangles."

"What do you mean?" asked Zac.

Sione replied, "You started using the fact that $\angle A \cong \angle R$ when you translated ΔABC so that vertex *A* coincides with vertex *R*. And you used the fact that $\overline{AB} \cong \overline{RS}$ when you rotated $\overline{AB}$ to coincide with $\overline{RS}$, but where did you use the fact that $\angle B \cong \angle S$?"

"Yeah, and what does it really mean to say that two angles are congruent?" Zac added. "Angles are more than just their vertex points."

5. How might thinking about Zac and Sione's questions help improve Zac's argument?

<u>Sione's Argument</u>

"I would start the same way you did, by translating point *A* until it coincides with point *R*, rotating $\overline{AB}$ about point *R* until it coincides with $\overline{RS}$, and then reflecting ΔABC across $\overleftrightarrow{RS}$," Sione said. "But then I would want to convince myself that points *C* and *T* coincide. I know that an angle is made up of two rays that share a common endpoint. Since I know that $\overrightarrow{AB}$ coincides with $\overrightarrow{RS}$ and $\angle A \cong \angle R$, that means that $\overrightarrow{AC}$ coincides with $\overrightarrow{RT}$. Likewise, I know that $\overrightarrow{BA}$ coincides with $\overrightarrow{SR}$ and $\angle B \cong \angle S$, so $\overrightarrow{BC}$ must coincide with $\overrightarrow{ST}$. Since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ intersect at point *C*, and $\overrightarrow{RT}$ and $\overrightarrow{ST}$ intersect at point *T*, points *C* and *T* must also coincide because the corresponding rays coincide. Therefore, $\overline{BC} \cong \overline{ST}$, $\overline{CA} \cong \overline{TR}$, and $\angle C \cong \angle T$ because both angles are made up of rays that coincide!"

At first Zac was confused by Sione's argument, but he drew diagrams and carefully marked and sketched out each of his statements until it started to slowly make sense.

6. Do the same kind of work that Zac did to make sense of Sione's argument. What parts of his argument are unclear to you? What ideas did sketching out the words of his proof help you to clarify?

Sione's argument suggests that ASA is sufficient criteria for determining if two triangles are congruent. Now Zac and Sione are wondering about other criteria, such as SAS or SSS, or perhaps even AAA (which Zac immediately rejects because he thinks two triangles can have the same angle measures but be different sizes).

7. Draw two triangles that have SAS congruence. Be sure to mark you triangles to show which sides and which angles are congruent.

8. Write out a sequence of transformations to show that the two triangles potentially coincide.

9. If Sione were to examine your work in #8, what questions would he wonder about?

10. How can you use the given congruence criteria (SAS) to resolve Simone's wonderings?

Repeat 7-10 for SSS congruence.

6.9 Congruent Triangles – Teacher Notes
A Solidify Understanding Task

Purpose: The purpose of this task is to establish ASA, SAS and SSS as sufficient criteria for claiming that two triangles are congruent and to show how the rigid-motion transformations, along with the given congruence criteria about the two triangles, allows us to prove that the two triangles are congruent. As students work on such proofs they often overlook or reveal misconceptions about how to use the given congruence criteria in their work. Consequently, they might create a sequence of transformations that they claim carries one triangle onto the other, similar to the work they did in the previous task *Can You Get There From Here*, but in doing so they often *assume* the triangles *are* congruent, rather than proving them *to be* congruent. Therefore, the purpose of this task is less about students creating their own arguments, and more about considering the details of how such arguments can be made. Students begin by analyzing a couple of different arguments about ASA criteria for congruent triangles—one that harbors some misconceptions and one that is more explicit about the details. Then they explore other criteria for congruent triangles, such as SSS and SAS, and begin to formulate their own arguments about how they might justify such criteria using transformations.

Note: As this module continues to unfold, students will be asked to continue formulating their own arguments or justifications that something is true, and this task provides a model for doing so. Recognize that students are developing their understandings about justification and proof, and that this work does not get fully formalized until Mathematics II. Based on the students in your class, you will need to decide how much of this work should be based on experimentation, perhaps using technology, and how much should be based on justifying the results of their experimentation.

Core Standards Focus:

G.CO.6 Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

See also Mathematics I note for G.CO.6, G.CO.7, and G.CO.8: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Related Standards: G.CO.5

Launch (Whole Class):
Clarify for students how this task is different from the previous one by saying something similar to the following: "In the previous task you found that if two figures are congruent you can always find a sequence of rigid-motion transformations that will carry one figure onto the other. In this task we will reverse this big idea: *If we can convince ourselves that a sequence of rigid-motion transformations has carried one figure onto another, then the two figures are congruent*. So, unlike in the previous task where we started with congruent figures, in this task we start with figures that we don't know are congruent and try to convince ourselves that they are."

You will need to introduce students to the use of notation for naming corresponding congruent parts of two triangles, such as SAS ("side-angle-side", meaning two sides and the included angle between the two sides of one triangle are congruent to corresponding sides and an included angle of another triangle), ASA ("angle-side-angle", meaning two angles and the included side between the two angles of one triangle are congruent to corresponding angles and an included side of another triangle), and SSS ("side-side-side", meaning the three sides of one triangle are congruent to the corresponding three sides of another triangle).

Decide what tools students will use to carry out the transformations and analyze the arguments made by Zac and Sione. For example, students could create two triangles on a computer screen that have ASA congruence, then perform Zac's transformations until the two triangles coincide. This would be an informal justification of the ASA congruence criteria. Simone's questions do not surface in such work because the technology obscures it. Similarly, using cut-outs of the figures, transparencies or tracing paper to perform the transformations will obscure the issues. Using rulers and protractors to redraw ΔABC after each transformation perhaps will reveal Simone's questions, since such drawings often are prone to inaccuracy. Making hand-drawn sketches of each transformation will certainly raise Simone's questions, since such drawings are based on one's visual imagination.

Explore (Small Group):
Since this lesson is about learning how to construct an argument based on transformations, it will be best to use a think-pair-share structure during much of the explore phase of the lesson. Here is the basic structure:

Have students individually answer questions 1 and 2, then discuss as a whole class. The point of question 2 is to recognize that we do not yet know the triangles are congruent, since we don't know anything about the congruence of the unmarked sides and angles.

Read Zac's argument and then have students discuss and respond to questions 3, 4 and 5 with a partner. Then discuss these questions as a whole class. The point of this discussion is to move away from an argument based on intuition ("it looks like the reflected triangle will coincide with the other triangle) and towards arguments made on the basis of preserved angles and distance.

During this part of the discussion students should focus on Sione's concerns about the shortcomings in Zac's argument.

Point out that Sione is going to make a more complete argument and note that it is going to initially be confusing to Zac—and perhaps to them as well—by reading the line immediately following Sione's argument: "*At first Zac was confused by Sione's argument, but he drew diagrams and carefully marked and sketched out each of her statements until it started to slowly make sense*." Often in geometry we need to follow another person's work step by step. Ask students to do the same thing Zac did—drawing and marking segments on the diagram that match with Sione's description. Give students a few minutes to explore Sione's argument on their own or with a partner, then discuss her ideas as a whole class.

The last part of the task gives students an opportunity to make "Sione-like" arguments for SAS and SSS congruence. Prompt students to draw appropriate auxiliary lines or circles to help make their arguments.

Here is an example of an argument for SSS congruence: Consider ΔABC and ΔDEF with corresponding sides congruent. Translate, rotate and reflect ΔABC as necessary until $\overline{AB}$ and $\overline{DE}$ coincide. We need to show that point *C* coincides with point *F* using the facts that $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$. One way to justify this claim is to argue that point *C* lies at the intersection of the circle with radius *BC* centered at *B* and the circle with radius *AC* centered at *A*. Likewise, point *F* lies at the intersection of the circle with radius *EF* centered at *E* and the circle with radius *DF* centered at *D*. Since the centers of circle *B* and circle *E* coincide and have the same size radii, and the centers of circle *A* and circle *D* coincide and have the same size radii, the point of intersection of these circles, *C* and *F* also coincide. Therefore, the triangles are superimposed on top of each other.

Discuss (Whole Class):
Share students' tentative arguments for why SAS and SSS congruence works. As students share their arguments, other students should take on the role of Sione, looking for places where the argument doesn't make sense or isn't convincing. This is hard work, and the nature of proof will be revisited in Mathematics II. You can feel that the lesson is successful if students are starting to see the need for greater precision in making arguments, and not basing arguments on "it looks like it works." Students should feel assured at the end of the lesson that ASA, SAS and SSS are sufficient criteria for determining that two triangles are congruent—that we don't need to know all pairs of consecutive angles and sides are congruent in order to know the triangles are congruent.

Aligned Ready, Set, Go: Congruence, Construction and Proof 8.9

Ready, Set, Go!

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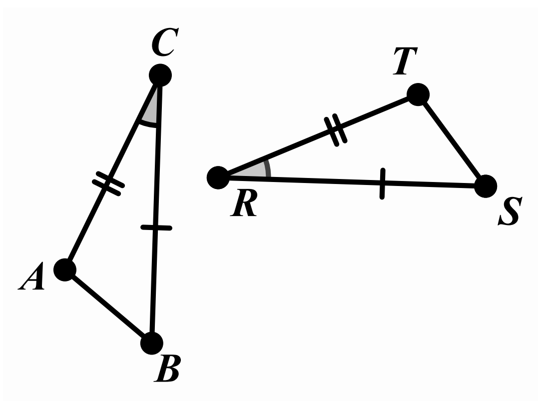
Ready

Topic: Corresponding parts of figures and transformations

Given the figures in each sketch with congruent angles and sides marked, first list the parts of the figures that correspond (For example, in #1, $\angle C \cong \angle R$) Then determine a reflection occurred as part of the sequence of transformations that was used to create the image.

1.

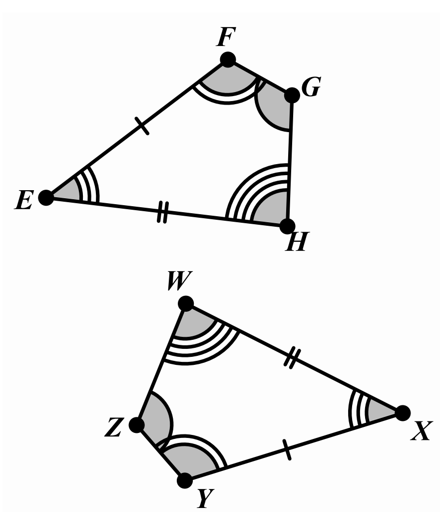


Congruencies

$\angle C \cong \angle R$

Reflected? Yes or No

2.



Congruencies

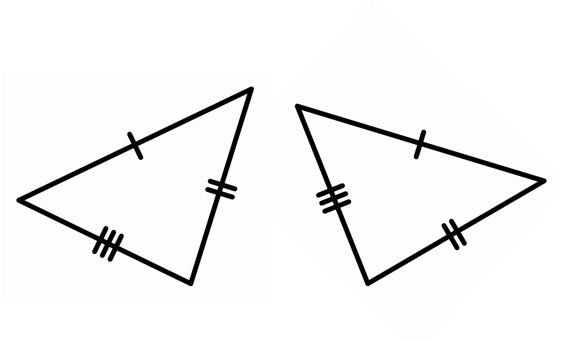
Reflected? Yes or No

Set

Topic: Triangle Congruencies

Explain whether or not the triangles are congruent, similar, or neither based on the markings that indicate congruence.

3.



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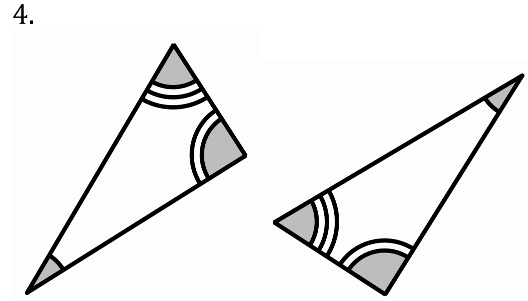

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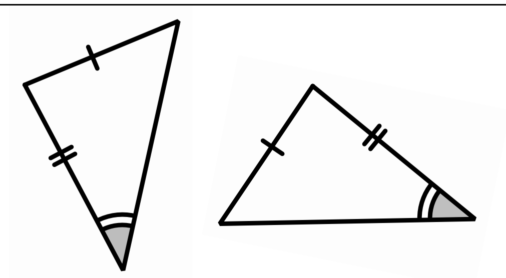

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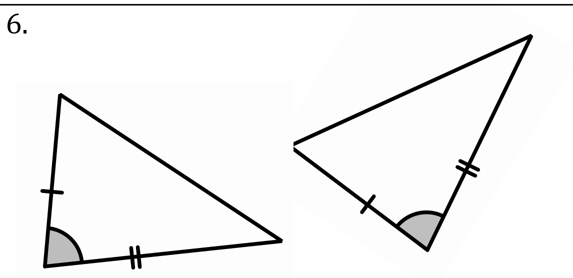
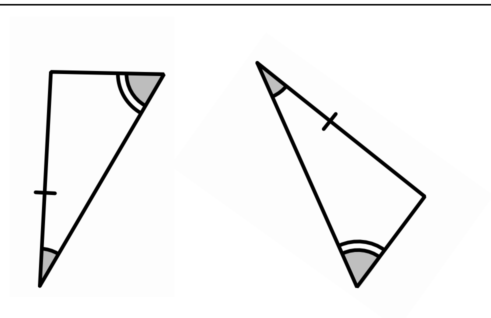
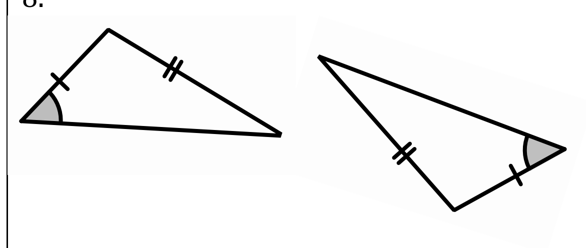


7.



8.



Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.

8. $\Delta ABC \cong \Delta PQR$

9. $\Delta XYZ \cong \Delta KLM$

Go

Topic: Review of solving equations and finding recursive rules for sequences.

Solve each equation for *t*.

10. $\frac{3t-4}{5} = 5$

11. $10 - t = 4t + 12 - 3t$

12. $P = 5t - d$

13. $xy - t = 13t + w$

Use the given sequence of number to write a recursive rule for the *n*th value of the sequence.

14. 5, 15, 45, …

15. $\frac{1}{2}$, 0, -$\frac{1}{2}$, -1, …

16. 3, -6, 12, -24, …

17. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, …

6.10 Congruent Triangles to the Rescue

A Practice Understanding Task

Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent.

Zac: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side.

Sione: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

Zac: But I've folded lots of isosceles triangles in half, and it always seems to work.

Sione: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the line through the vertex point of the angle made up of the two congruent sides and the midpoint of the third side in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could she use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "crease line"? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral *A*, *B*, *C*, and *D*, and label the point of intersection of the two diagonals as point *N.*

2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent.

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$

3. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

- the diagonals bisect each other
- the diagonals are congruent
- the diagonals are perpendicular to each other
- the diagonals bisect the angles of the quadrilateral

Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

6.10 Congruent Triangles to the Rescue – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to provide students with practice in identifying the criteria they might use—ASA, SAS or SSS—to determine if two triangles embedded in another geometric figure are congruent, and then to use those congruent triangles to make other observations about the geometric figures based on the concept that corresponding parts of congruent triangles are congruent. A secondary purpose of this task is to allow students to continue to examine what it means to make an argument based on the definitions of transformations, as well as based on properties of congruent triangles. The focus should be on using congruent triangles and transformations to identify other things that can be said about a geometric figure, rather than on the specific properties of triangles or quadrilaterals that are being observed. These observations will be more formally proved in Secondary II. The observations in this task also provide support for the geometric constructions that are explored in the next sequence of tasks.

Core Standards Focus:

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

See also Mathematics I note for G.CO.6, G.CO.7, G.CO.8: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Related Standards: G.CO.10

Launch (Whole Class):

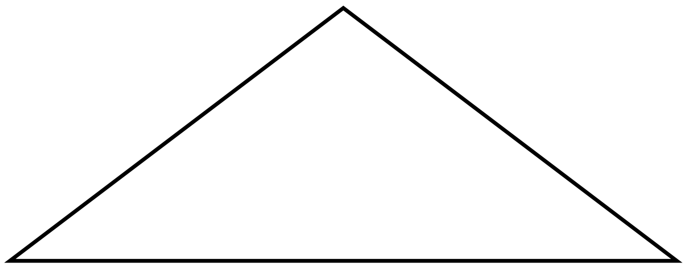
Make sure that students know the definition of an isosceles triangle and give them several isosceles triangles to fold—essentially recreating Zac's paper-folding experiment as described in part 1 of the task (see attached handout of isosceles triangles). Ask students if they see any congruent triangles inside of the folded isosceles triangle, and what criteria for congruent triangles—ASA, SAS or SSS—they could use to convince themselves that these interior triangles are congruent. Work through the additional questions in part 1 with the class, giving students time to think about each question individually or with a partner.

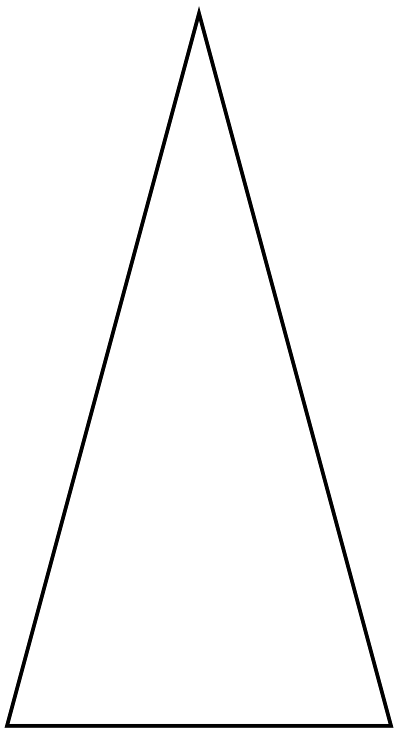
Help students see the difference between verifying Zac's claim ("every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side") through experimentation—paper folding—and a justification based on transformations and congruent triangle criteria. It appears from folding one side of the isosceles triangle onto the other that two congruent triangles are formed. This can be justified using the SSS triangle congruence criterion: the line through the vertex and the midpoint of the opposite side is common to both interior triangles (S1); the midpoint of the opposite side forms two corresponding congruent segments in the interior triangles (S2); and by definition of an isosceles triangle the other pair of sides in the interior triangles are congruent (S3). Since the interior triangles are congruent by SSS, we can also conclude that the three corresponding angles are congruent. This leads to such additional properties as: the base angles of the isosceles triangle are congruent; the vertex angle is bisected by the line through the vertex and midpoint of the opposite side; and the line through the vertex and midpoint of the opposite side is perpendicular to the base since the angles formed are congruent and together form a straight angle. Collectively, these statements justify Zac's claim that every isosceles triangle has a line of symmetry.

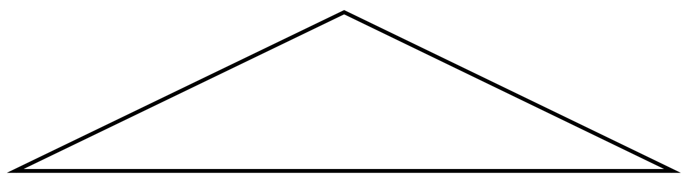
Explore (Small Group):
The guided discussion of part 1 of this task will prepare students to work more independently on part 2. You may want to assign different groups to a particular quadrilateral, so all of the quadrilaterals get explored. Center the exploration time on part 2, question 2—looking for congruent triangles and listing the criteria that was used to claim that the triangles are congruent. Fast finishers can work on part 2, question 3—justifying other properties of quadrilaterals based on corresponding parts of congruent triangles.

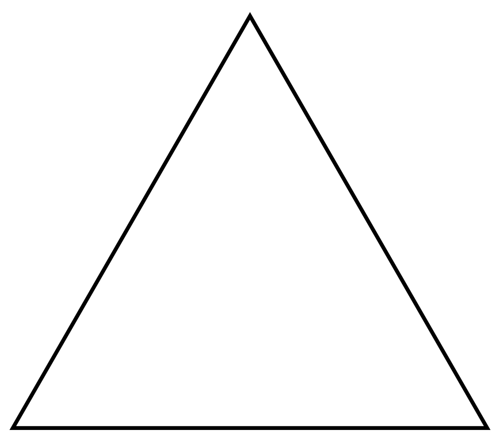
Discuss (Whole Class):
The focus of the discussion should be on part 2, question 2—identifying congruent triangles formed in different types of quadrilaterals by drawing in the diagonals. As students claim two triangles are congruent, ask them to explain the triangle congruence criteria—ASA, SAS or SSS—they used to justify their claim. As time allows, discuss some of the other claims that can be made about the quadrilaterals based on corresponding parts of congruent triangles.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.10









Ready, Set, Go!

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Ready

Topic: Defining bisectors of angles and perpendicular bisectors.

1. Based on the meaning of "bisect", which means to split into two equal parts, what would it mean to *bisect* an angle? Describe in words and also provide visuals to communicate the meaning of angle bisector.

2. What does it mean if you have a *perpendicular bisector* of a line segment? Provide both written explanation and visual sketches to communicate the meaning of perpendicular bisector.

Set

Topic: Use congruent triangle criteria and transformations to justify conjectures.

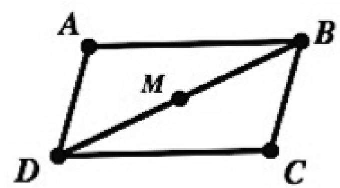
In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

3. True statements:

   Point ***M*** is the midpoint of $\overline{DB}$

   $\angle ABD \cong \angle BDC$

   $\overline{AB} \cong \overline{DC}$



Conjecture: $\angle A \cong \angle C$

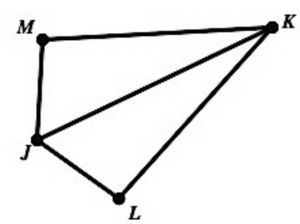
a. Is the conjecture correct?

b. Argument to prove you are right:

4. True statements

   $\angle KJL \cong \angle KJM$

   $\overline{JL} \cong \overline{JM}$



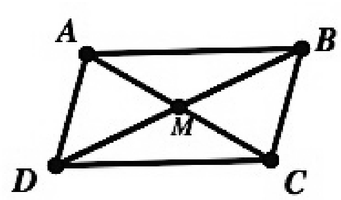
Conjecture: $\overline{JK}$ bisects $\angle MKL$

a. Is the conjecture correct?

b. Argument to prove you are right:

5. True statements

   ΔADM is a 180°
   rotation of ΔCMB



Conjecture: $\Delta ABM \cong \Delta CDM$

a. Is the conjecture correct?

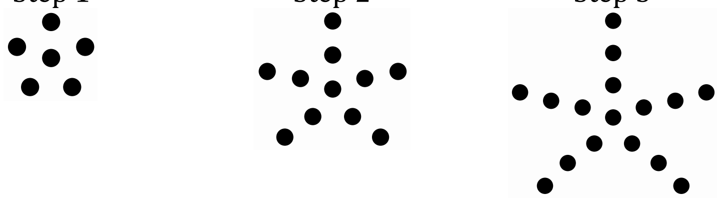
b. Argument to prove you are right:

Go

Topic: Create both explicit and recursive rules for the visual patterns.

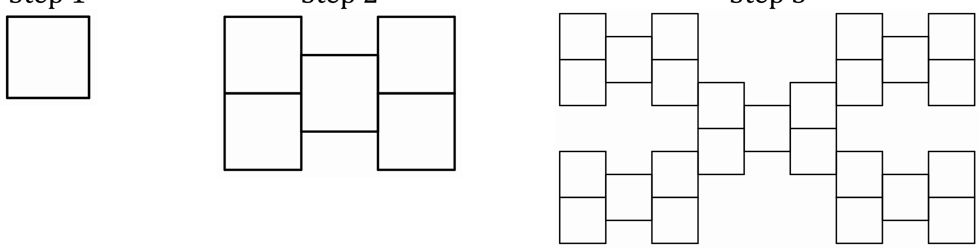
6. Find an explicit function rule and a recursive rule for dots in step *n*.

Step 1 Step 2 Step 3



7. Find an explicit function rule and a recursive rule for squares in step *n.*

Step 1 Step 2 Step 3



Find an explicit function rule and a recursive rule for the values in each table.

8.

Step	Value
1	1
2	11
3	21
4	31

9.

n	f(n)
2	16
3	8
4	4
5	2

10.

n	f(n)
1	-5
2	25
3	-125
4	625

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6.11 Under Construction

A Develop Understanding Task

Anciently, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools you can construct all kinds of geometric shapes.

Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the following sketch.

Now the hard work begins. We can't just keep drawing line segments, because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

Constructing a rhombus

Knowing what you know about circles and line segments, how might you locate point *C* on the ray in the diagram above so the distance from *B* to *C* is the same as the distance from *B* to *A*?

1. Describe how you will locate point *C* and how you know $\overline{BC} \cong \overline{BA}$, then construct point *C* on the diagram above.

Now that we have three of the four vertices of the rhombus, we need to locate point *D*, the fourth vertex.

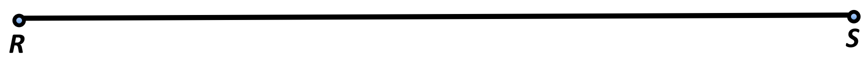
2. Describe how you will locate point *D* and how you know $\overline{CD} \cong \overline{DA} \cong \overline{AB}$, then construct point *D* on the diagram above.

Constructing a Square (A rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given $\overline{RS}$ below, fold and crease the paper so that point *R* is reflected onto point *S*. Based on the definition of reflection, what do you know about this "crease line"?



You have "constructed" a perpendicular bisector of $\overline{RS}$ by using a paper-folding strategy. Is there a way to construct this line using a compass and straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the "crease line". When you have located at least two points on the "crease line" use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of $\overline{RS}$.

Now that you have created a line perpendicular to $\overline{RS}$ we will use the right angle formed to construct a square.

5. Label the midpoint of $\overline{RS}$ on the diagram above as point *M.* Using segment $\overline{RM}$ as one side of the square, and the right angle formed by segment $\overline{RM}$ and the perpendicular line drawn through point *M* as the beginning of a square. Finish constructing this square on the diagram above. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

6.11 Under Construction – Teacher Notes
A Develop Understanding Task

Purpose: Compass and straightedge constructions can be justified based on properties of quadrilaterals, corresponding parts of congruent triangles, and the definitions of the rigid-motion transformations. While we have used dynamic geometric software and paper folding in previous tasks, in this learning cycle we restrict the work to compass and straightedge in order to generate ideas about defending how these constructions result in the desired objects—a goal which is sometimes obscured by other tools. In this task students invent strategies for constructing a rhombus and a square. Embedded in this work are smaller constructions, such as copying a segment (using radii of congruent circles) or creating the perpendicular bisector of a segment. These constructions also contain the potential for building more sophisticated constructions, such as bisecting an angle by constructing a diagonal of a rhombus.

Core Standards Focus:

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Related Standards:

Launch (Whole Class):
Read the introductory paragraphs of the task with students to suggest why we are going to restrict ourselves to using two tools: a compass and a straightedge. Remind students of the definition of a circle that we encountered in *Leap Year*: the set of all points in a plane that are equidistant from a fixed point called the center of the circle. Point out that students may need to think about this definition as they work on this task. Ask students what they would need to know about two different circles in order to know that the circles are congruent?

Read through the remainder of the first page of the task up to the words "we have to stop drawing and start constructing." Tell students to follow the prompts as they work on the two tasks of constructing a rhombus and a square.

Explore (Small Group):
Allow students sufficient time to explore both constructions.

The key to constructing the rhombus is creating a set of congruent circles, centered at each of the vertices of the evolving rhombus. Since radii of congruent circles are congruent line segments, we can use these congruent segments to define the sides of the rhombus.

The key to locating points on the perpendicular bisector is to construct congruent circles on each endpoint of the segment, with the radius larger than half of the segment. The points where these circles intersect, along with the endpoints of the segment, form pairs of congruent triangles. Ask students to identify some congruent triangles based on the ASA, SAS or SSS criteria, and then explain why these congruent triangles justify that the line through the points of intersection of the circles is a perpendicular bisector of the segment.

Since finishing the square is the same as the rhombus construction once the right angle has been constructed, you may move to the whole class discussion of the task once all students have explored a strategy for constructing the perpendicular bisector of the given segment.

Discuss (Whole Class):
Select students to present each of the constructions. Highlight the key issues underlying each construction, as described above, as they emerge in students' work during the presentations.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.11

Ready, Set, Go!

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Ready

Topic: Tools for construction and geometric work.

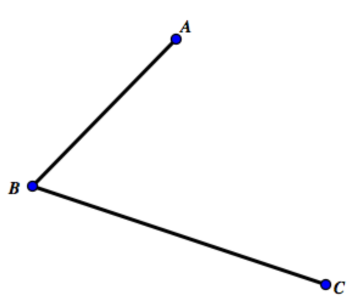
1. Using your compass draw several concentric circles that have point A as a center and then draw those same sized concentric circles that have B as a center. What do you notice about where all the circles with center A intersect all the corresponding circles with center B?

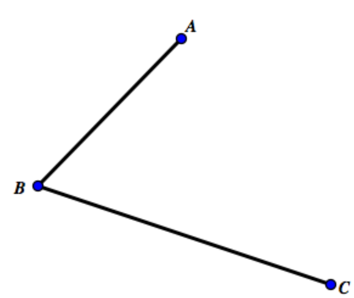
2. In the problem above you have demonstrated one way to find the midpoint of a line segment. Explain another way that a line segment can be bisected without the use of circles.

Set

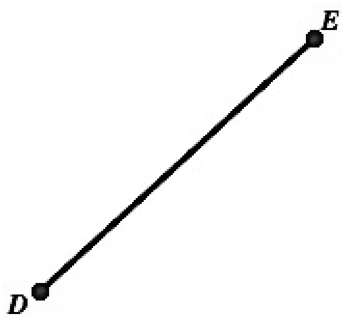
Topic: Constructions with compass and straight edge.

3. Bisect the angle below do it with compass and straight edge as well as with paper folding.

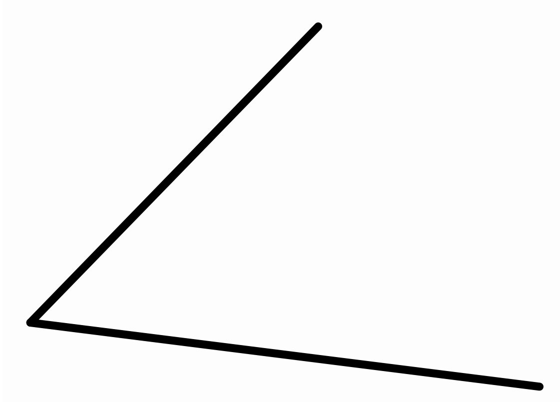




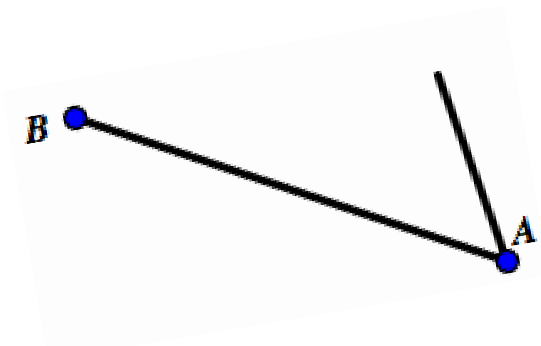
4. Copy the segment below using construction tools of compass and straight edge, label the image ***D'E'***.



5. Copy the angle below using construction tool of compass and straight edge.

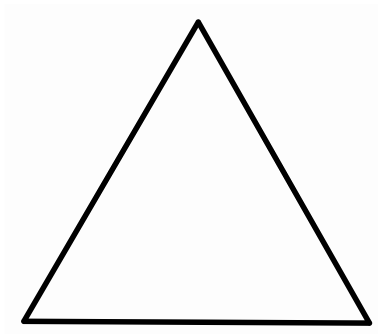


6. Construct a rhombus on the segment AB that is given below and that has point A as a vertex. Be sure to check that your final figure is a rhombus.



7. Construct a square on the segment CD that is given below. Be sure to check that your final figure is a square.

8. Given the equilateral triangle below, find the center of rotation of the triangle using compass and straight edge.



Go

Topic: Solving systems of equations review.

Solve each system of equations. Utilize substitution, elimination, graphing or matrices.

9. $\begin{cases} x = 11 + y \\ 2x + y = 19 \end{cases}$

10. $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

11. $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

12. $\begin{cases} y = -x + 1 \\ y = 2x + 1 \end{cases}$

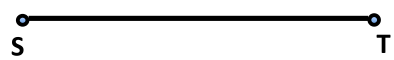
13. $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$

14. $\begin{cases} 4x - y = 7 \\ -6x + 2y = 8 \end{cases}$

6.12 More Things Under Construction
A Develop Understanding Task

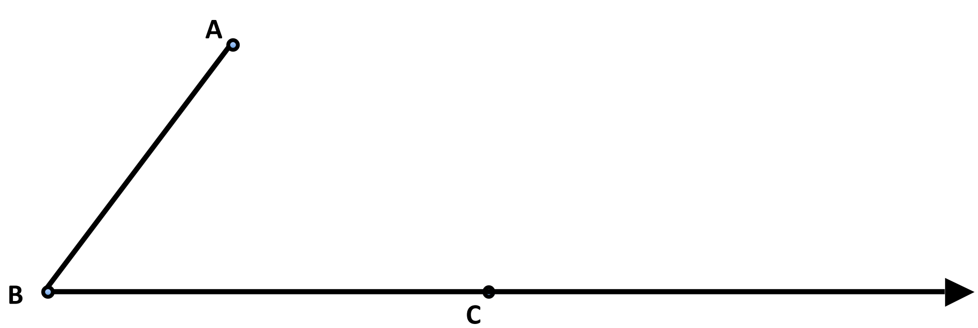
Constructing an Equilateral Triangle

Like a rhombus, an equilateral triangle has three congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given $\overline{ST}$ below as one side of the equilateral triangle.



Constructing a Parallelogram

To construct a parallelogram we will need to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to segment $\overline{AB}$ through point C on the diagram below. Since we have observed that parallel lines have the same slope, the line through point C will be parallel to $\overline{AB}$ only if the angle formed by the line and $\overrightarrow{CD}$ is congruent to $\angle ABC$. Can you describe and illustrate a strategy that will construct an angle with vertex at point C and a side parallel to $\overline{AB}$? (Hint: We know that corresponding parts of congruent triangles are congruent, so perhaps we can begin by constructing some congruent triangles.)



<u>Constructing a Hexagon Inscribed in a Circle</u>

Because regular polygons have rotational symmetry, they can be *inscribed* in a circle. The *circumscribed* circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

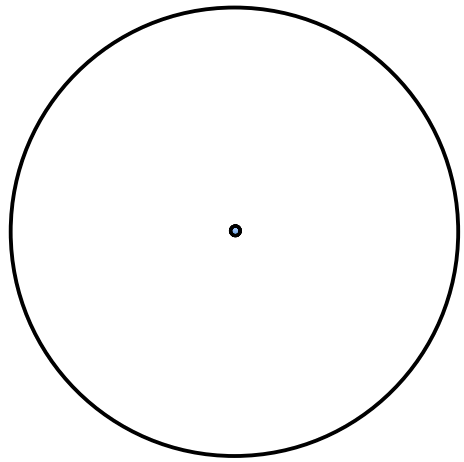
We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

1. Sketch a diagram of such a decomposition.

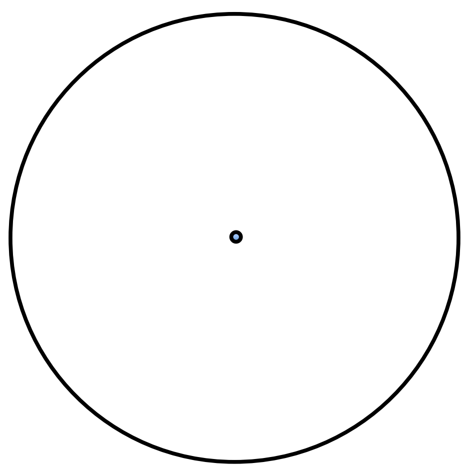
2. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?

3. The six vertices of the hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are *chords* of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? Be able to justify how you know this is so.

4. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the circle given below.



Modify your work with the hexagon to construct an equilateral triangle inscribed in the circle given below.



Describe how you might construct a square inscribed in a circle.

6.12 More Things Under Construction – Teacher Notes

A Develop Understanding Task

Purpose: This task continues the construction work of the previous task, allowing students to "invent" some additional construction strategies and to solidify the idea that these constructions are based on properties of quadrilaterals, corresponding parts of congruent triangles, and the definitions of the rigid-motion transformations. In this task students invent strategies for constructing an equilateral triangle, a parallelogram and a hexagon inscribed in a circle. Embedded in this work are smaller constructions, such as copying an angle (by constructing congruent triangles) and creating a line parallel to another line through a given point. These constructions also contain the potential for building more sophisticated constructions, such as inscribing a square or equilateral triangle in a circle.

Core Standards Focus:

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Related Standards:

Launch (Whole Class):
Give students a few minutes individually to construct the equilateral triangle. If students need prompting to do this, remind them that an equilateral triangle, like a rhombus or a square, contains three congruent sides. How can they use the ideas of the rhombus construction in this work? Once students are back into "construction mode" you can set them to work on the other constructions in this task.

Explore (Small Group):
Allow students sufficient time to explore both constructions.

The key to constructing the parallelogram is to create parallel lines by copying an angle (so the slant or slope will be the same). In this task, we copy an angle by constructing a congruent triangle in which the angle we desire to copy is embedded.
The key to inscribing a hexagon inside a circle is to notice that a hexagon can be decomposed into six equilateral triangles by drawing the lines of symmetry of the hexagon that pass through opposite vertices. If the hexagon is inscribed in a circle, these lines of symmetry form diameters of the circle. These lines of symmetry intersect the circle at six equally spaced intervals, and the line segments between these points of intersection are the same length as the radii of the circle. Therefore, we can use a compass setting equal to the radius of the circle to mark off these six points where the vertices of the hexagon can be located.

Discuss (Whole Class):
Select students to present each of the constructions. Highlight the key issues underlying each construction, as described above, as they emerge in students' work during the presentations.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.12

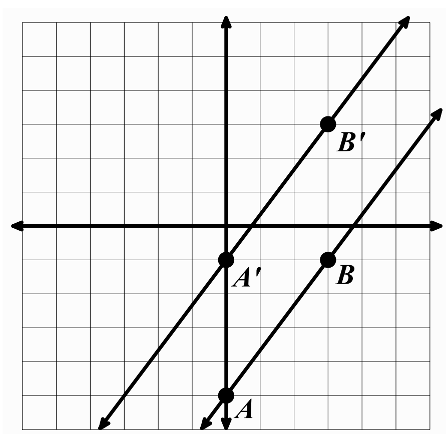
Ready, Set, Go!

Ready

Topic: Transformations of lines, algebraic and geometric thoughts.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, label them, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

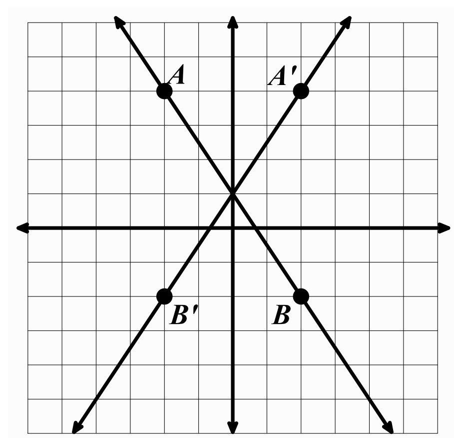


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

2.

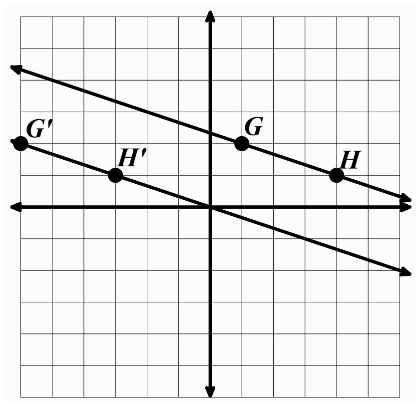


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

3.

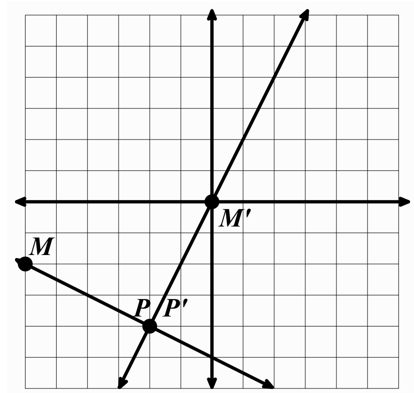


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4.



a. Description of Transformation:

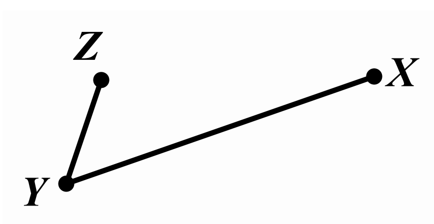
b. Equation for pre-image:

c. Equation for image:

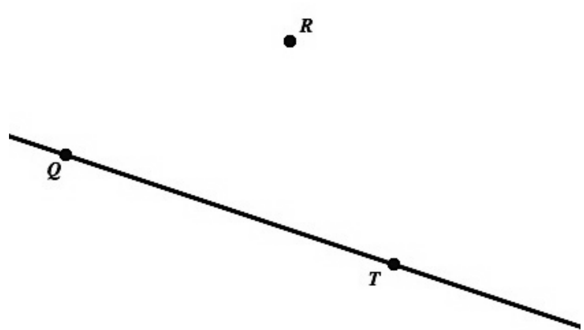
Set

Topic: Geometric Constructions using compass and straight edge.

5. Construct a parallelogram given sides $\overline{XY}$ and $\overline{YZ}$ and $\angle XYZ$.



6. Construct a line parallel to $\overline{QT}$ and through point ***R.***



7. Given segment $\overline{AB}$ show all points ***C*** such that $\triangle ABC$ is an isosceles triangle.

8. Given segment $\overline{AB}$ show all points C such that $\triangle ABC$ is a right triangle.

Go

Topic: Triangle congruence and properties of polygons.

9. What is the minimum amount of information needed to determine that two triangles are congruent? List all possible combinations of needed criteria.

10. What is a line of symmetry and what is a diagonal? Are they the same thing? Could they be the same in a polygon? If so give an example, if not explain why not.

11. How is the number of lines of symmetry for a *regular* polygon connected to the number of sides of the polygon? How is the number of diagonals for a polygon connected to the number of sides?

12. What do right triangles have to do with finding distance between points on a coordinate grid?

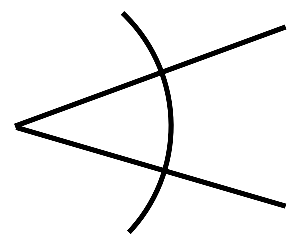
6.13 Justifying Constructions

A Solidify Understanding Task

Compass and straightedge constructions can be justified using such tools as:

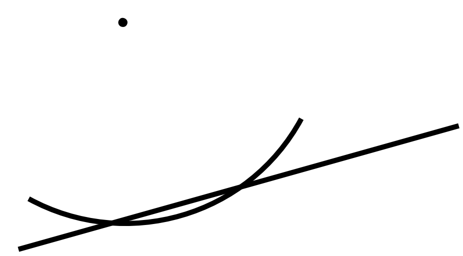
- the definitions and properties of the rigid-motion transformations
- identifying corresponding parts of congruent triangles
- using observations about sides, angles and diagonals of special types of quadrilaterals

Study the steps of the following procedure for *constructing an angle bisector*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

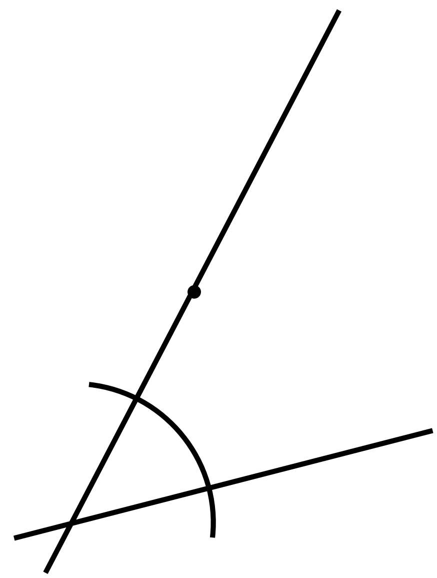
Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for *constructing a line perpendicular to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.	
Without changing the span of the compass, locate a second point not on the given line, by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for constructing a line parallel to a given line through a given point.

Steps	Illustration
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc, and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

6.13 Justifying Constructions – Teacher Notes

A Solidify Understanding Task

Purpose: In the two previous tasks students have invented strategies for constructing various polygons: rhombuses, squares, parallelograms, equilateral triangles, and regular hexagons. Embedded within these constructions are the ideas of the standard constructions: copying a segment, copying an angle, bisecting a segment, bisecting an angle, constructing parallel and perpendicular lines through given points. In this task students examine the details of the standard constructions and justify them based on properties of quadrilaterals, corresponding parts of congruent triangles, and the definitions of the rigid-motion transformations.

Core Standards Focus:

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Related Standards:

Launch (Whole Class):
Point out that in each chart students have been given an incomplete illustration of the construction steps that are outlined in the first column of the chart. They are to follow the steps to complete the illustration, then write a justification as to why the construction works.

Explore (Small Group):
Monitor students as they complete the illustrations of each construction. Listen for the ways students justify why the construction works. You may need to help students "see" the underlying congruent triangles or quadrilaterals on which the constructions are based. For example, the construction of the angle bisector is based on constructing a rhombus and its diagonal. Since we have conjectured that the diagonal of a rhombus bisects the angles through which it is drawn, we can justify this construction on that basis. Alternatively, we could look for congruent triangles formed by this line, or use this line as a line of reflection, in order to justify this construction in

other ways. Select students to present who have developed thoughtful arguments justifying each construction.

Discuss (Whole Class):
Have selected students demonstrate how the constructions steps lead to the desired objects, and then justify the construction based on congruent triangles, properties of quadrilaterals, or definitions of transformations.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.13

Ready, Set, Go!

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Ready

Topic: Rotation symmetry for regular polygons and transformations

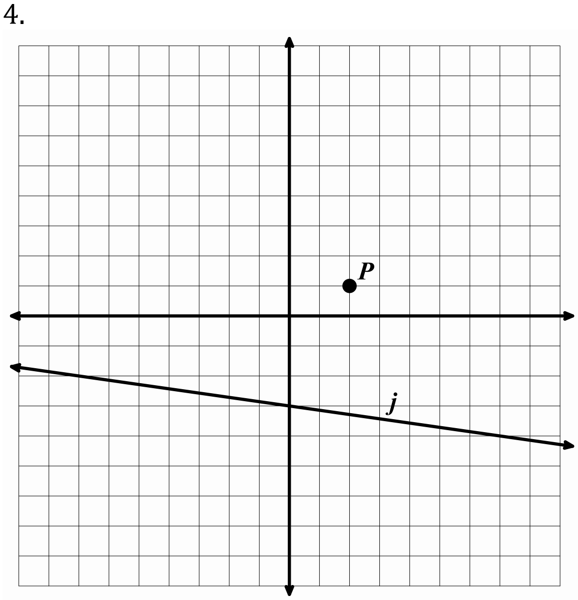
1. What angles of rotational symmetry are there for a pentagon?

2. What angles of rotational symmetry are there for a hexagon?

3. If a regular polygon has an angle of rotational symmetry that is 40^0, how many sides does the polygon have?

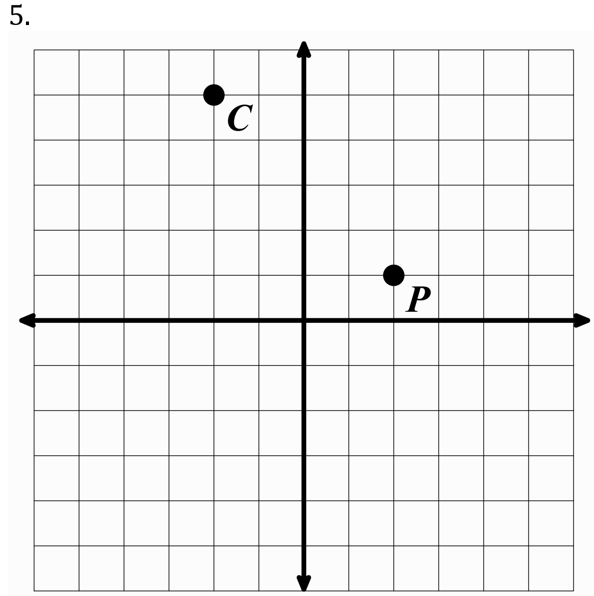
On each given coordinate grid below perform the indicated transformation.

4.



Reflect point ***P*** over line ***j***.

5.

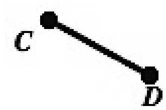


Rotate point ***P*** 90^0 clockwise around point ***C***.

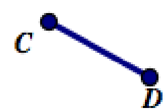
Set

Topic: Constructing regular polygons inscribed in a circle.

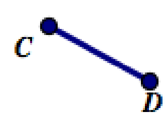
6. Construct an isosceles triangle that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the triangle.



7. Construct a hexagon that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the hexagon.



8. Construct a square that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the square.



Go

Topic: Finding distance and slope.

For each pair of given coordinate points find distance between them and find the slope of the line that passes through them. Show all your work.

9. (-2, 8) , (3, -4)
   a. Slope: b. Distance:

10. (-7, -3) , (1, 5))
   a. Slope: b. Distance:

11. (3, 7) , (-5, 9)
   a. Slope: b. Distance:

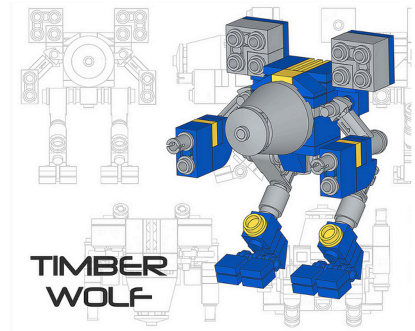
12. (1, -5) (-7, 1)
   a. Slope: b. Distance:

13. (-10, 31) (20, 11)
   a. Slope: b. Distance:

14. (16, -45) (-34, 75)
   a. Slope: b. Distance:

6.14 Construction Blueprints

A Practice Understanding Task



For each of the following straightedge and compass constructions, illustrate or list the steps for completing the construction and give an explanation for why the construction works. You explanations may be based on rigid-motion transformations, congruent triangles, or properties of quadrilaterals.

Purpose of the construction	Illustration and/or steps for completing the construction	Justification of why this construction works
Copying a segment	1. Set the span of the compass to match the distance between the two endpoints of the segment. 2. Without changing the span of the compass, draw an arc on a ray centered at the endpoint of the ray. The second endpoint of the segment is where the arc intersects the ray.	The given segment and the constructed segment are radii of congruent circles.
Copying an angle		
Bisecting a segment		
Bisecting an angle		
Constructing a perpendicular bisector of a line segment		
Constructing a perpendicular to a line through a given point		

Constructing a line parallel to a given line through a given point		
Constructing an equilateral triangle		
Constructing a regular hexagon inscribed in a circle		

6.14 Construction Blueprints – Teacher Notes
A Practice Understanding Task

Purpose: The purpose of this task is to give students additional practice with the standard constructions by asking them to outline the steps of each construction in their own words, and to justify each construction in terms of the geometric ideas on which it is based.

Core Standards Focus:

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

See also Mathematics I note for G.CO.12, G.CO.13: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Related Standards:

Launch (Whole Class):
Remind students that in the previous task they were given prescribed steps for completing some standard constructions and asked to follow those steps to figure out how the construction works. Now they have the opportunity to create their own "construction blueprints." The details of each construction have been explored throughout the past three tasks, and this is an opportunity for students to summarize their learning.

Explore (Small Group):
Monitor students as they work on this task to verify that they can adequately describe a set of construction steps that accomplishes each purpose. If students are struggling with a particular construction, point them back to the polygon constructions (e.g., rhombus, square, parallelogram) in which the particular construction was embedded.

Discuss (Whole Class):
A whole class discussion may not be necessary, unless there are particular constructions that are difficult for many students. Otherwise, give individual feedback and prompts as students work on this task.

Aligned Ready, Set, Go: Congruence, Construction and Proof 6.14

Ready, Set, Go!

Ready

Topic: Connecting tables with transformations.

For each function find the outputs that fill in the table. Then describe the relationship between the outputs in each table.

1. $f(x) = 3x$ $g(x) = 3x - 5$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

2. $t(x) = 2x$ $h(x) = 2x - 5$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

Relationship between $t(x)$ and $h(x)$:

3. $f(x) = 2x$ $g(x) = 2(x - 3)$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

4. $t(x) = 4x$ $h(x) = 4^{(x-3)}$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

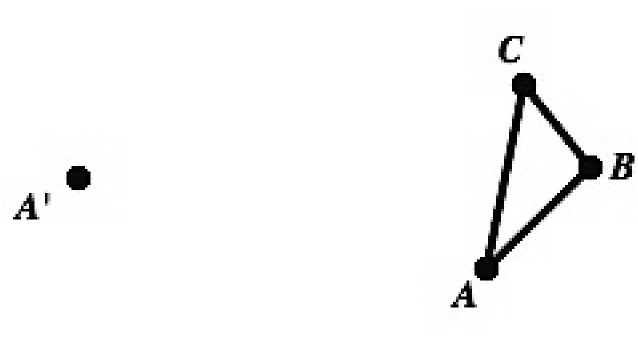
Relationship between $t(x)$ and $h(x)$:

Set

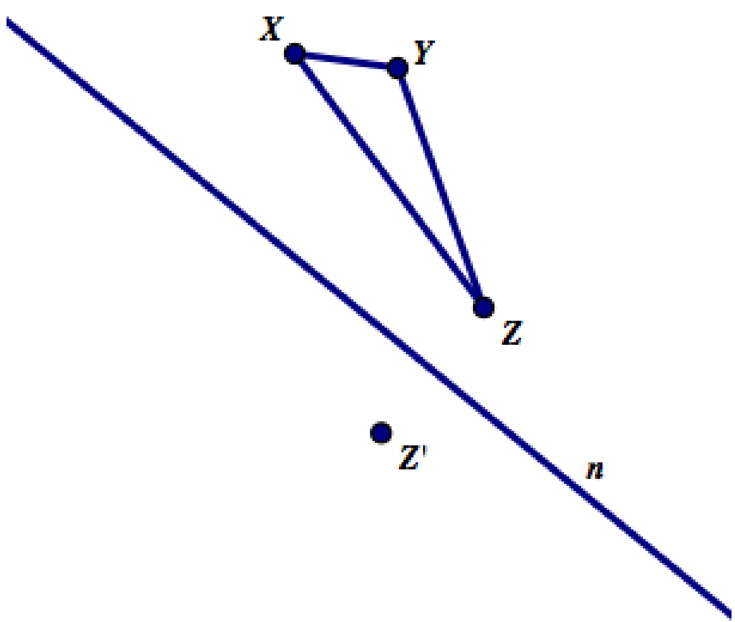
Topic: Constructing transformations

In each problem below use compass and straight edge to construct the transformation that is described.

5. Construct D A'B'C' so that it is a translation of D ABC. (Hint: parallel lines may be useful.)



6. Construct D X'Y'Z' so that it is a reflection of D XYZ over line ***m***.
(Hint: perpendicular lines may be useful.)



Go

Topic: Transformations and triangle congruence.

Determine whether or not the statement is true or false. If true, explain why. If false, explain why not or provide a counterexample.

7. If one triangle can be transformed so that one of its angles and one of its sides coincide with another triangles angle and side then the two triangles are congruent.

8. If one triangle can be transformed so that two of its sides and any one of its angles will coincide with two sides and an angle from another triangle then the two triangles will be congruent.

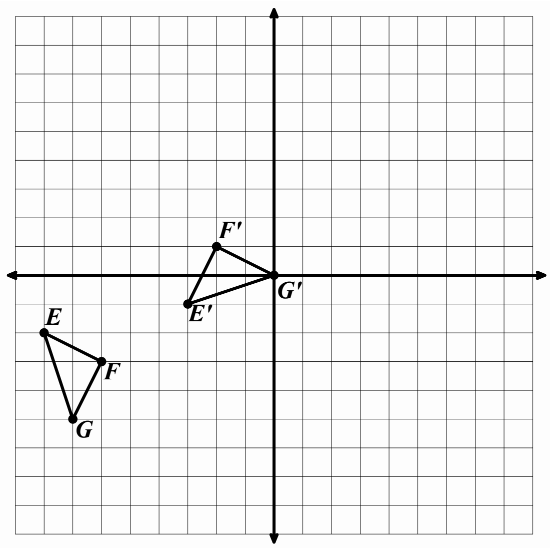
9. If all three angles of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

10. If all three sides of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

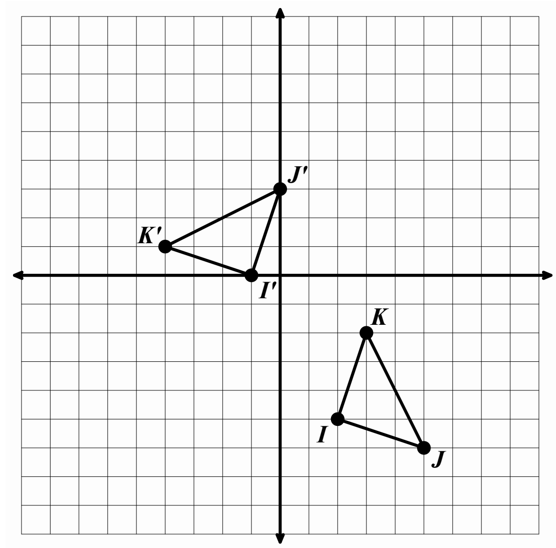
11. For any two congruent polygons there is a sequence of transformations that will transform one of the polygons onto the other.

Find the point of rotation for each of the figures below.

12.

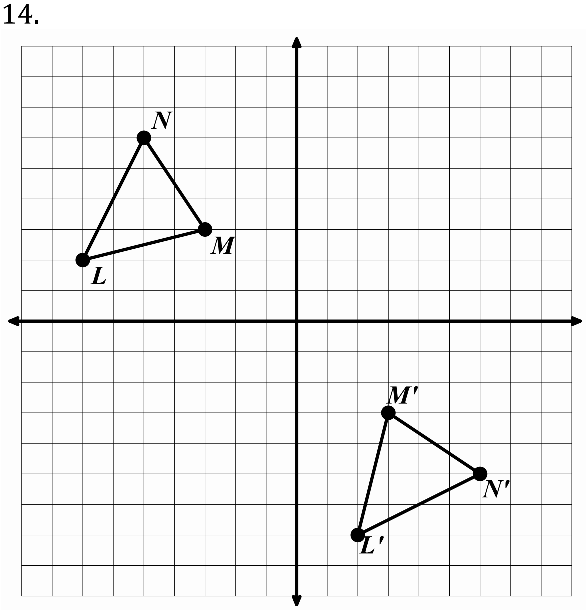


13.



Find the line of reflection for each of the figures drawn below.

14.



15.