Secondary One Mathematics: An Integrated Approach

Module 7 (Honors) Connecting Algebra and Geometry

By

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www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 7 – Connecting Algebra and Geometry

Classroom Task: 7.1 Go the Distance- A Develop Understanding Task
Use coordinates to find distances and determine the perimeter of geometric shapes **(G.GPE.7)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.1

Classroom Task: 7.2 Slippery Slopes – A Solidify Understanding Task
Prove slope criteria for parallel and perpendicular lines **(G.GPE.5)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.2

Classroom Task: 7.3 Prove It! – A Solidify Understanding Task
Use coordinates to algebraically prove geometric theorems **(G.GPE.4)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.3

Classroom Task: 7.4 Training Day– A Solidify Understanding Task
Write the equation f(t)= m(t) + k by comparing parallel lines and finding k **(F.BF.3, F.BF.1, F.IF.9)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.4

Classroom Task: 7.5 Training Day Part II – A Practice Understanding Task
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Ready, Set, Go Homework: Connecting Algebra and Geometry 7.5

Classroom Task: 7.6 Shifting Functions – A Practice Understanding Task
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Ready, Set, Go Homework: Connecting Algebra and Geometry 7.6

Classroom Task: 7.7H The Arithmetic of Vectors – A Solidify Understanding Task
Defining and operating with vectors as quantities with magnitude and direction **(N.VM.1, N.VM.2, N.VM.3, N.VM.4, N.VM.5)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.7H

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Ready, Set, Go Homework: Connecting Algebra and Geometry 7.8H

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Ready, Set, Go Homework: Connecting Algebra and Geometry 7.9H

Classroom Task: 7.10H Solving Systems with Matrices, Revisited – A Solidify Understanding Task
Solving a system of linear equations using the multiplicative inverse matrix **(A.REI.1, UT Honors Standard: Solve systems of linear equations using matrices)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.10H

Classroom Task: 7.11H Transformations with Matrices – A Solidify Understanding Task
Using matrix multiplication to reflect and rotate vectors and images **(N.VM.11, N.VM.12)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.11H

Classroom Task: 7.12H Plane Geometry – A Practice Understanding Task
Solving problems involving quantities that can be represented by vectors **(N.VM.3, N.VM.4a, N.VM.12)**
Ready, Set, Go Homework: Connecting Algebra and Geometry 7.12H

7.1 Go the Distance
A Develop Understanding Task

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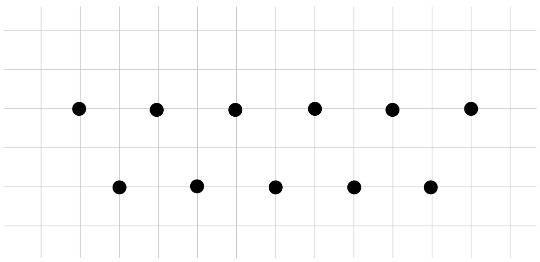
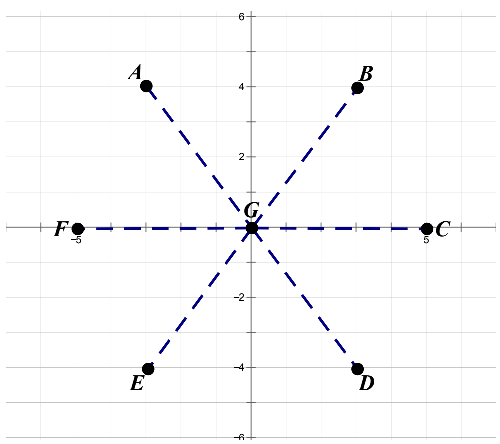
The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one girl in the middle to six other girls. On the grid, their pattern looks like this:

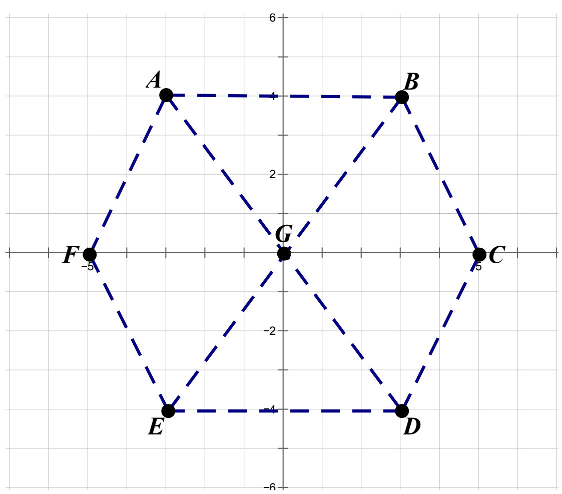


The question the girls have is how long to make the ribbons. Some girls think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

2. Explain how you found the length of each ribbon.

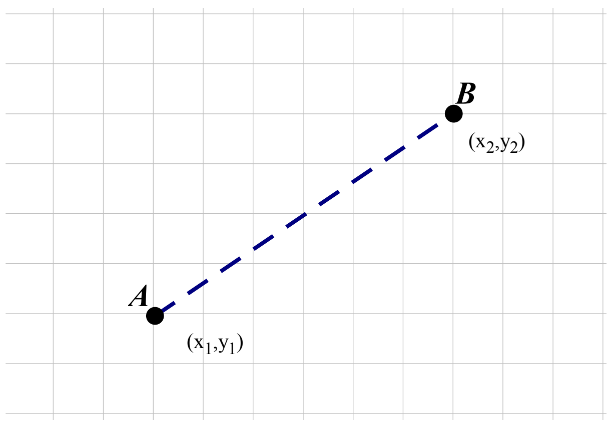
When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1, y_1) and B (x_2, y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, using points A and B.

5. Use the process you came up with in #4 to find the distance between two points located at (-1, 5) and (2, -6)

6. Use you process to find the perimeter of the hexagon pattern shown in #3.

7.1 Go the Distance – Teacher Notes
A Develop Understanding Task

Note to Teachers: Calculators should be available for this task.

Purpose: The purpose of this task is to develop the distance formula, based upon students' understanding of the Pythagorean theorem. In the task, students are asked to calculate distances between points using triangles, and then to formalize the process to the distance formula. At the end of the task, students will use the distance formula to find the perimeter of a hexagon.

Core Standards Focus:

G. GPE Use coordinates to prove simple geometric theorems algebraically.

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Related Standards:

G.GPE.4. Use coordinates to prove simple geometric theorems algebraically.

Launch (Whole Class):

Begin the task by ensuring that student understand the problem situation. Project the drawing in #1 and ask students which ribbon looks longer, $\overline{GB}$ or $\overline{GC}$. Ask how they can test their claims. Some students may suggest using the Pythagorean Theorem to find the length of GB. Ask what they would need to use the Pythagorean Theorem. At this point, set students to work on the task.

Explore (Small Group):

During the exploration period, watch for students that are stuck on the first part of the problem. You may ask them to draw the triangle that will help them to use the Pythagorean Theorem and how they might find the length of the legs of the triangle so they can find the hypotenuse. As you monitor student thinking on #3, watch for students who are noticing how to find the length of the legs of the triangle when it has been moved away from the origin. Look for students that have written a good step-by-step procedure for #4. It will probably be difficult for them to use the symbols appropriately, so watch for words that appropriate describe the procedure.

Discuss (Whole Class):

Start the discussion by having a group show how they found the length of $\overline{BC}$ in problem #3. Move next to #4 and have a group that has written a step by step procedure. Try walking through the group's procedure with the numbers from problem #3 and see if it gives the appropriate answer. If necessary, work with the class to modify the procedure so that the list of steps is correct. Once the steps are outlined in words, go through the steps using points A (x_1, y_1) and B (x_2, y_2) and formalize the procedures with the symbols. An example:

Steps in words	Steps in symbols
Find the length of the horizontal leg of the triangle	$x_2 - x_1$
Find the length of the vertical leg of the triangle	$y_2 - y_1$
Use the Pythagorean Theorem to write an equation	$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$
Simplify the left side of the equation	$(x_2 - x_1)^2 + (y_2 - y_1)^2 = c^2$
Take the square root of both sides of the equation	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{c^2}$
Simplify	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = c$ (c being the desired distance)

After going through this process, you should end with the distance formula. Apply the formula using the points in #5.

Aligned Ready, Set, Go: ***Connecting Algebra and Geometry 7.1***

Name: Connecting Algebra and Geometry 7.1

Ready, Set, Go!

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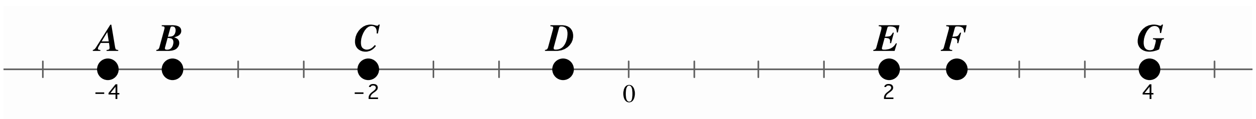
Ready

Topic: Finding the distance between two points

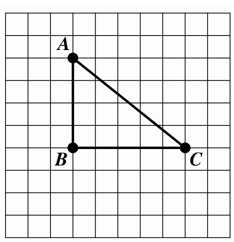
Use the number line to find the distance between the given points. (The notation AB means the distance between points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8.



a. Find AB

b. Find BC

c. Find AC

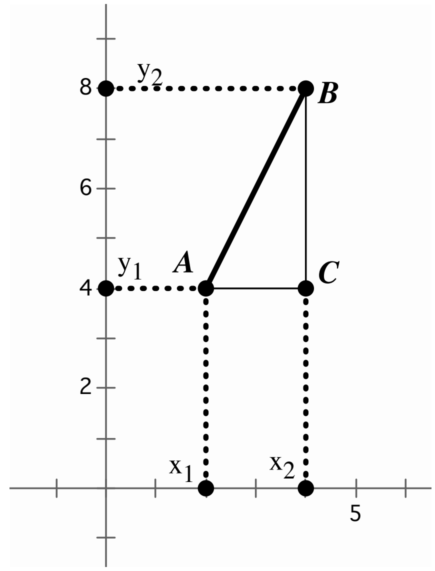
9. Why is it easier to find the distance between points A and B and points B and C than it is to find the distance between A and C?

10. Explain how to find the distance between points A and C.

Name: Connecting Algebra and Geometry 7.1

Set

Topic: Slope triangles and the distance formula.



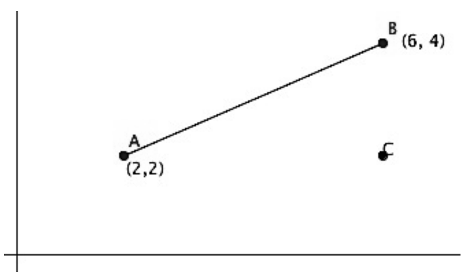
Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2 - y_1}{x_2 - x_2}$ *where m is the slope.*

11a. What does the value $(y_2 - y_1)$ tell you?

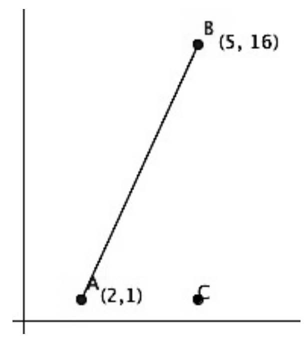
b. What does the value $(x_2 - x_1)$ tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and performing the Pythagorean Theorem. In this exercise try to develop a more efficient method of finding the length of a line segment by using the meaning of $(y_2 - y_1)$ *and* $(x_2 - x_1)$ *combined with the Pythagorean Theorem.*

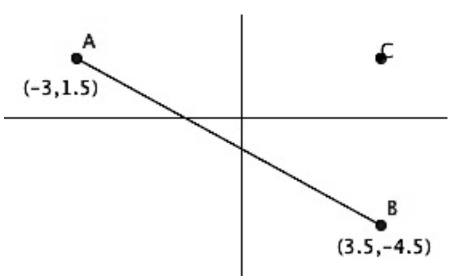
12. Find AB.



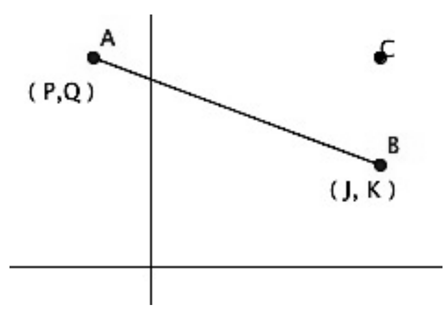
13. Find AB.



14. Find AB.



15. Find AB.



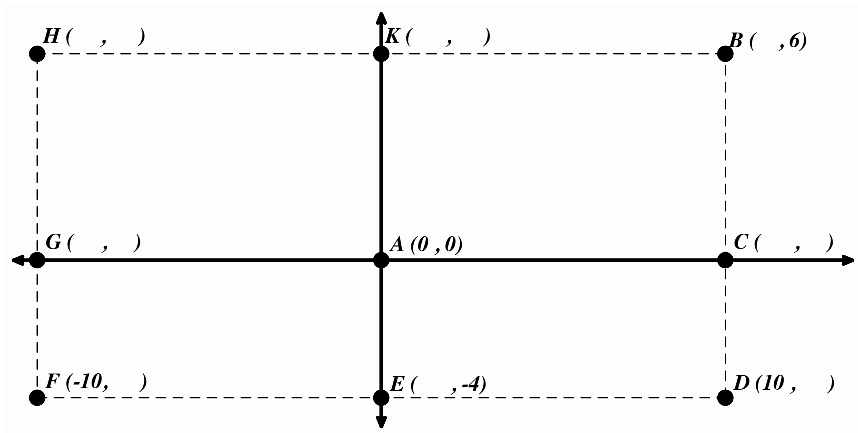
Name: Connecting Algebra and Geometry 7.1

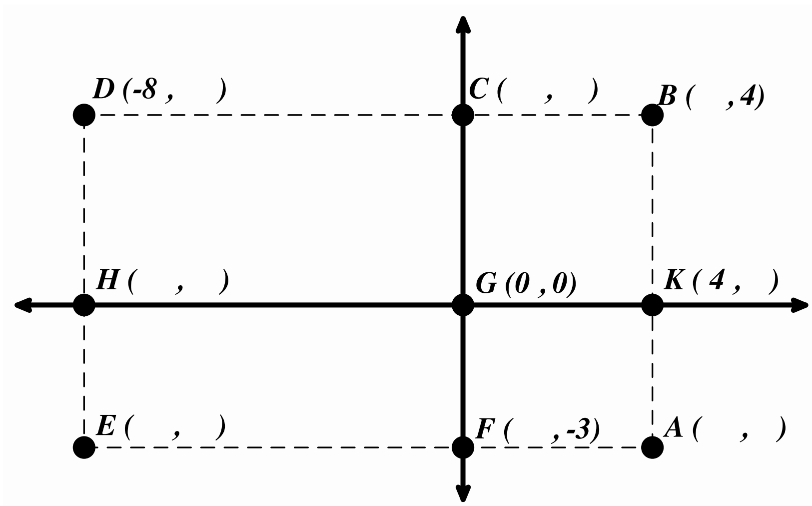
Go Topic: Rectangular coordinates

Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

16a. Find HB

b. Find BD





17a. Find DB

b. Find CF

Need Help? Check out these related videos:

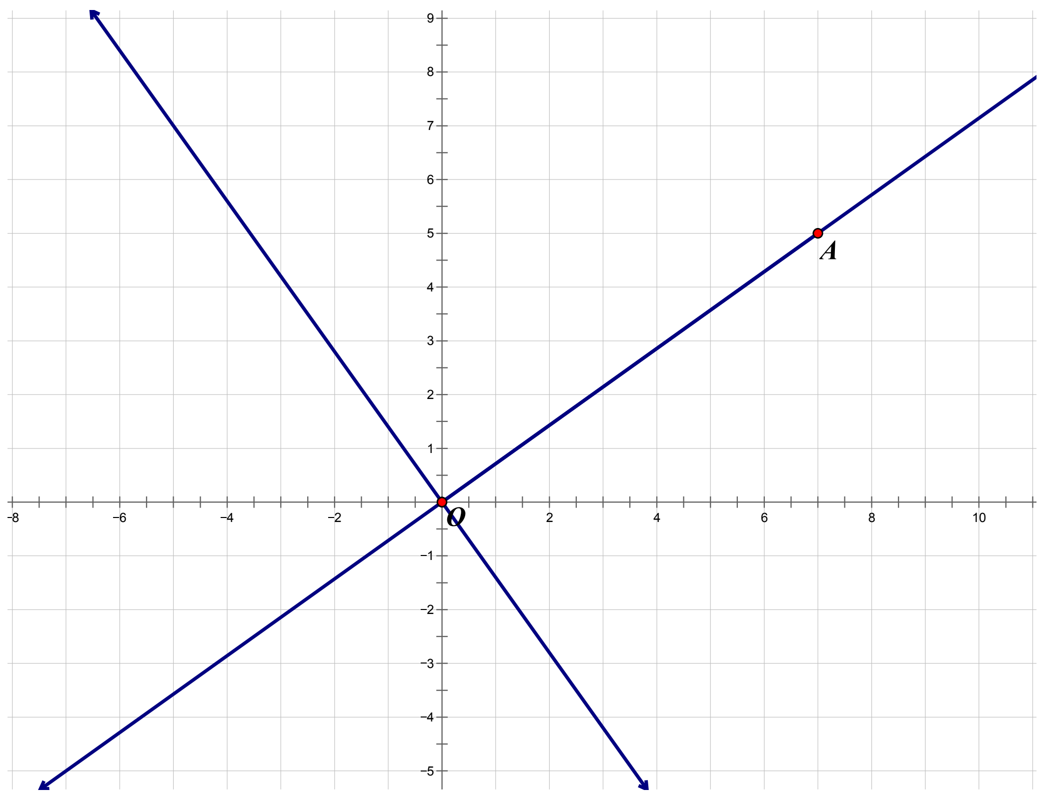
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/the-coordinate-plane

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/distance-formula

7.2 Slippery Slopes

A Solidify Understanding Task

While working on "Is It Right?" in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

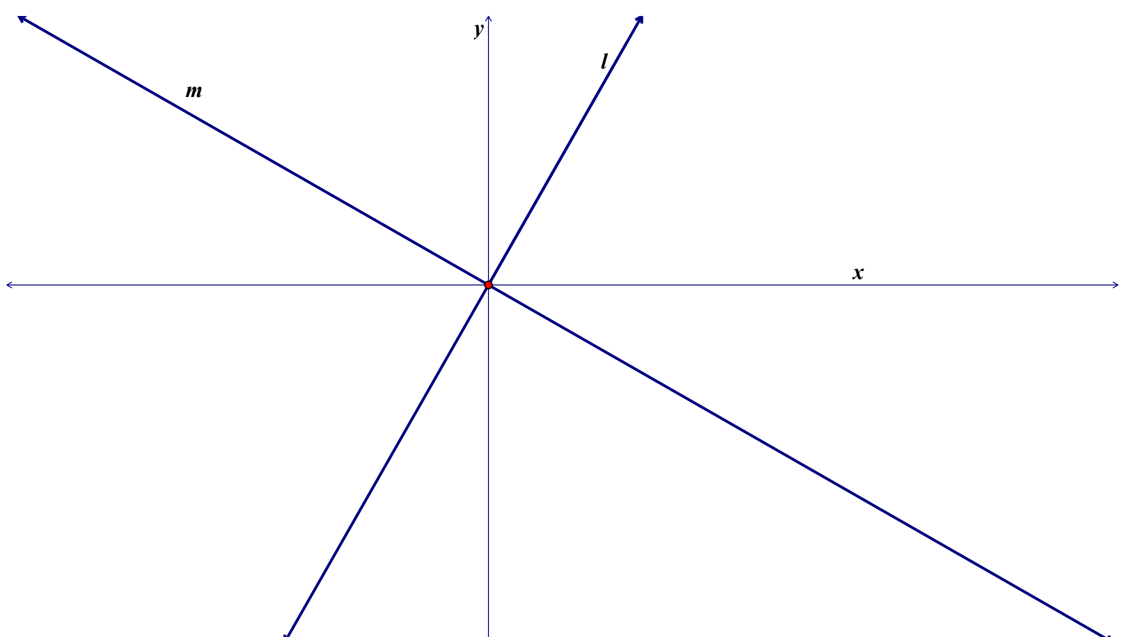
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with this theorem.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle.

6. What is the slope of line *l*?

Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

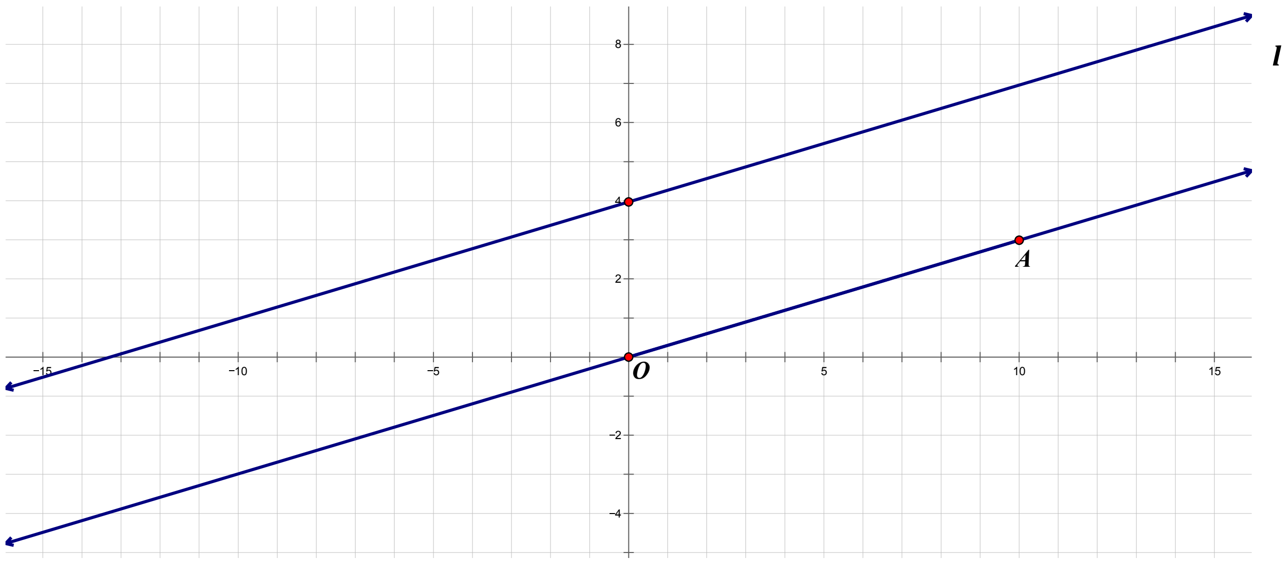
10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

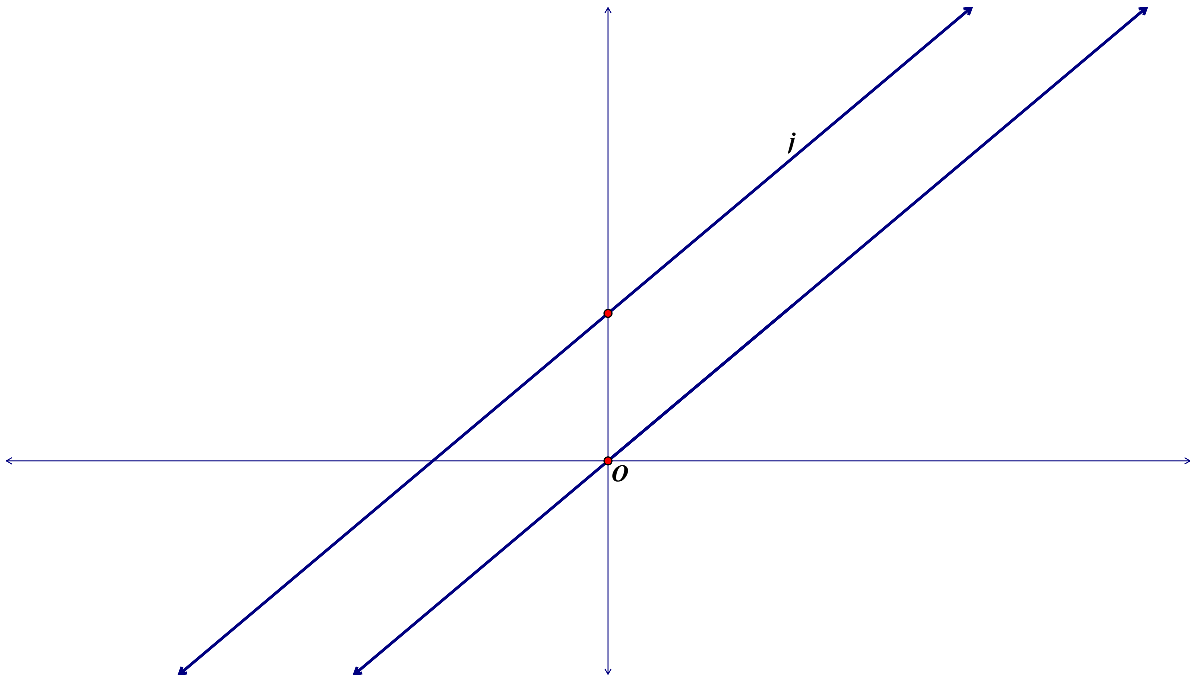
Draw the slope triangle from point A. What is the slope of $\overleftrightarrow{OA}$?



What translation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto line l?

What must be true about the slope of line l? Why?

Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

7.2 Slippery Slopes – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to prove that parallel lines have equal slopes and that the slopes of perpendicular lines are negative reciprocals. Students have used these theorems previously, without proof. The proofs use the ideas of slope triangles, rotations, and translations. Both proofs are preceded by a specific case that demonstrates the idea before students are asked to follow the logic using variables and thinking more generally.

Core Standards Focus:

G. GPE Use coordinates to prove simple geometric theorems algebraically.

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Related Standards: G.CO.4, G.CO.5

Launch (Whole Class):

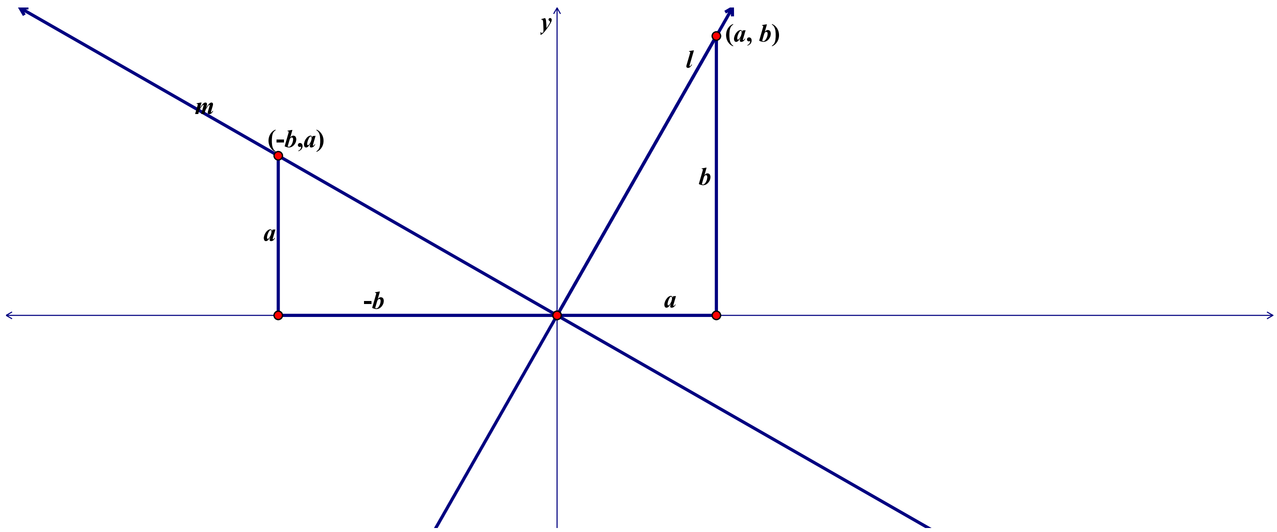
If students haven't been using the term "slope triangle", start the discussion with a brief demonstration of slope triangles and how they show the slope of the line. Students should be familiar with performing a 90 degree rotation from the previous module, so begin the task by having students work individually on questions 1, 2, 3, and 4. When most students have drawn a conclusion for #4, have a discussion of how they know the two lines are perpendicular. Since the purpose is to demonstrate that perpendicular lines have slopes that are negative reciprocals, emphasize that the reason that we know that the lines are perpendicular is that they were constructed based upon a 90 degree rotation.

Explore (Small Group):

The proof that the slopes of perpendicular lines are negative reciprocals follows the same pattern as the example given in the previous problem. Monitor students as they work, allowing them to select a point, label the coordinates and then the sides of the slope triangles. Refer students back to the previous problem, asking them to generalize the steps symbolically if they are stuck. When students are finished with questions 6-12, discuss the proof as a whole group and then have students complete the task.

Discuss (Whole Class):

The setup for the proof is below:



The slope of line l is $\frac{b}{a}$ and the slope of line m is $\frac{a}{-b}$ or - $\frac{a}{b}$. The product of the two slopes is -1, therefore they are negative reciprocals. If the lines are translated so that the intersection is not at the origin, the slope triangles will remain the same. Discuss with the class how questions 6-12 help us to consider all the possible cases, which is necessary in a proof.

After students have finished the task, go through the brief proof that the slopes of parallel lines are equal.

Aligned Ready, Set, Go: ***Connecting Algebra and Geometry 7.2***

Name: Connecting Algebra and Geometry | 7.2

Ready, Set, Go!

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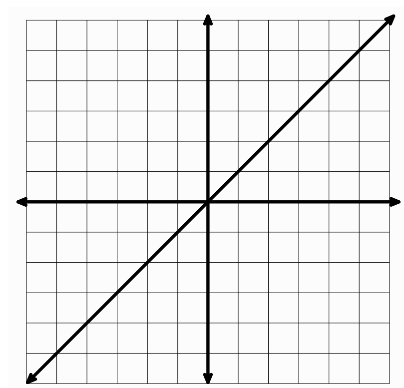
Ready

Topic: Graphing lines.

The graph at the right is of the line $f(x) = x$.



1a. On the same grid, graph a parallel line that is 3 units above it.

b. Write the equation of the new line. ___________________________

c. Write the y-intercept of the new line as an ordered pair.

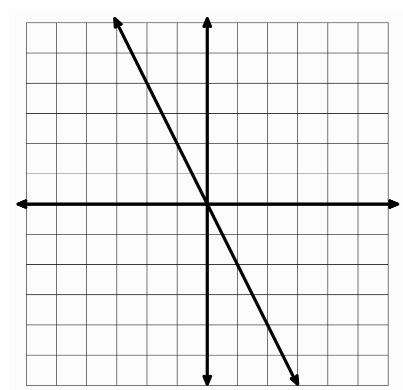
d. Write the x-intercept of the new line as an ordered pair.

e. Write the equation of the new line in point-slope form using the y-intercept.

f. Write the equation of the new line in point-slope form using the x-intercept.

g. Explain in what way the equations are the same and in what way they are different.

The graph at the right is of the line $f(x) = -2x$.



2a. On the same grid, graph a parallel line that is 4 units below it.

b. Write the equation of the new line. ___________________________

c. Write the y-intercept of the new line as an ordered pair.

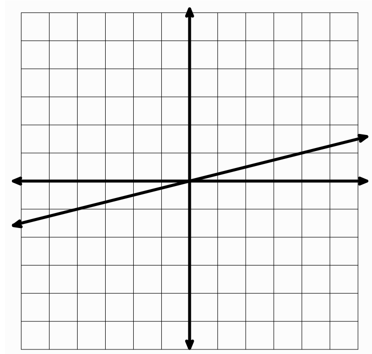
d. Write the x-intercept as an ordered pair.

e. Write the equation of the new line in point-slope form using the y-intercept

f. Write the equation of the new line in point-slope form using the x-intercept.

g. Explain in what way the equations are the same and in what way they are different.

Name: Connecting Algebra and Geometry 7.2



The graph at the right is of *f(x)* = ¼ x

3a. Graph a parallel line 2 units below.

b. Write the equation of the new line.

c. Write the y-intercept as an ordered pair.

d. Write the x-intercept as an ordered pair.

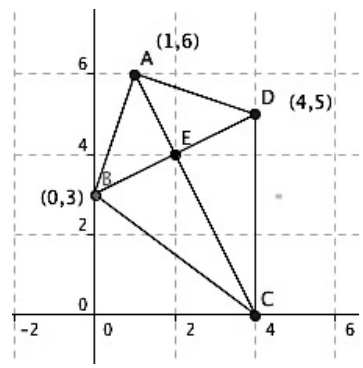
e. Write the equation of the new line in point-slope form using the y-intercept

f. Write the equation of the new line in point-slope form using the x-intercept

g. Explain in what way the equations are the same and in what way they are different.

Set

Topic: Verifying and Proving Geometric Relationships



The quadrilateral at the right is called a **kite.**

Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically.
(A symbol may be used more than once.)

$\cong$ $\perp$ $\parallel$ $<$ $>$ $=$

Proof

4. $\overline{BC}$ ________ $\overline{DC}$ ______________________________

5. $\overline{BD}$ ________ $\overline{AC}$ ______________________________

6. $\overline{AB}$ ________ $\overline{BC}$ ______________________________

7. ΔABC _______ ΔADC ____

____

8. $\overline{BE}$ ________ $\overline{ED}$ ____

____

9. $\overline{AE}$ ________ $\overline{ED}$ ____

____

10. $\overline{AC}$ _________ $\overline{BD}$ ____

____

Go

Topic: Writing equations of lines.

Write the equation of the line in standard form using the given information.

11. Slope: -¼ point (12, 5)

12. A (11, -3) , B (6, 2)

13. x-intercept: -2, y-intercept: -3

14. All x values are -7, y can be anything

15. Slope: ½ x-intercept: 5

16. E (-10, 17) , G (13, 17)

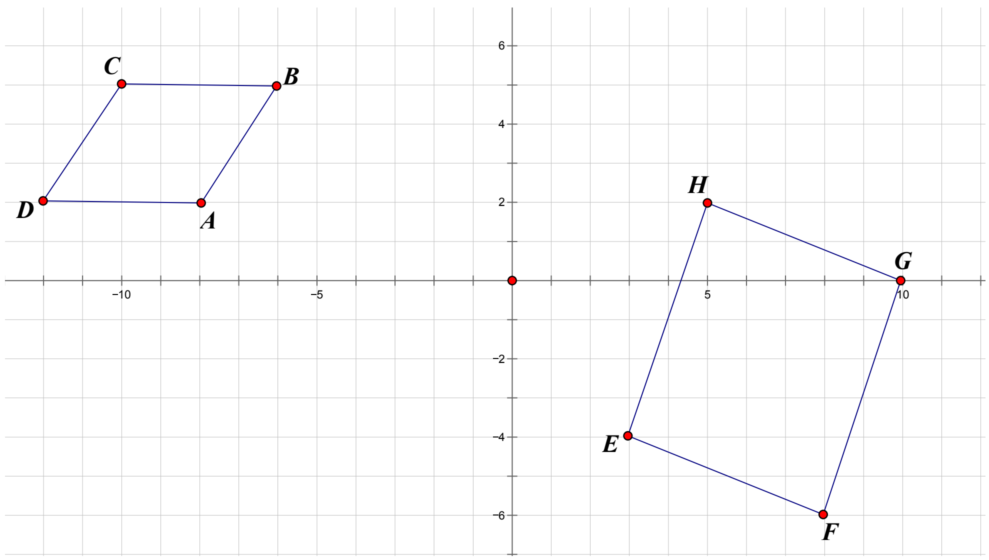
Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-using-x-and-y-intercepts

www.flickr.com/photos/safari_vacation

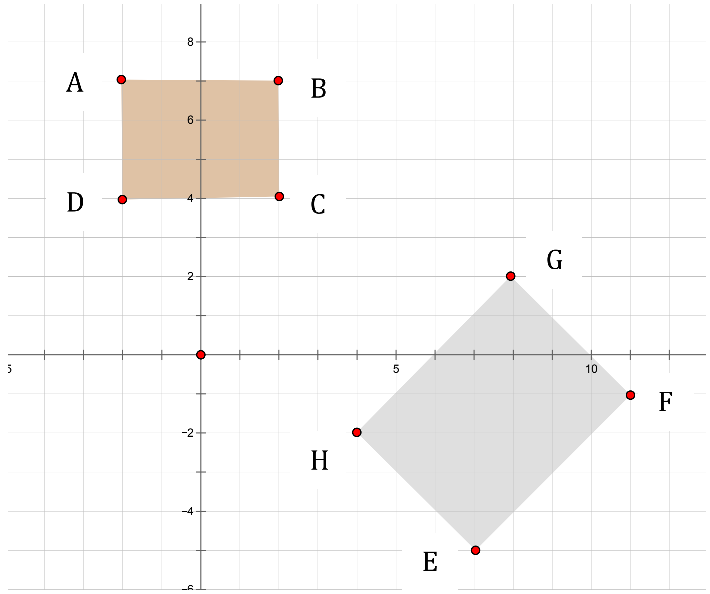
7.3 Prove It!
A Solidify Understanding Task

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.



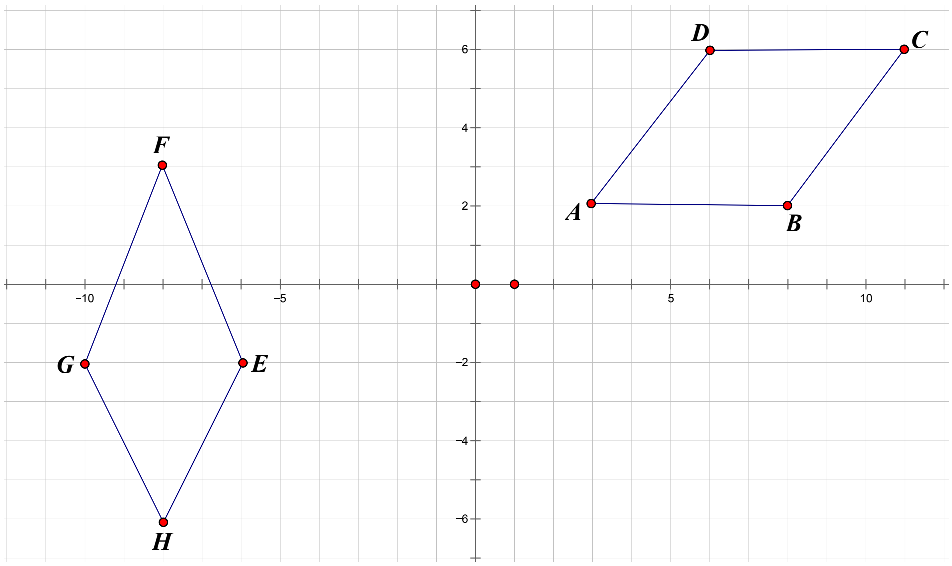
Is ABCD a parallelogram? Explain how you know.

Is EFGH a parallelogram? Explain how you know.



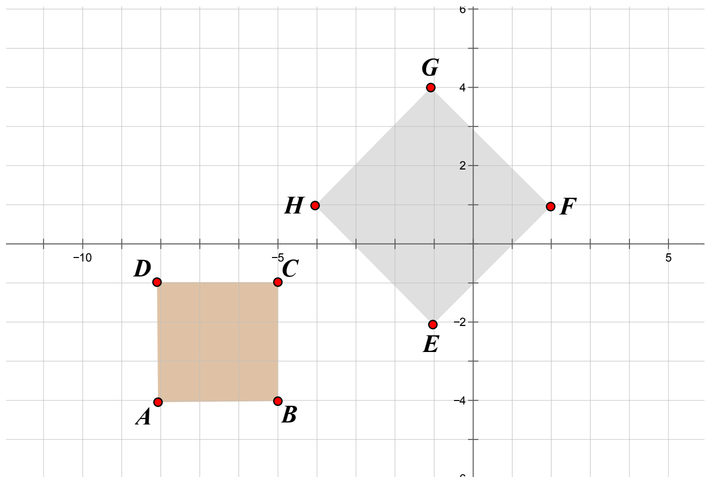
Is ABCD a rectangle? Explain how you know.

Is EFGH a rectangle? Explain how you know.



Is ABCD a rhombus? Explain how you know.

Is EFGH a rhombus? Explain how you know.



Is ABCD a square? Explain how you know.

Is EFGH a square? Explain how you know.

7.3 Prove It! – Teacher Notes

A Solidify Understanding Task

Purpose:

The purpose of this task is to solidify student understanding of quadrilaterals and to connect their understanding of geometry and algebra. In the task they will use slopes and distance to show that particular quadrilaterals are parallelograms, rectangles, rhombi or squares. This task will also strengthen student understanding of justification and proof, and the need to put forth a complete argument based upon sound mathematical reasoning.

Core Standards Focus:

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.

Related Standards: G.GPE.5, G.GPE.7

Launch (Whole Class):

Launch the task with a discussion of what students know about the properties of quadrilaterals, for instance that a rhombus has two pairs of parallel sides (making it a parallelogram), congruent sides, and perpendicular diagonals. Discuss what you would need to show to prove a claim that a figure is a particular quadrilateral. For instance, it is not enough to show that a shape is a rhombus by showing that the two pairs of sides are parallel, but it would be enough to show that the diagonals are perpendicular. Why?

Explore (Small Group):

Monitor students as they work. It may be helpful to recognize that each set of problems is set up so there is a simple case and a more complicated case. The simple case is designed to help students get ideas for how to prove the more complicated case. Keep track of the various approaches that students use to verify their claims and press them to organize their work so that it communicates to an outside observer. Students should be showing sides or diagonals are parallel or perpendicular using the slope properties, and the distance formula to show that sides or diagonals are congruent. You may also see students try to show one figure to be a particular quadrilateral and then use transformations to show that the second figure is the same type, particularly in the case of the squares. Select one group of students that have articulated a clear argument for each type of quadrilateral. Be sure to also select a variety of approaches so that students see several methods in the discussion.

Discuss (Whole Class):

The discussion should proceed in the same order as the task, with different groups demonstrating their strategies for one parallelogram, one rectangle, one rhombus, and one square. Select the shape that does not have sides that on a grid line, so that students are demonstrating the more challenging cases. One recommended sequence for the discussion would be:

1. Parallelogram EFGH demonstrated by showing two pairs of parallel sides using slopes.
2. Rectangle EFGH demonstrated by using the distance formula to show that the diagonals are congruent.
3. Showing that quadrilateral EFGH is not a rhombus because the sides are not congruent.
4. Square EFGH demonstrated using a translation and dilation of square ABCD. (They must demonstrate that ABCD is a square before using this strategy.)

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.3

Name: Connecting Algebra and Geometry | 7.3

www.flickr.com/photos/safari_vacation

Ready, Set, Go!

Ready

Topic: Tables of value

Find the value of *f (x)* for the given domain. Write *x* and *f (x)* as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

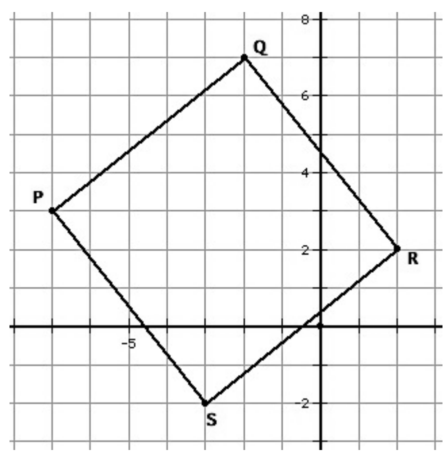
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

Set

Topic: Characteristics of rectangles and squares

4a. Is the figure below a rectangle? (Justify your answer)

b. Is the figure a square? (Justify your answer)

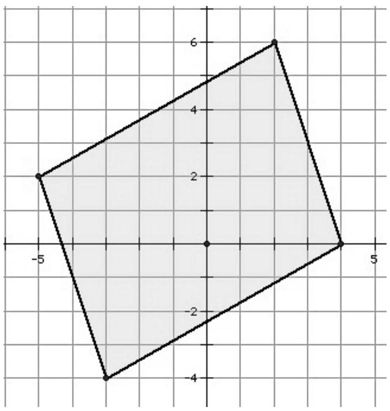


Name: Connecting Algebra and Geometry | 7.3

Go

Find the perimeter of each figure below. Round to the nearest hundredth.

5.



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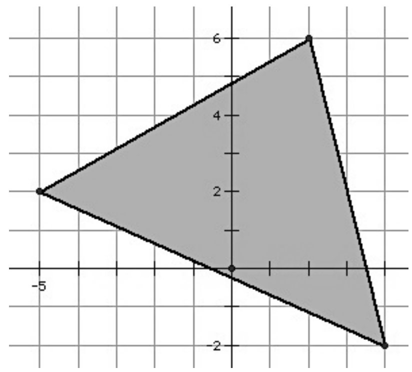

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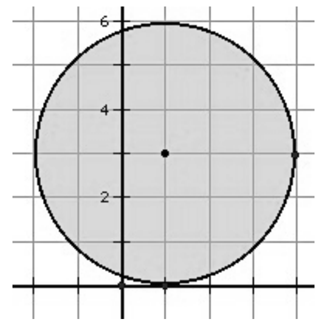

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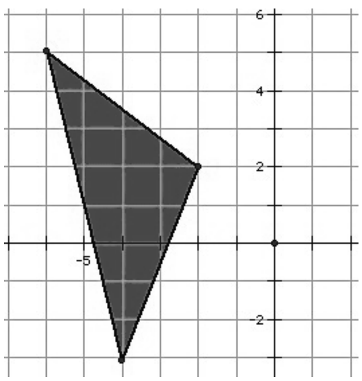

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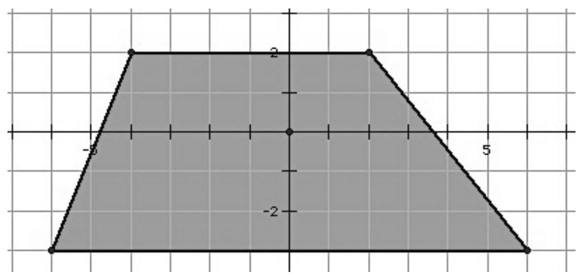

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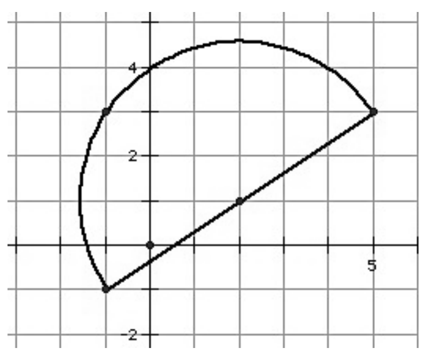


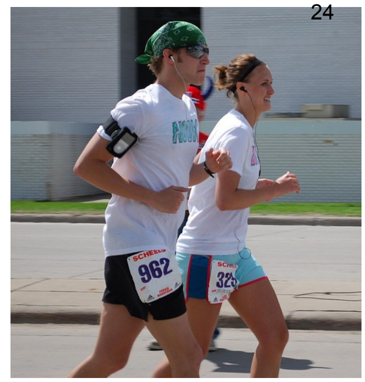
Need Help? Check out these related videos:

http://www.khanacademy.org/math/geometry/basic-geometry/v/perimeter-and-area-of-a-non-standard-polygon

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/distance-formula

7.4 Training Day

A Develop Understanding Task



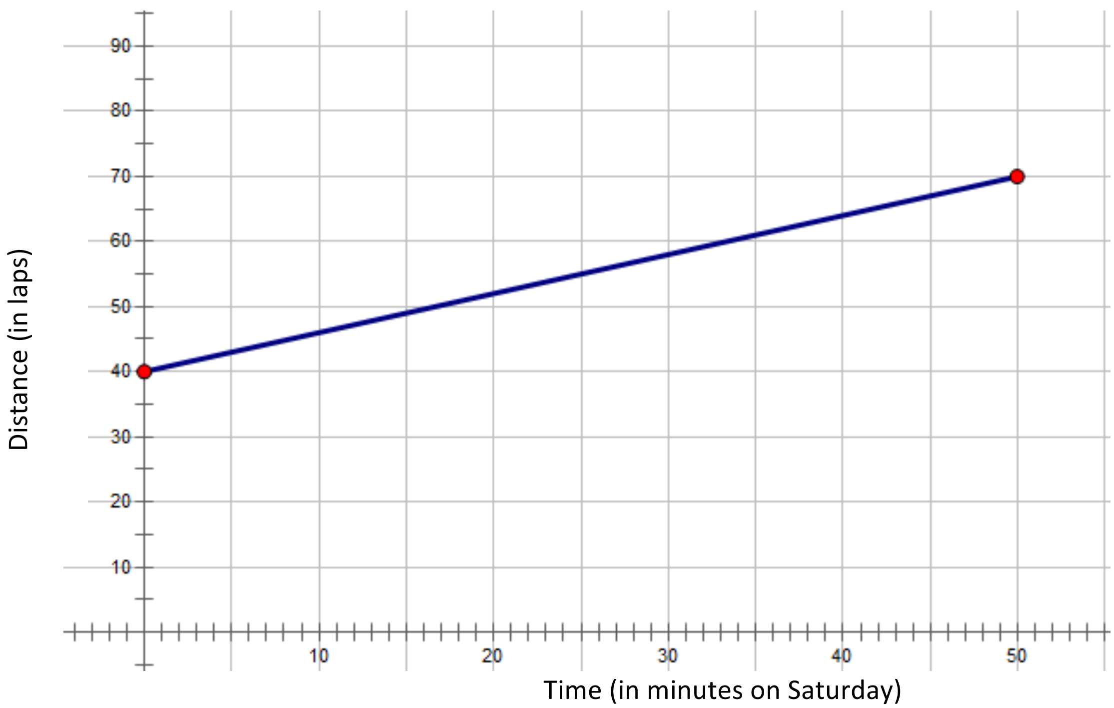
http://www.flickr.com/photos/fargomoorheadcvb/

Fernando and Mariah are training for six weeks to run in the Salt Lake half- marathon. To train, they run laps around the track at Eastland High School. Since their schedules do not allow them to run together during the week, they each keep a record of the total number of laps they run throughout the week and then always train together on Saturday morning. The following are representations of how each person kept track of the total number of laps that they ran throughout the week plus the number of laps they ran on Saturday.

Fernando's data:

Time (in minutes on Saturday)	0	10	20	30	40	50
Distance (in laps)	60	66	72	78	84	90

Mariah's data:



What observations can be made about the similarities and differences between the two trainers?

1. Write the equation, *m(t)*, that models Mariah's distance.

2. Fernando and Mariah both said they ran the same rate during the week when they were training separately. Explain in words how Fernando's equation is similar to Mariah's. Use the sentence frame: The rate of both runners is the same throughout the week, however, Fernando ____.

3. In mathematics, sometimes one function can be used to build another. Write Fernando's equation, *f(t)*, by starting with Mariah's equation, *m(t)*.

   f(t) =

4. Use the mathematical representations given in this task (table and graph) to model the equation you wrote for number 3. Write in words how you would explain this new function to your class.

7.4 Training Day Teacher Notes

A Develop Understanding Task

Purpose: Students have had a lot of experience with linear functions and their relationships. They have also become more comfortable with function notation and features of functions. In this task, students first make observations about the rate of change and the distance traveled by the two runners. Using their background knowledge of linear functions, students start to surface ideas about vertical translations of functions and how to build one function from another.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Note: Focus on vertical translations of graphs of linear and exponential functions.)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.1 Write a function that describes a relationship between two quantities.★

Related Standards: F.BF.1b, F.IF.1, F.IF.2, A.CED.3

Launch (Whole Class):

To begin this task, read the scenario as a whole group, then ask students to write their answer to the question: “What observations can be made about the similarities and differences between the two trainers?” After a couple of minutes, have students share their observations with a partner. Listen for students to discuss the meaning of the slope and the y-intercept in both situations. If needed, ask the group questions to clarify that the y-intercept is the number of laps each person runs during the week before they meet on Saturday morning and that the slope is the same in both situations. Since the purpose of the lesson is to see how one function can be built from another similar function, these are the two most important ideas to come out of launch conversation.

Explore (Small Group):

As you monitor, listen for student reasoning about the relationship between the amount of laps run by Mariah and Fernando. Encourage students to explain their reasoning to each other using prior academic vocabulary while working through solutions to problems. If students are incorrect in their thinking, be redirect their thinking by asking them to explain how their function relates to the situation.

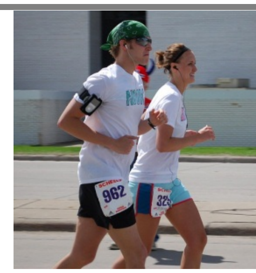
Discuss (Whole Class):

During the monitoring phase, select students to share their results to strengthen the whole group understanding of the relationship between the 'original function' (*m(t)*) and the 'transformed function' (*f(t)*). You may wish to start the whole group discussion by choosing someone who has graphically shown Fernando and Mariah's graph on the same axes. Have this student share the relationship between the two graphs and press to bring out that, at any given time, *f(t)* is always '20 laps more' than *m(t).* Likewise, have a student share who can explain the relationship using a table. After both representations (table and graph) are shown, ask the whole group to see what connections they can make between the equation, table, and graph.

Aligned Ready, Set, Go: Connecting 7.4

Name: Connecting Algebra and Geometry | 7.4

Ready, Set, Go!

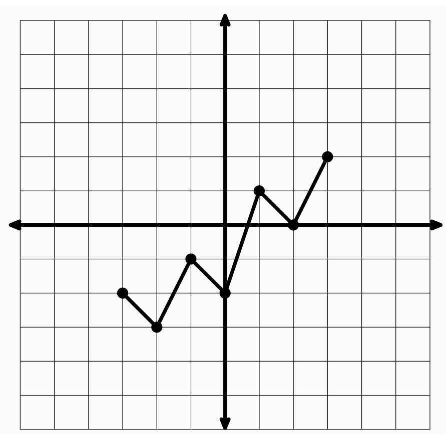


http://www.flickr.com/photos/fargomoorheadcvb

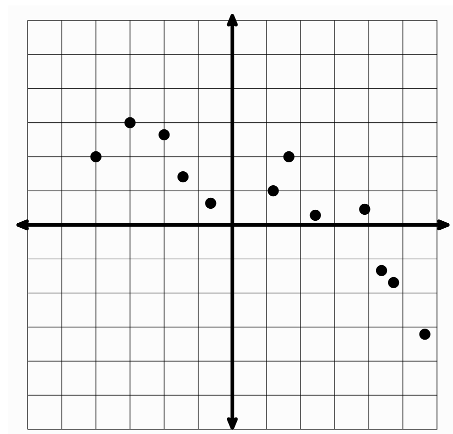
Ready

Topic: Vertical transformations of graphs

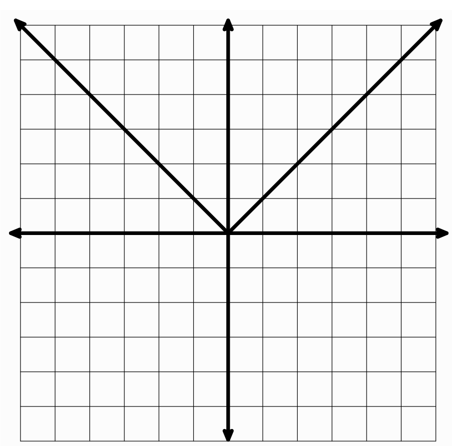
1. Use the graph below to draw a new graph that is translated up 3 units.



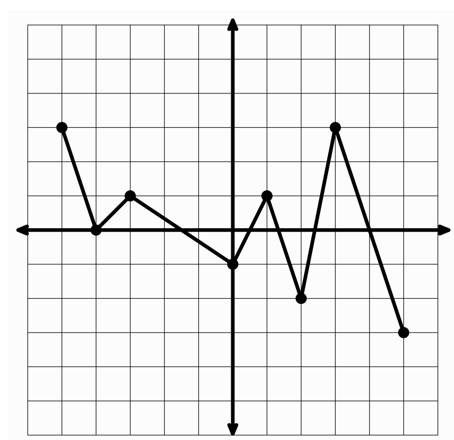
2. Use the graph below to draw a new graph that is translated down 1 unit.



3. Use the graph below to draw a new graph that is translated down 4 units.



4. Use the graph below to draw a new graph that is translated down 3 units.



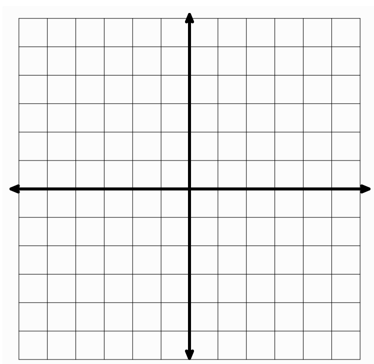
Name: Connecting Algebra and Geometry | 7.4

Set

You are given the equation of *f(x)* and the transformation *g(x) = f(x) + k.* Graph both *f (x)* and *g (x) and* the linear equation for *g(x)* below the graph.

5. $f(x) = 2x - 4$

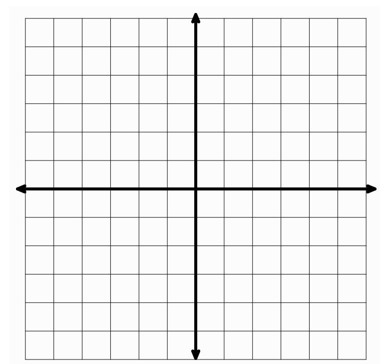
$g(x) = f(x) + 3$



$g(x)$ = ______________________

6. $f(x) = 0.5x$

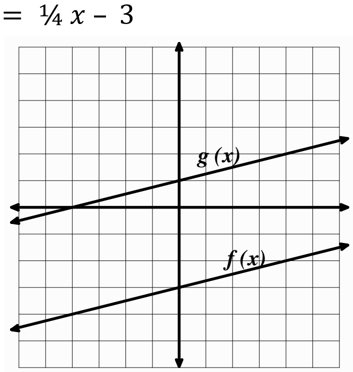
$g(x) = f(x) - 3$



$g(x)$ = ______________________

Based on the given graph, write the equation of *g(x)* in the form of *g(x) = f(x) + k.* Then simplify the equation of *g(x)* into slope-intercept form. The equation of *f(x)* is given.

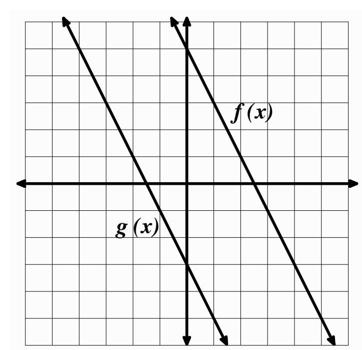
7. $f(x) = \frac{1}{4}x - 3$



a. $g(x)$ = ______________________
Translation form

b. $g(x)$ = ______________________
Slope-Intercept form

8. $f(x) = -2x + 5$



a. $g(x)$ = ______________________
Translation form

b. $g(x)$ = ______________________
Slope-Intercept form

Go

9. Fernando and Mariah are training for a half marathon. The chart below describes their workout for the week just before the half marathon. If four laps are equal to one mile, and if there are 13.1 miles in a half marathon, do you think Mariah and Fernando are prepared for the event? Describe how you think each person will perform in the race. Include who you think will finish first and what each person's finish time will be. Use the data to inform your conclusions and to justify your answers.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Fernando: Distance (in laps)	34	45	52	28	49	36
Time per day (in minutes)	60	72	112	63	88	58
Mariah: Distance (in laps)	30	48	55	44	38	22
Time per day (in minutes)	59	75	119	82	70	45

7.5 Training Day Part II

A Practice Understanding Task

http://www.flickr.com/photos/pdgoodman

Fernando and Mariah continued training in preparation for the half marathon. For the remaining weeks of training, they each separately kept track of the distance they ran during the week. Since they ran together at the same rate on Saturdays, they took turns keeping track of the distance they ran and the time it took. So they would both keep track of their own information, the other person would use the data to determine their own total distance for the week.

Week 2: Mariah had completed 15 more laps than Fernando before they trained on Saturday.

a. Complete the table for Mariah.

Time (in minutes on Saturday)	0	10	20	30	40	50	60
Fernando: Distance (in laps)	50	56	62	68	74	80	86
Mariah: Distance (in laps)							

b. Write the equation for Mariah as a transformation of Fernando. Equation for Mariah: $m(t) = f(t)$ ________

Week 3: On Saturday morning before they started running, Fernando saw Mariah's table and stated, "My equation this week will be $f(t) = m(t) + 30$."

a. What does Fernando's statement mean?
b. Based on Fernando's translated function, complete the table.

Time (in minutes on Saturday)	0	20	40	60	70
Fernando: Distance (in laps)					
Mariah: Distance (in laps)	45	57	69	81	87

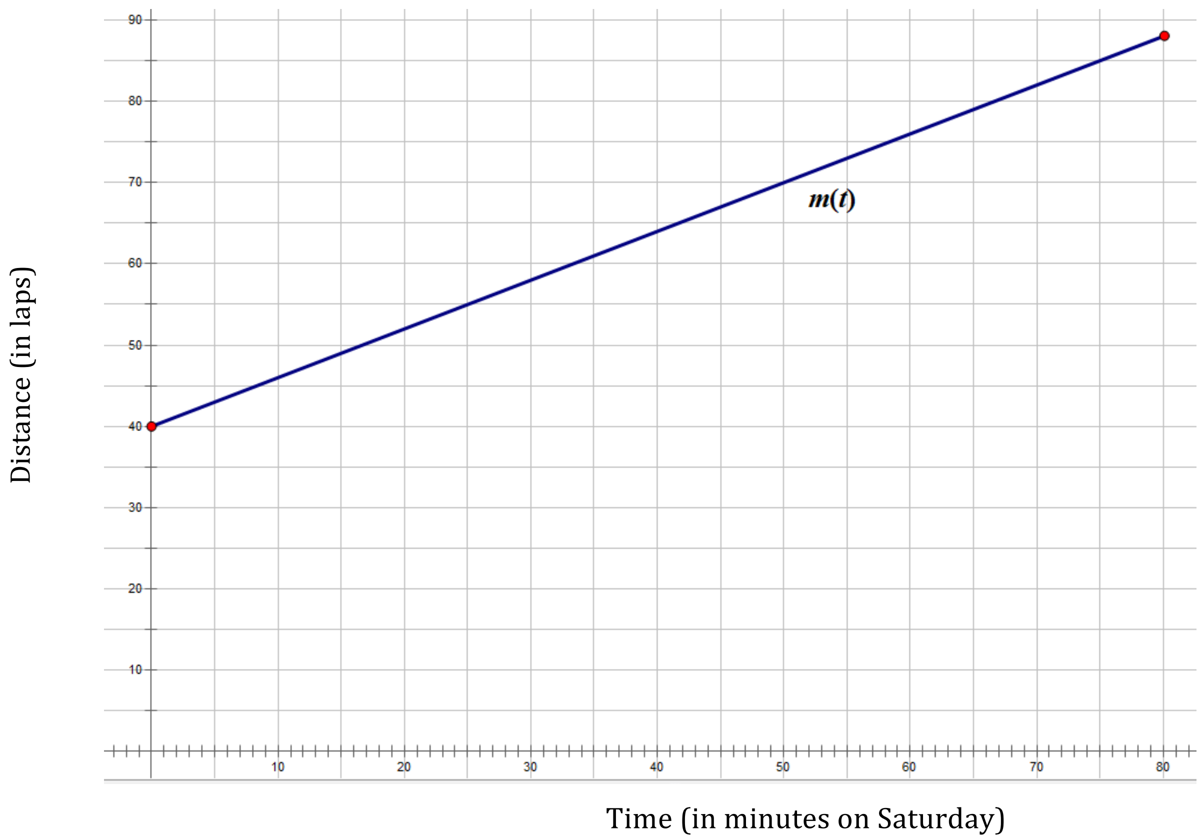
c. Write the equation for both runners:

d. Write the equation for Mariah, transformed from Fernando.

e. What relationship do you notice between your answers to parts c and d?

Week 4: The marathon is only a couple of weeks away!

a. Use Mariah's graph to sketch *f(t)*. *f(t) = m(t)* – 10



b. Write the equations for both runners.
c. What do you notice about the two graphs? Would this always be true if one person ran "*k*" laps more or less each week?

Week 5: This is the last week of training together. Next Saturday is the big day. When they arrived to train, they noticed they had both run 60 laps during the week.

a. Write the equation for Mariah given that they run at the same speed that they have every week.
b. Write Fernando's equation as a transformation of Mariah's equation.

What conjectures can you make about the general statement: "*g(x) = f(x)+k*" when it comes to linear functions?

7.5 Training Day Part 2 – Teacher Notes
A Solidify Understanding Task

Purpose: Students will solidify their understanding of vertical transformations of linear functions in this task. Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form y = $f(x)+k$ represents a vertical translation, with the output values changing while the input values stay the same.
- Understanding that a vertical shift of a linear function results in a line parallel to the original.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Note: Focus on vertical translations of graphs of linear and exponential functions.)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.1 Write a function that describes a relationship between two quantities.★

Related Standards: F.IF.1, F.IF.2 , A.CED.3

<u>Launch (Whole Class):</u>

Read the initial story and ask students to clarify what this means. If not stated, clarify that the two runners are going the same rate each time they run together on Saturday morning. Students should be able to get started on this task without additional support, since it is similar in nature to the work they did on "Training Day".

Explore (Small Group):

Watch for students who confuse input/output values as well as for students who struggle with making sense of using function notation in the first two problems (weeks 2 and 3). Listen for students who make the connection that the 'shift' is about 'adjusting the output values' and are visually showing the connection to the table and the equations (the distance between the number of laps of the two runners is the same in the table as it is in the "shift" or "+K" value).

Week 4 has students visually see the shift of "k" units with a graphical representation. This is another way students see that each output value is exactly *k* units away from the other function. It can be noted that the lines are parallel, however, make sure the discussion talks about the distance of the output values (since with future functions, they will not be 'parallel lines'… but that they do maintain a distance of K units.

Discuss (Whole Class):

The goal of this whole group discussion is to highlight the different ways to see vertical translations of linear functions. Have different students go over each week of training, showing how the vertical shift of one function relates to the other. For each week, have students show connections between the context, the mathematical representation, and the transformation function.

Aligned Ready, Set, Go: Connections 7.5

Name: Connecting Algebra and Geometry | 7.5

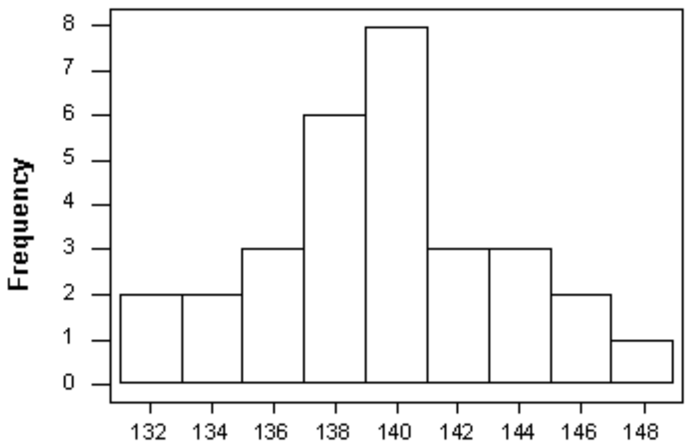
http://www.flickr.com/photos/pdgoodman

Ready, Set, Go!

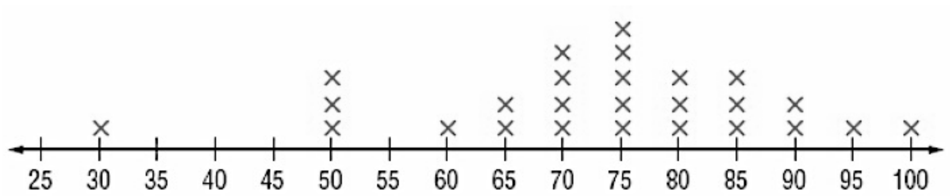
Ready

Topic: Identifying spread.

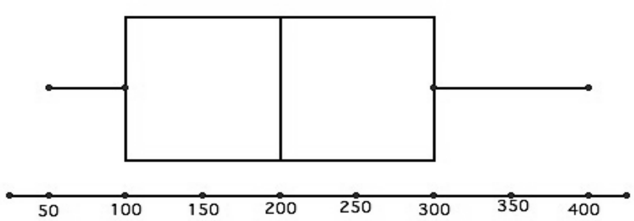
1. Describe the spread in the histogram below.



2. Describe the spread in the line plot below.



3. Describe the spread in the box and whisker plot.



Name: Connecting Algebra and Geometry | 7.5

Set

You are given information about $f(x)$ and $g(x)$. Rewrite $g(x)$ in translation form: $g(x) = f(x) + k$

4. $f(x) = 7x + 13$
   $g(x) = 7x - 5$

   $g(x)$ = ____________________
   Translation form

5. $f(x) = 22x - 12$
   $g(x) = 22x + 213$

   $g(x)$ = ____________________
   Translation form

6. $f(x) = -15x + 305$
   $g(x) = -15x - 11$

   $g(x)$ = ____________________
   Translation form

7.

x	f(x)	g(x)
3	11	26
10	46	61
25	121	136
40	196	211

$g(x)$ = ____________________
Translation form

8.

x	f(x)	g(x)
-4	5	-42
-1	-1	-48
5	-13	-60
20	-43	-90

$g(x)$ = ____________________
Translation form

9.

x	f(x)	g(x)
-10	4	-15.5
-3	7.5	-12
22	20	0.5
41	29.5	10

$g(x)$ = ____________________
Translation form

Go

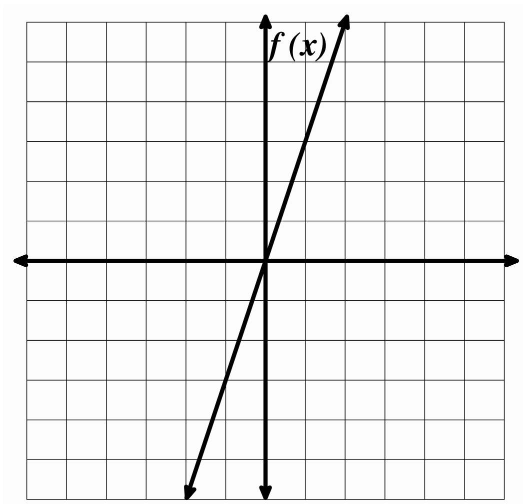
Topic: Vertical and horizontal translations

10. Use the graph of $f(x) = 3x$ to answer the following questions.

   a. Sketch the graph of $g(x) = 3x - 2$ on the same grid.

   b. Sketch the graph of $h(x) = 3(x - 2)$.

   c. Describe how $f(x)$, $g(x)$, and $h(x)$ are different and how they are the same.



d. Explain in what way the parentheses affect the graph. Why do you think this is so?

7.6 Shifting Functions

A Practice Understanding Task

http://www.flickr.com/photodean/21202433@N08,

Part I: Transformation of an exponential function.

The table below represents the property value of Rebekah's house over a period of four years.

Rebekah's Home

Time (years)	Property Value	Common Ratio
0	150,000	
1	159,000	
2	168,540	
3	178,652	
4	189,372	

Rebekah says the function $P(t) = 150{,}000(1.06)^t$ represents the value of her home.

1. Explain how this function is correct by using the table to show the initial value and the common ratio between terms.

Jeremy lives close to Rebekah and says that his house is always worth $20,000 more than Rebekah's house. Jeremy created the following table of values to represent the property value of his home.

Jeremy's Home

Time (years)	Property Value	Relationship to Rebekah's table
0	170,000	
1	179,000	
2	188,540	
3	198,652	
4	209,372	

When Rebekah and Jeremy tried to write an exponential function to represent Jeremy's property value, they discovered there was not a common ratio between all of the terms.

2. Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.

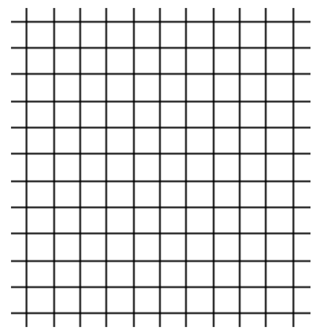
Part 2: Shifty functions.

Given the function *g(x)* and information about *f(x),*

- write the function for *f(x),*
- graph both functions on the set of axes, and
- show a table of values that compares *f(x)* and *g(x)*.

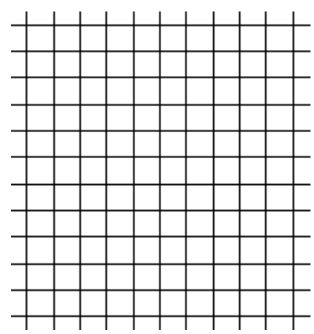
3. $If\ g(x) = 3(2)^x\ and\ \ f(x) =\ g(x) - 5, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



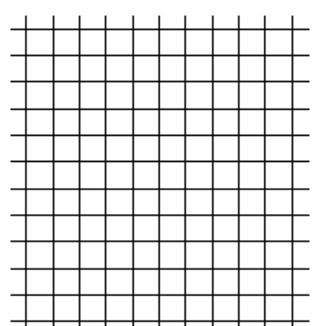
4. $If\ g(x) = 4(.5)^x\ and\ \ f(x) =\ g(x) + 3,\ then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



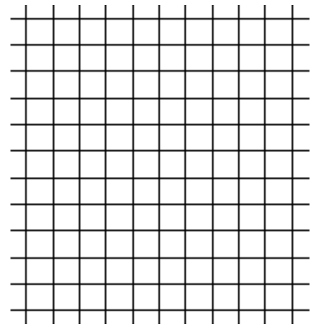
5. $If\ g(x) = 4x + 3\ and\ \ f(x) =\ g(x) + 7, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



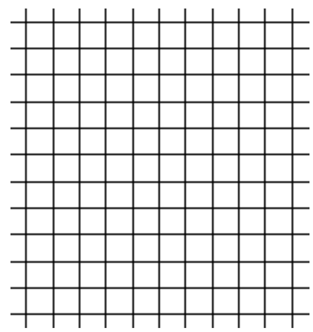
6. $If\ g(x) = 2x + 1\ and\ f(x) = g(x) - 4, then\ f(x) =$ ______________________________

x				
f(x)				
g(x)				



7. $If\ g(x) = -x\ and\ f(x) = g(x) + 3, then\ f(x) =$ ______________________________

x				
f(x)				
g(x)				



Part III: Communicate your understanding.

8. If *f(x) = g(x) + k,* describe the relationship between *f(x)* and *g(x).* Support your answers with tables and graphs.

7.6 Shifting Functions – Teacher Notes
A Practice Understanding Task

Purpose: Students will solidify their understanding of vertical transformations of exponential functions then practice shifting linear and exponential functions in this task. Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form y = *f(x)+ k* represents a change of *k* units in output values while the input values stay the same.
- Understanding that a vertical shift of a function creates a function that is exactly *k* units above or below the original function.
- Connecting equations, graphs, and table values and how the value of *k* shows up in each representation.

Core Standards Focus:

F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k,k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (Note: Focus on vertical translations of graphs of linear and exponential functions.)

F.BF.1 Write a function that describes a relationship between two quantities.★

Related Standards: F.BF.1b, F.IF.1, F.IF.2, F.IF.9, A.CED.3

Launch (Whole Class):

Prior to the task, you may wish to access student background knowledge by asking students how they would determine the equation of an exponential function given a table of values or a pair of points. You may wish to have an example to help clarify:

x	*0*	*1*	*2*	*3*	*4*
f(x)	*5*	*10*	*20*	*40*	*80*

Introduce this task by talking about the scenario presented, then let students work independently at first, then with a partner to answer questions 1 and 2 from Part I.

Explore (Small Group):

As you monitor Part I of this task, look for student understanding that exponential functions in the form of $f(x) = a(b)^x$ have a common ratio between terms (If there is confusion, correct students- the goal of this task is to deepen student understanding of vertical transformations of linear and exponential functions, not to determine that there is a common ratio between terms). Bring

students back together, then ask someone to explain why this function, $J(t) = R(t) + 20,000$ is a solution to problem 2 (Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.)
Once you discuss how this is similar to the linear and geometric transformations, have students complete the rest of the task, monitoring for student understanding.

Discuss (Whole Class):

Students work toward becoming fluent with transforming linear and exponential functions. For the whole group discussion, choose problems to discuss that were difficult for students to complete.
Goals of this task include:

- Writing function transformations using function notation.
- Recognizing that the general form $y = f(x) + k$ represents a change of k units in output values while the input values stay the same.
- Understanding that a vertical shift of a function creates a function that is exactly k units above or below the original function.
- Connecting equations, graphs, and table values and how the value of k shows up in each representation.

Aligned Ready, Set, Go: Connecting 7.6

Name: Connecting Algebra and Geometry | 7.6

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Ready, Set, Go!

Ready

Topic: Finding percentages.

Mrs. Gonzalez noticed that her new chorus class had a lot more girls than boys in it. There were 32 girls and 17 boys. (Round answers to the nearest %.)

1. What percent of the class are girls?

2. What percent are boys?

3. 68% of the girls were sopranos.
 a.How many girls sang soprano?

 b. What percent of the entire chorus sang soprano?

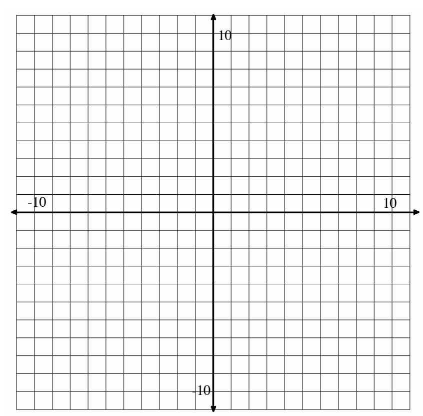
4. Only 30% of the boys could sing bass.
 a. How many boys were in the bass section?

 b. What percent of the entire chorus sang bass?

5. Compare the number of girls who sang alto to the number of boys who sang tenor. Which musical section is larger? Justify your answer.

Set

Topic: Graphing exponential equations



6. Think about the graphs of $y = 2^x$ and $y = 2^x - 4$.
 a. Predict what you think is the same and what is different.

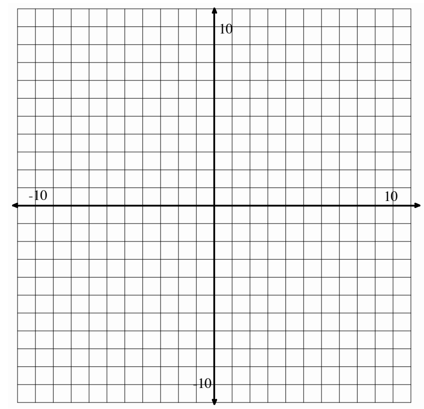
 b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed when you subtracted 4. Identify in what way it changed.

Name: Connecting Algebra and Geometry | 7.6

7. Think about the graphs of $y = 2^x$ and $y = 2^{(x-4)}$

   a. Predict what you think is the same and what is different.

   b. Use your calculator to graph both equations on the same grid.
   Explain what stayed the same and what changed.
   Identify in what way it changed.



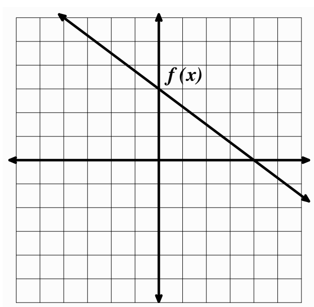
Go

Topic: Vertical translations of linear equations

The graph of $f(x)$ and the translation form equation of $g(x)$ are given. Graph $g(x)$ on the same grid and write the slope-intercept equation of $f(x)$ and $g(x)$.

8. $g(x) = f(x) - 5$

a.

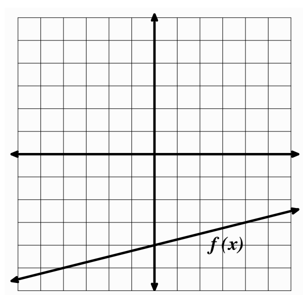


b. $f(x) =$ ________________

c. $g(x) =$ __________________
Slope-Intercept form

9. $g(x) = f(x) + 4$

a.

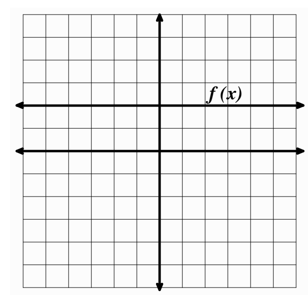


b. $f(x) =$ ________________

c. $g(x) =$ __________________
Slope-Intercept form

10. $g(x) = f(x) - 6$

a.



b. $f(x) =$ ________________

c. $g(x) =$ __________________
Slope-Intercept form

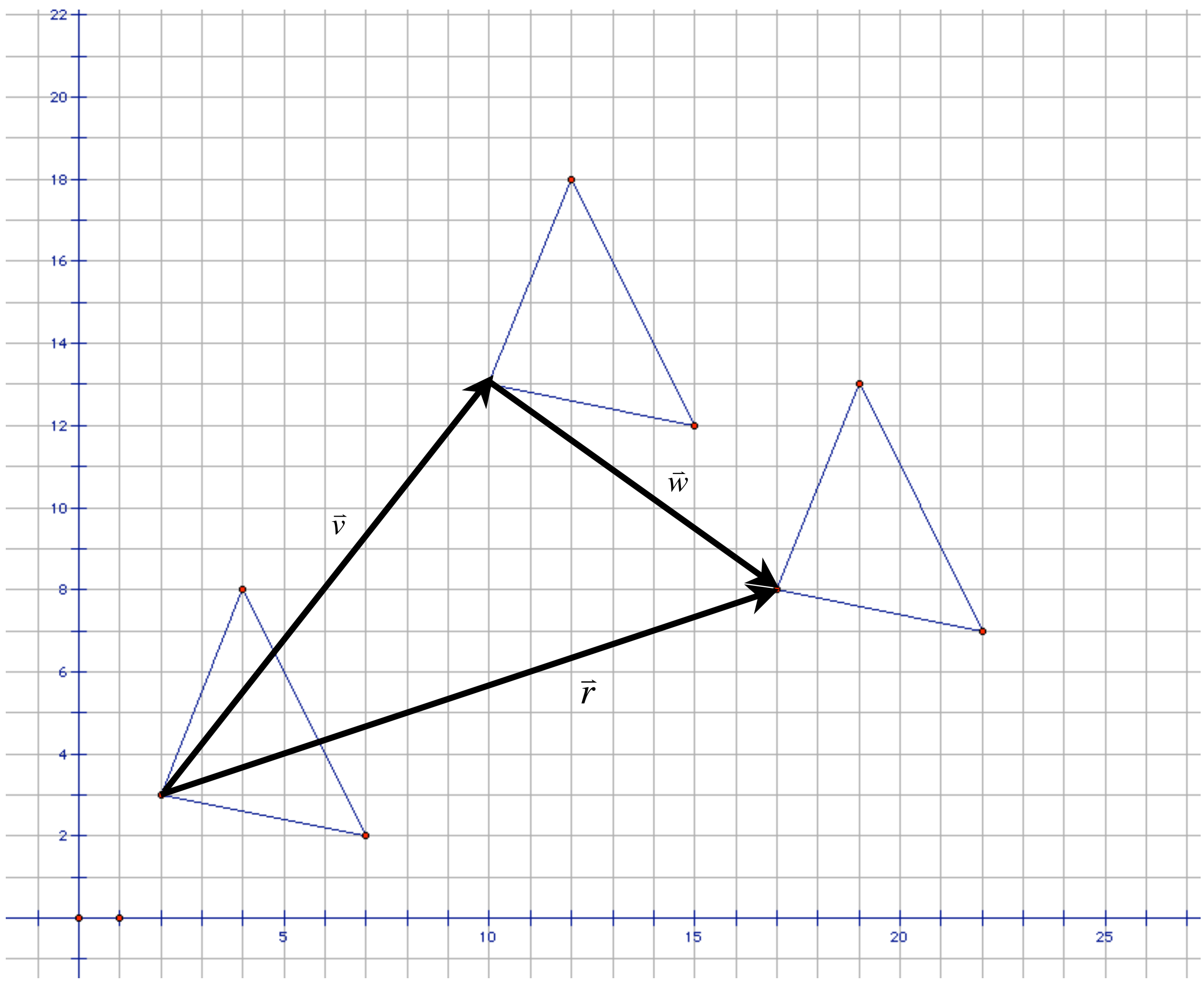
Need Help? Check out these related videos:

http://www.khanacademy.org/math/arithmetic/percents/v/identifying-percent-amount-and-base

7.7H The Arithmetic of Vectors

A Solidify Understanding Task

The following diagram shows a triangle that has been translated to a new location, and then translated again. Arrows have been used to indicate the movement of one of the vertex points through each translation. The result of the two translations can also be thought of as a single translation, as shown by the third arrow in the diagram.



Draw arrows to show the movement of the other two vertices through the sequence of translations, and then draw an arrow to represent the resultant single translation. What do you notice about each set of arrows?

A **vector** is a quantity that has both **magnitude** and **direction**. The arrows we drew on the diagram represent both translations as vectors—each translation has *magnitude* (the distance moved) and *direction* (the direction in which the object is moved). Arrows, or *directed line segments*, are one way of representing a vector.

Addition of Vectors

1. In the example above, two vectors $\vec{v}$ and $\vec{w}$ were combined to form vector $\vec{r}$. This is what is meant by "adding vectors". Study each of the following methods for adding vectors, then try each method to add vectors $\vec{s}$ and $\vec{t}$ given in the diagram below to find $\vec{q}$, such that $\vec{s}+\vec{t}=\vec{q}$.

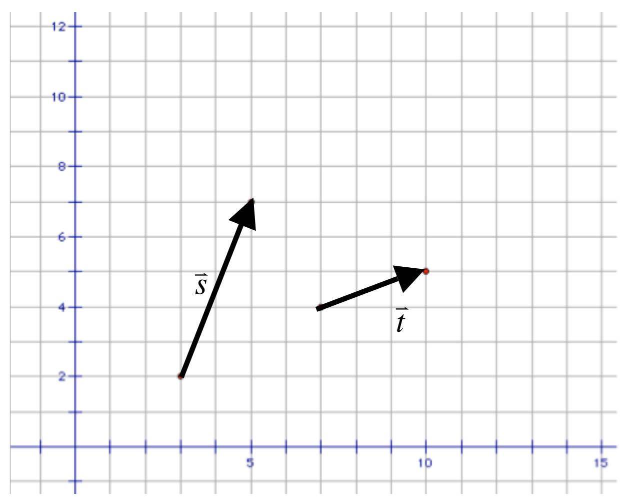
2. Explain why each of these methods gives the same result.

Method 1: *End-to-end*

The diagram given above illustrates the end-to-end strategy of adding two vectors to get a resultant vector that represents the sum of the two vectors. In this case, the resulting vector shows that a single translation could accomplish the same movement as the combined sum of the two individual translations, that is $\vec{v}+\vec{w}=\vec{r}$.

Method 2: *The parallelogram rule*

Since we can relocate the arrow representing a vector, draw both vectors starting at a common point. Often both vectors are relocated so they have their *tail* ends at the origin. These arrows form two sides of a parallelogram. Draw the other two sides. The resulting sum is the vector represented by the arrow drawn from the common starting point (for example, the origin) to the opposite vertex of the parallelogram. Question to think about: How can you determine where to put the missing vertex point of the parallelogram?



Method 3: *Using horizontal and vertical components*

Each vector consists of a horizontal component and a vertical component. For example, vector $\vec{v}$ can be thought of as a movement of 8 units horizontally and 13 units vertically. This is represented with the notation $\langle 8,13\rangle$. Vector $\vec{w}$ consists of a movement of 7 units horizontally and -5 units vertically, represented by the notation $\langle 7,-5\rangle$. Question to think about: How can the components of the individual vectors be combined to determine the horizontal and vertical components of the resulting vector $\vec{r}$?

3. Examine vector $\vec{s}$ given above. While we can relocate the vector, in the diagram the *tail* of the vector is located at (3, 2) and the *head* of the vector is located at (5, 7). Explain how you can determine the horizontal and vertical components of a vector from just the coordinates of the point at the tail and the point at the head of the vector? That is, how can we find the horizontal and vertical components of movement without counting across and up the grid?

Magnitude of Vectors

The symbol $\|\vec{v}\|$ is used to denote the magnitude of the vector, in this case the length of the vector.

Devise a method for finding the magnitude of a vector and use your method to find the following. Be prepared to describe your method for finding the magnitude of a vector.

4. $\|\vec{v}\|$

5. $\|\vec{w}\|$

6. $\|\vec{v} + \vec{w}\|$

Scalar Multiples of Vectors

We can stretch a vector by multiplying the vector by a scale factor. For example, $2\vec{v}$ represents the vector that has the same direction as $\vec{v}$, but whose magnitude is twice that of $\vec{v}$.

Draw each of the following vectors on a coordinate graph:

7. $3\vec{s}$

8. $-2\vec{t}$

9. $3\vec{s} + (-2\vec{t})$

10. $3\vec{s} - 2\vec{t}$

Other Applications of Vectors

We have illustrated the concept of a vector using translation vectors in which the magnitude of the vector represents the distance a point gets translated. There are other quantities that have magnitude and direction, but the magnitude of the vector does not always represent length.

For example, a car traveling 55 miles per hour along a straight stretch of highway can be represented by a vector since the speed of the car has magnitude, 55 miles per hour, and the car is traveling in a specific direction. Pushing on an object with 25 pounds of force is another example. A vector can be used to represent this push since the force of the push has magnitude, 25 pounds of force, and the push would be in a specific direction.

10. A swimmer is swimming across a river with a speed of 20 ft/sec and at a 45° angle from the bank of the river. The river is flowing at a speed of 5 ft/sec. Illustrate this situation with a vector diagram and describe the meaning of the vector that represents the sum of the two vectors representing the motion of the swimmer and the flow of the river.

11. Two teams are participating in a tug-of-war. One team exerts a combined force of 200 pounds in one direction while the other team exerts a combined force of 150 pounds in the other direction. Illustrate this situation with a vector diagram and describe the meaning of the vector that represents the sum of the vectors representing the efforts of the two teams.

7.7H The Arithmetic of Vectors – Teacher Notes

A Solidify Understanding Task

Teacher Note: This task contains many conceptual ideas, strategic procedures, and representational ways of thinking about vectors, and you may find it to be too much to attempt in one class period. If you choose to separate the task into two parts, the first day should consist of the three methods for adding vectors, a discussion on why the three methods produce the same results (question 2), and end with a discussion of how to find the horizontal and vertical components of a vector (question 3). The second day would include strategies for finding the magnitude of a vector, drawing scalar multiples of vectors and defining subtraction as adding the vector that faces the opposite direction. The second day would also examine vector quantities such as velocity or force, where the length of the directed line segment used to represent the vector quantity measures something other than distance.

Purpose: Students already have an intuitive understanding of one application of vectors—the translation vector—based on their work with translations of figures in a plane. The purpose of this task is to make the concept of a translation vector explicit (i.e., a translation vector has both magnitude and direction), and then to use translation vectors to examine some of the arithmetic of vectors: adding and subtracting vectors, scalar multiplication of vectors. Three methods for adding vectors are introduced: end-to-end, the parallelogram rule, and using horizontal and vertical components of the vector. The last part of the task considers other possible applications of vectors—quantities that have both magnitude and direction—such as velocity and force.

Core Standards Focus:

N.VM.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, *v*).

N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.4 Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
c. Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

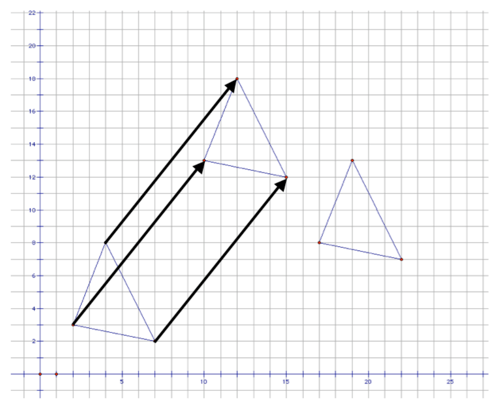
N.VM.5 Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
b. Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|**v**. Compute the direction of c**v** knowing that when |c|**v** ≠ 0, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

Launch (Whole Class): [questions 1-3]

Discuss the diagram on the first page of the task, including the additional arrows students are asked to draw in question 1. The discussion should bring out the following points:

- A **vector** is a quantity that has both magnitude and direction. The arrows we drew on the diagram represent the translations as vectors—each translation has *magnitude* (the distance moved) and *direction* (the direction in which the object is moved).
- Arrows, or *directed line segments*, are one way of representing a vector.
- For each translation, the arrows connecting a pre-image point to its image point are the same length and parallel to each other.
- We can draw an infinite set of such arrows for each translation since each point in the plane—including points in the interior and exterior of the triangle, as well as points on each of the line segments forming the triangle—get translated the same distance and same direction.
- While the vector can be represented by many different parallel line segments of the same length, there is only one vector. That is, a vector has magnitude and direction, but no location. In the diagram, each arrow drawn represents the same vector, even though the arrows are drawn in three separate locations.



Before moving to the explore, point out ways of naming vectors using the "harpoon" over the top of the variable, or using boldfaced text. Also point out the component form of representing a vector used in addition method 3 of the task. Assign students to work on the first part of the task, through question 3.

Explore (Small Group): [questions 1-3]

After clarifying that a vector is a quantity that has both magnitude and direction, and that vectors can be represented by directed line segments (arrows) drawn anywhere in the plane, turn students attention to the strategies for adding vectors as described in the task. Allow students time to try out each strategy and select students to present who can articulate how they applied each strategy to find $\vec{s} + \vec{t}$. This work should lead students to the observation that in each of these methods the horizontal components of movement of the individual vectors are combined together and the vertical components are also combined. Along with the third method for adding vectors using components, students should consider question 3: How can we find these components without counting the horizontal and vertical movement across and up the grid?

Discuss (Whole Class): [questions 1-3]
Discuss each of the strategies for adding vectors by having selected students present their work. Discuss the questions at the end of the sections describing the parallelogram rule and the component-wise methods for adding vectors. These questions should point students' attention towards adding the horizontal components together and then the vertical components together from each of the two addend vectors in order to find the horizontal and vertical components of the resultant vector. The presentations should include a summary discussion as to why these three methods produce the same results.

Question 3 should suggest a method for writing a vector in component form. Students may observe that if the coordinates of the point at the tail of the vector are (x_1, y_1) and the coordinates of the point at the head of the vector are (x_2, y_2), then the horizontal component of the vector is given by $x_2 - x_1$ and the vertical component is given by $y_2 - y_1$. Therefore, the component form of a vector is given by $\langle x_2 - x_1, y_2 - y_1 \rangle$.

Launch (Whole Class): [questions 4-11]
Following the presentations and discussions about adding vectors, assign students to work on the remainder of the task with a partner.

Explore (Small Group): [questions 4-11]
If students are having difficulty developing a strategy for finding the magnitude (i.e., length) of a vector in questions 4-6, remind them of previous work in which they have treated a non-vertical, non-horizontal line segment as the hypotenuse of a right triangle so they could apply the Pythagorean Theorem to find its length. They might also draw upon recent work with the distance formula.

The work with scalar multiples in questions 7-10 will surface the idea of changing the direction of a vector when multiplying by a negative scalar factor, and defining subtraction as adding the vector facing the opposite direction. Watch for students who are surfacing these ideas in their discussions with their peers.

Students who have successfully added vectors using all three methods and have worked out a strategy for finding the magnitude of a vector and for drawing scalar multiples of vectors can work on the additional applications of vectors in questions 10 and 11. These applications give students a sense of how vectors can represent quantities that have magnitude and direction, but for which the magnitude of the vector represents something other than distance, such as speed or force.

Discuss (Whole Class): [questions 4-11]
Have students present their strategies for finding the magnitude or length of a vector, which should be based on the Pythagorean Theorem or the distance formula. Also, discuss scalar multiples of a vector, including the reversal of direction when multiplying by a negative scalar. End this discussion of scalars by defining subtraction of vectors as adding the additive inverse (i.e., the vector with the same magnitude but opposite direction).

If there is time, examine the additional application problems as a whole class. It is important that students scale their vectors so that the length of the arrow used to represent each quantity is proportionally correct. For example, in the swimming problem the vector representing the speed of the swimmer should be 4 times longer than the vector representing the speed of the water and drawn at a 45° angle to the vector representing the downstream flow of the water.

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.7H

Name: Connecting Algebra and Geometry | 7.7H

Ready, Set, Go!

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Ready

Topic: Solving Equations Using Properties of Arithmetic

1. Here are the steps Zac used to solve the following equation. State or describe the properties of arithmetic or the properties of equality he is using in each step.

$2(x + 5) + 7x = 4x + 15$	
	the distributive property
$(2x + 10) + 7x = 4x + 15$	
	a.
$2x + (10 + 7x) = 4x + 15$	
	b.
$2x + (7x + 10) = 4x + 15$	
	c.
$(2x + 7x) + 10 = 4x + 15$	
	d.
$(2 + 7)x + 10 = 4x + 15$	
	e.
$9x + 10 = 4x + 15$	
	f.
$9x + 10 - 10 = 4x + 15 - 10$	
	g.
$9x + 0 = 4x + 5$	
	h.
$9x = 4x + 5$	

	i.
$9x - 4x = 4x + 5 - 4x$	
	j.
$(9 - 4)x = 4x + 5 - 4x$	
	k.
$5x = 4x + 5 - 4x$	
	l.
$5x = 4x - 4x + 5$	
	m.
$5x = 0 + 5$	
	n.
$5x = 5$	
	o.
$\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 5$	
	p.
$1x = 1$	
	q.
$x = 1$	

Solve each of the following equations for *x*, carefully recording each step. Then state or describe the properties of arithmetic (for example, the distributive property, or the associative property of multiplication, etc.) or properties of equality (for example, the addition property of equality) that justify each step.

2. $2(3x + 5) = 4(2x - 1)$

3. $\frac{4}{5}x + 3 = 2x - 1$

Name: Connecting Algebra and Geometry | 7.7H

Set

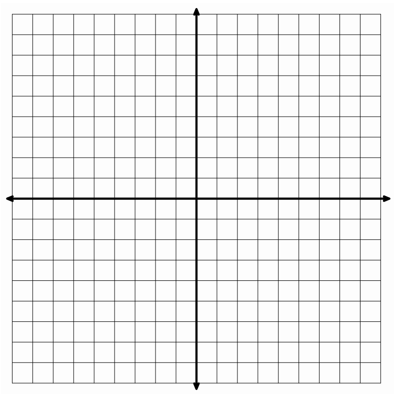
Topic: Adding vectors

Two vectors are described in component form in the following way:

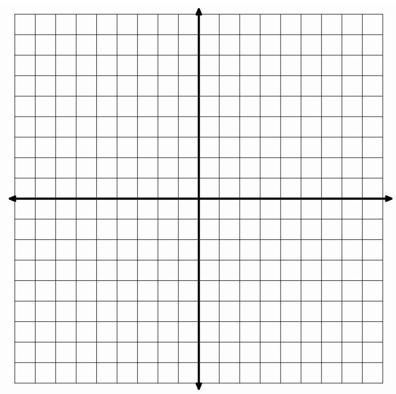
$$\vec{v} : \langle -2 , 3 \rangle \text{ and } \vec{w} : \langle 3 , 4 \rangle$$

On the grids below, create vector diagrams to show:

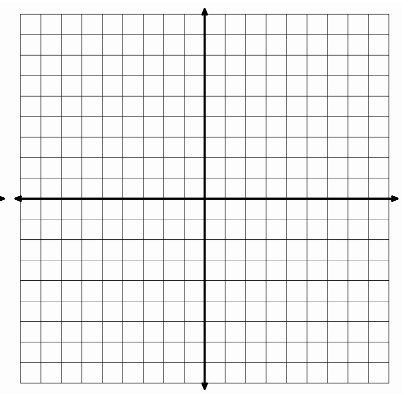
4. $\vec{v} + \vec{w} =$



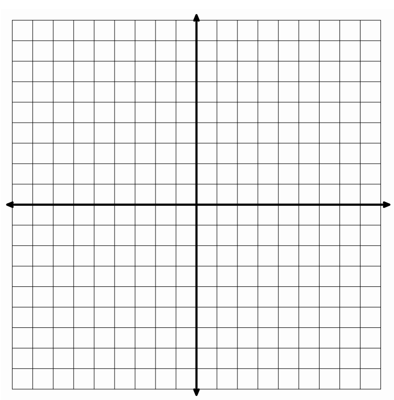
5. $\vec{v} - \vec{w} =$



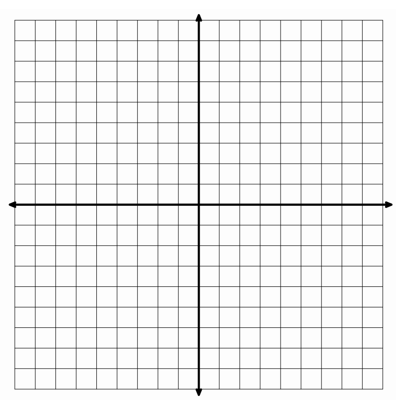
6. $3\vec{v} =$



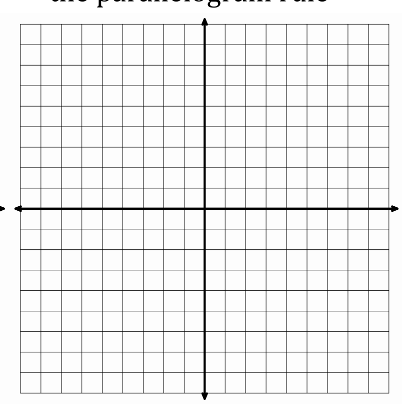
7. $-2\vec{w} =$



8. $3\vec{v} - 2\vec{w} =$



9. Show how to find $\vec{v} + \vec{w}$ using the parallelogram rule



Name: Connecting Algebra and Geometry | 7.7H

Go

Topic: The arithmetic of matrices

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix},\ B = \begin{bmatrix} 2 & 5 \\ -3 & 2 \end{bmatrix},\ \textbf{and}\ C = \begin{bmatrix} 4 & 2 & -1 \\ 5 & 2 & 3 \end{bmatrix}$$

Find the following sums, differences or products, as indicated. If the sum, difference or product is undefined, explain why.

10. $A + B$

11. $A + C$

12. $2A - B$

13. $A \cdot B$

14. $B \cdot A$

15. $A \cdot C$

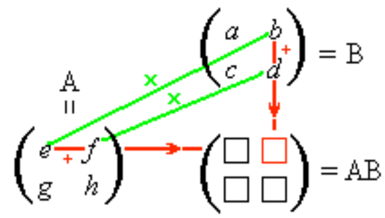
16. $C \cdot A$

Need Help? Check out these related videos:

http://www.khanacademy.org/science/physics/mechanics/v/visualizing-vectors-in-2-dimensions

7.8 H More Arithmetic of Matrices

A Solidify Understanding Task



http://commons.wikimedia.org/wiki/File:Matriz_A_por_B.png

In this task you will have an opportunity to examine some of the properties of matrix addition and matrix multiplication. We will restrict this work to square 2×2 matrices.

The table below defines and illustrates several properties of addition and multiplication for real numbers and asks you to determine if these same properties hold for matrix addition and matrix multiplication. While the chart asks for a single example for each property, you should experiment with matrices until you are convinced that the property holds or you have found a counter-example to show that the property does not hold. Can you base your justification on more that just trying out several examples?

Property	Example with Real Numbers	Example with Matrices
Associative Property of Addition $(a + b) + c = a + (b + c)$		
Associative Property of Multiplication $(ab)c = a(bc)$		
Commutative Property of Addition $a + b = b + a$		
Commutative Property of Multiplication $ab = ba$		
Distributive Property of Multiplication Over Addition $a(b + c) = ab + ac$		

In addition to the properties listed in the table above, addition and multiplication of real numbers include properties related to the numbers 0 and 1. For example, the number 0 is referred to as the *additive identity* because $a + 0 = 0 + a = a$, and the number 1 is referred to as the *multiplicative identity* since $a \cdot 1 = 1 \cdot a = a$. Once the additive and multiplicative identities have been identified, we can then define additive inverses a and $-a$ since $a + -a = 0$, and multiplicative inverses a and $1/a$ since $a \cdot \frac{1}{a} = 1$. To decide if these properties hold for matrix operations, we will need to determine if there is a matrix that plays the role of 0 for matrix addition, and if there is a matrix that plays the role of 1 for matrix multiplication.

The Additive Identity Matrix

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of 0, or the additive identity matrix, for the following matrix addition. Will this same matrix work as the additive identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

The Multiplicative Identity Matrix

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of 1, or the multiplicative identity matrix, for the following matrix multiplication. Will this same matrix work as the multiplicative identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

Now that we have identified the additive identity and multiplicative identity for 2×2 matrices, we can search for additive inverses and multiplicative inverses of given matrices.

Finding an Additive Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of the additive inverse of the first matrix. Will this same process work for finding the additive inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Finding a Multiplicative Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix containing these variables plays the role of the multiplicative inverse of the first matrix. Will this same process work for finding the multiplicative inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

7.8H More Arithmetic of Matrices – Teacher Notes
A Solidify Understanding Task

Purpose: In previous modules students have learned how to add, subtract and multiply matrices. The purpose of this task is to examine the similarities between the properties of operations with real numbers and the properties of operations with matrices. Students will examine the associative, distributive and commutative properties to determine if they hold for matrix addition and multiplication. They will also examine the properties of additive and multiplicative identities and inverses for matrix operations. As part of this task they will develop a strategy for finding the multiplicative inverse of a square matrix. The inverse of a matrix will be used in a subsequent task to give students an alternative strategy for solving systems of linear equations.

Core Standards Focus:

N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.

Related Standards: N.VM.8

Launch (Whole Class):
In this task students will be using language like *additive identity* or *multiplicative inverse* or *the associative property of multiplication*. You might begin this task by reviewing what these properties mean for addition and multiplication of real numbers. Use the first column of the chart in the task to review and illustrate these properties. Then ask students if vector addition, as defined in the previous task has some of these same properties. That is, ask questions such as the following and allow students time to express why they think these properties hold. Accept generic diagrams of vectors with non-specific lengths as “representation-based proof” of their claims about these properties.

- Is vector addition associative?
- Is vector addition commutative?
- Is there a vector that plays the role that 0 plays in the addition of real numbers? That is, is there an additive identity for vector addition?
- Do vectors have additive inverses?
- Is scalar multiplication distributive over vector addition?

Once these properties have been reviewed in terms of operations with real numbers, and explored in terms of operations with vectors, inform students that they will be considering these properties in terms of adding and subtracting square matrices.

Explore (Small Group):
Students should experiment with square matrices of their own choosing as they explore each of the properties listed in the chart for operations with square matrices. Encourage them to find counter-examples, or to convince themselves why counter-examples may not exist. Listen for "proof-like" arguments, such as "the associative property of addition holds because we are just adding the three elements in the same position of the three matrices which is the same as the associative property for three real numbers", rather than justification based solely on trying out specific cases.

The second and third pages of the task ask students to find the elements of identity and inverse matrices. This work should be fairly easy based on what students already know about matrix addition and multiplication, with the exception of finding the multiplicative inverse of a matrix. All other cases can be resolved by guess and check. In the case of the multiplicative inverse, suggest that students multiply out the matrices on the left side and set the resulting expressions equal to the appropriate elements of the matrix on the right side of the equation. This should lead to the following four equations:

$3a + c = 1$
$3b + d = 0$
$4a + 2c = 0$
$4b + 2d = 1$

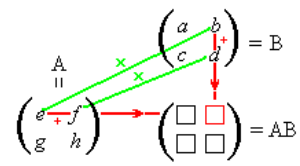
Students can then solve for *a, b, c* and *d* by pairing equations that contain the same two variables into systems of equations.

Discuss (Whole Class):
Have students share their justifications that matrix addition and matrix multiplication share the same properties of operations as the corresponding operations with real numbers, with the exception of the commutative property of multiplication. Have students present a counter-example for this exception. Make sure students can find the multiplicative inverse of a matrix by setting up the matrix equation with variables as the elements of the inverse matrix, turning the matrix equation into two systems of linear equations, and solving for the variables that represent the elements of the inverse matrix. If technology is available, you might show students how to obtain the multiplicative inverse matrix using technology. We will use the multiplicative inverse matrix in a later task to provide an alternative way for solving systems using matrices.

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.8H

Name: Connecting Algebra and Geometry | 7.8H

Ready, Set, Go!



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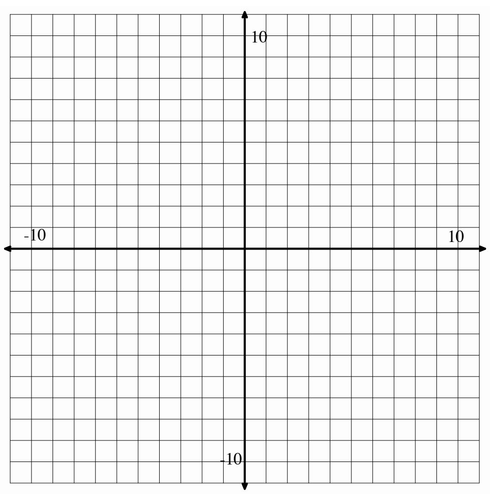
Ready

Topic: Solving systems of linear equations

1. Solve the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

a. By Graphing:



b. By substitution:

c. By elimination:

Set

Topic: Inverse matrices

2. Given: Matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

a. Find the additive inverse of matrix A

b. Find the multiplicative inverse of matrix A

Name: Connecting Algebra and Geometry 7.8H

3. Given: Matrix $B = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$

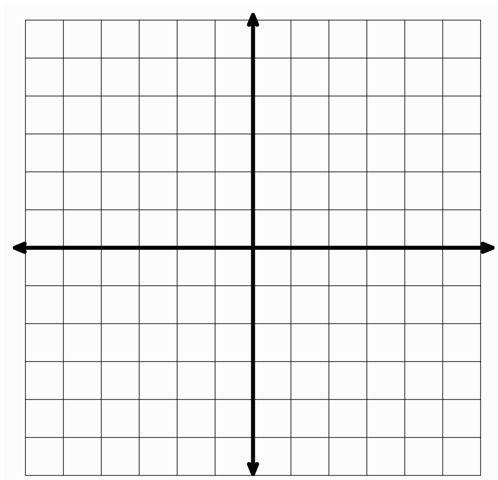
a. Find the additive inverse of matrix *B*.

b. Find the multiplicative inverse of matrix *B*

Go

Topic: Parallel lines, perpendicular lines and length from a coordinate geometry perspective

Given the four points: *A* (2, 1), *B* (5, 2), *C* (4, 5), and *D* (1, 4)



4. Is *ABCD* a parallelogram? Provide convincing evidence for your answer.

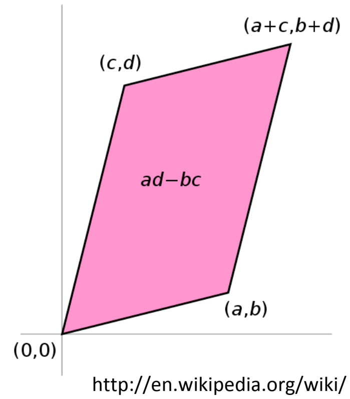
5. Is *ABCD* a rectangle? Provide convincing evidence for your answer.

6. Is *ABCD* a rhombus? Provide convincing evidence for your answer.

7. Is *ABCD* a square? Provide convincing evidence for your answer.

7.9H The Determinant of a Matrix

A Solidify Understanding Task



http://en.wikipedia.org/wiki/
File:Area_parallellogram_as_determinant.svg

In the previous task we learned how to find the multiplicative inverse of a matrix. Use that process to find the multiplicative inverse of the following two matrices.

1. $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. Were you able to find the multiplicative inverse for both matrices?

There is a number associated with every square matrix called the **determinant**. If the determinant is not equal to zero, then the matrix has a multiplicative inverse.

For a 2×2 matrix the determinant can be found using the following rule: (note: the vertical lines, rather than the square brackets, are used to indicate that we are finding the determinant of the matrix)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

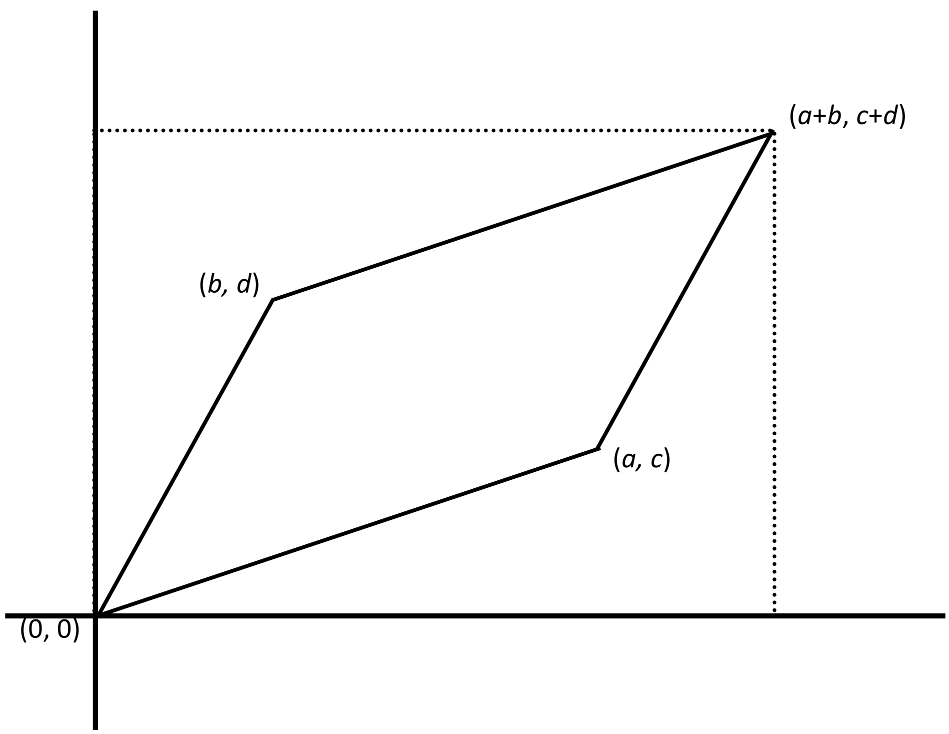
4. Using this rule, find the determinant of the two matrices given in problems 1 and 2 above.

The absolute value of the determinant of a 2×2 matrix can be visualized as the area of a parallelogram, constructed as follows.

- Draw one side of the parallelogram with endpoints at (0, 0) and (a, c).
- Draw a second side of the parallelogram with endpoints at (0, 0) and (b, d).
- Locate the fourth vertex that completes the parallelogram.

(Note that the elements in the columns of the matrix are used to define the endpoints of the vectors that form two sides of the parallelogram.)

5. Use the following diagram to show that the area of the parallelogram is given by $ad - bc$.



6. Draw the parallelograms whose areas represent the determinants of the two matrices listed in questions 1 and 2 above. How does a zero determinant show up in these diagrams?

7. Create a matrix for which the determinant will be negative. Draw the parallelogram associated with the determinant of your matrix and find the area of the parallelogram.

The determinant can be used to provide an alternative method for finding the inverse of 2×2 matrix.

8. Use the process you used previously to find the inverse of a generic 2×2 matrix whose elements are given by the variables *a, b, c* and *d.* For now, we will refer to the elements of the inverse matrix as M_1, M_2, M_3 and M_4 as illustrated in the following matrix equation. Find expressions for M_1, M_2, M_3 and M_4 in terms of the elements of the first matrix, *a, b, c* and *d*.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

M_1 =

M_2 =

M_3 =

M_4 =

Use your work above to explain this strategy for finding the inverse of a 2×2 matrix: (note: the $^{-1}$ superscript is used to indicate that we are finding the multiplicative inverse of the matrix)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ where $ad - bc$ is the determinant of the matrix

7.9H The Determinant of a Matrix – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to introduce the determinant of a square matrix, and to connect the determinant of a 2×2 matrix to the area of a parallelogram. The "area of a parallelogram" representation will be used to illustrate why some square matrices have a determinant of 0. Students will observe that if the determinant of a matrix is equal to zero, the matrix will not have a multiplicative inverse. In the next task, *Solving Systems with Matrices, Revisited*, non-zero determinants will be used as an indicator that a system of linear equations has a unique solution.

In the last part of the task students are introduced to an alternative way for finding the inverse of a 2×2 matrix using the determinant. This will support the work of the next task where students will solve systems of linear equations using inverse matrices. Once students have a clear understanding of what an inverse matrix means and have found a few inverses of 2×2 matrices using both of the methods of this task, you may also want to introduce them to the use of technology to find inverse matrices. (A third algebraic method for finding the inverse of a matrix by using row reduction is not introduced in this sequence of tasks, but could be explored with more advanced students.)

Core Standards Focus:

N.VM.10 Understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM.12 Work with 2 × 2 matrices and interpret the absolute value of the determinant in terms of area.

Launch (Whole Class):

Give students a few minutes individually to work on finding the inverse of the matrices in questions 1 and 2 using the strategy developed in the previous task. That is, students should look for the values of *a, b, c* and *d* that make the following matrix equations true.

For question 1: $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Students can multiply the two matrices on the left side of this equation and set the resulting expressions equal to the elements in the equation on the right side of the equation. This will lead to the following two systems that can be solved for *a, b, c,* and *d*.

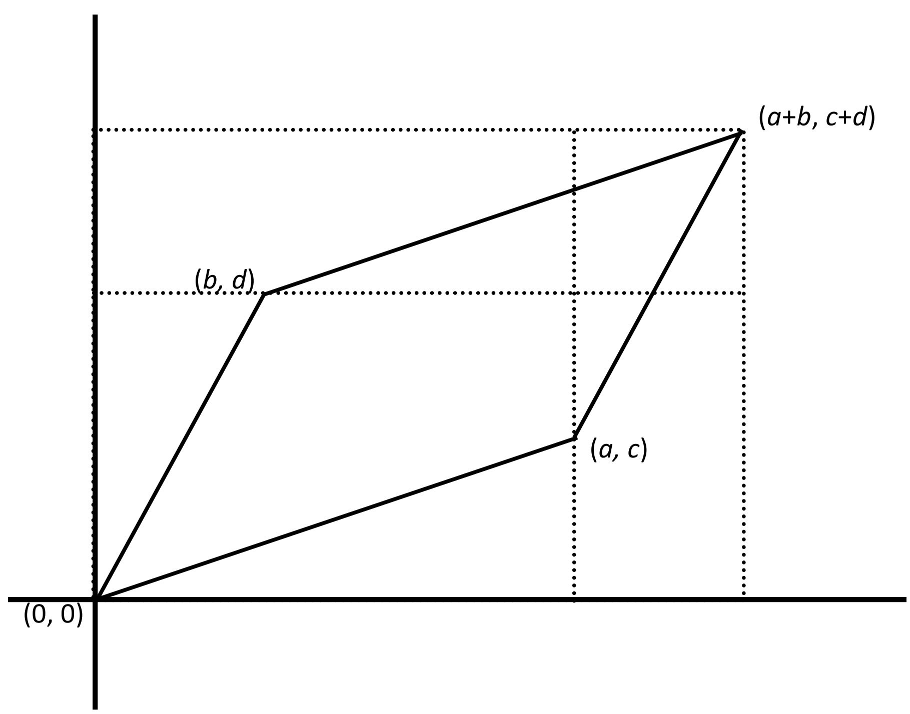
$\begin{cases} 5a + c = 1 \\ 6a + 2c = 0 \end{cases}$ and $\begin{cases} 5b + d = 0 \\ 6b + 2d = 1 \end{cases}$.

For question 2: $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ As students examine the related systems, such as $\begin{cases} 6a + 2b = 1 \\ 3a + b = 0 \end{cases}$, they will find that the systems have no solutions and therefore, the given matrix does not have a multiplicative inverse. Introduce the determinant as a number that will tell us if a square matrix has a multiplicative inverse before going through the work of trying to calculate what the inverse matrix is, and have students apply the notation given prior to question 4 to find the determinant of the two given matrices. Then assign students to work on the remainder of the task.

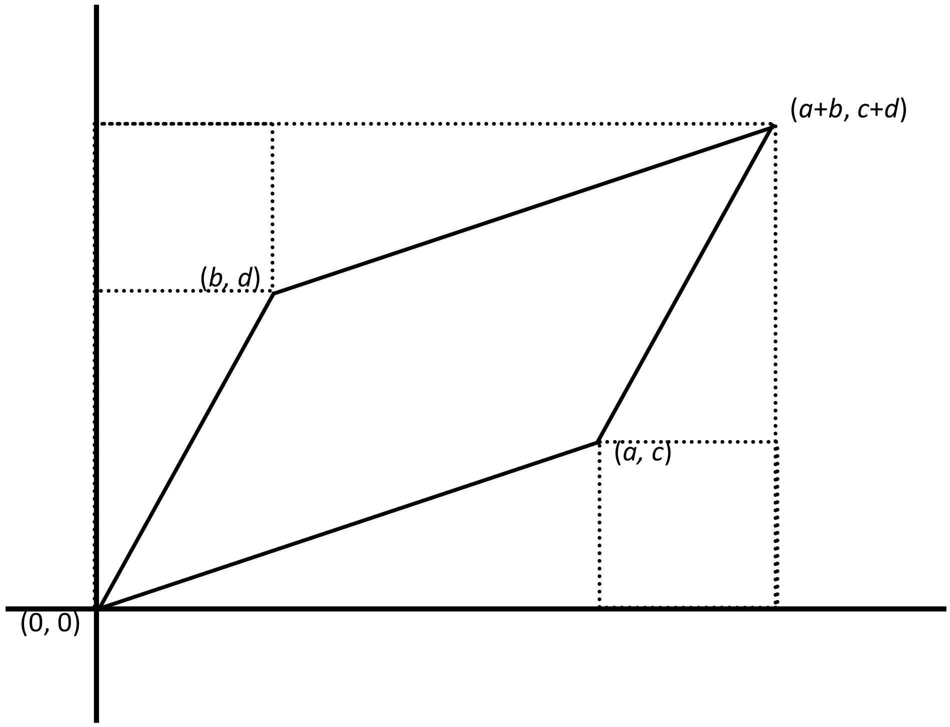
Explore (Small Group):

Associating the determinant of a 2×2 matrix with the area of a parallelogram will help students see what happens when the determinant of a matrix is zero. This occurs when the parallelogram degenerates into two overlapping line segments. In this drawing of the parallelogram, the columns of the matrix are treated as defining two vectors that form two sides of a parallelogram—the same parallelogram formed in the task *The Arithmetic of Vectors* to add vectors using the parallelogram rule. To show that the area of this parallelogram is given by $ad - bc$, students may choose to find the area of the surrounding rectangle, and then subtract the area of the extra dart-shaped pieces, leaving the area of the parallelogram.

The area of the surrounding rectangle is given by $(a + b)(c + d)$. Since students may not know how to multiply these two binomials together, they may need to decompose the larger rectangle into smaller rectangles, as shown in the following diagram.



The area of the extra pieces that need to be subtracted from the surrounding rectangle can be found by decomposing the extra dart-shaped pieces into rectangles and triangles, as in the following diagram.



In question 8 students develop an alternative method for finding the inverse of a 2×2 matrix, if the inverse exists. Students will first set up two systems of equations to solve for M_1, M_2, M_3 and M_4. Each of these values will consist of a fraction whose denominator can be represented by $ad - bc$, the value of the determinant. Consequently, the value $\frac{1}{ad - bc}$ can be factored out of the matrix as a scalar multiple, leading to the rule given in the task for the inverse matrix.

Discuss (Whole Class):
Have selected students present their work showing that the area of the parallelogram is given by the value of the determinant $ad - bc$. In the cases where the determinant is negative, the area of the parallelogram is $|ad - bc|$. This will come out as students share their work on question 7, and the issue with a 0 determinant will come out of the work on question 6.

Students may need assistance with the algebra in question 8. If so, work through this derivation as a whole class, rather than with each individual group. The derivation starts by forming the following two systems:

$$\begin{cases} aM_1 + bM_3 = 1 \\ cM_1 + bM_3 = 0 \end{cases} \quad \text{and} \quad \begin{cases} aM_2 + bM_4 = 0 \\ cM_2 + dM_4 = 1 \end{cases}$$

Solving these systems for M_1, M_2, M_3 and M_4 leads to:

$$M_1 = \frac{d}{ad - bc}$$

$$M_2 = \frac{-b}{ad - bc}$$

$$M_3 = \frac{-c}{ad - bc}$$

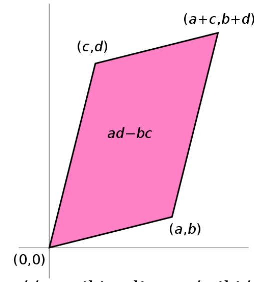
$$M_4 = \frac{a}{ad - bc}$$

Therefore, the inverse matrix is $\begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$ or, after factoring out the scalar factor $\frac{1}{ad-bc}$, the inverse matrix is given by $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.9H

Name: Connecting Algebra and Geometry | 7.9H

Ready, Set, Go!



http://en.wikipedia.org/wiki/File:Area_parallellogram_as_determinant.svg

Ready

Topic: Solving systems of linear equations using row reduction

Given the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

1. Zac started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -5 & -17 \\ 2 & 1 & 10 \end{bmatrix}$

Describe what Zac did to get from the matrix on the left to the matrix on the right.

2. Lea started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -3 & 3 \\ 1 & \frac{1}{2} & 5 \end{bmatrix}$

Describe what Lea did to get from the matrix on the left to the matrix on the right.

3. Using either Zac's or Lea's first step, continue solving the system using row reduction. Show each matrix along with notation indicating how you got from one matrix to another. Be sure to check your solution.

Name: Connecting Algebra and Geometry | 7.9H

Set

Topic: The determinant of a 2×2 matrix

4. Use the determinant of each 2×2 matrix to decide which matrices have multiplicative inverses, and which do not.

a. $\begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

5. Find the multiplicative inverse of each of the matrices in 4, provided the inverse matrix exists.

a.

b.

c.

6. Generally matrix multiplication is not commutative. That is, if A and B are matrices, typically $A \cdot B \neq B \cdot A$. However, multiplication of inverse matrices is commutative. Test this out by showing that the pairs of inverse matrices you found in question 7 give the same result when multiplied in either order.

Name: Connecting Algebra and Geometry | 7.9H

Go

Topic: Parallel and perpendicular lines

Determine if the following pairs of lines are parallel, perpendicular or neither. Explain how you arrived at your answer.

7. $3x + 2y = 7$ and $6x + 4y = 9$

8. $y = \frac{2}{3}x - 5$ and $y = -\frac{2}{3}x + 7$

9. $y = \frac{3}{4}x - 2$ and $4x + 3y = 3$

10. Write the equation of a line that is parallel to $y = \frac{4}{5}x - 2$ and has a y-intercept at (0, 4).

11. Write the equation of a line that is perpendicular to $y = -\frac{2}{3}x + 3$ and passes through the point (2, 5).

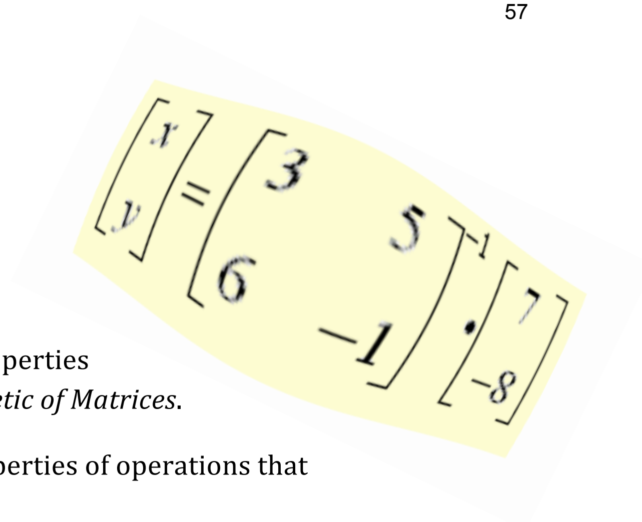
12. Write the equation of a line that is parallel to $y = -\frac{2}{3}x + 3$ and passes through the point (2, 5).

Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/equations-of-parallel-and-perpendicular-lines

7.10H Solving Systems with Matrices, Revisited

A Solidify Understanding Task



When you solve linear equations, you use many of the properties of operations that were revisited in the task *More Arithmetic of Matrices*.

1. Solve the following equation for *x* and list the properties of operations that you use during the equation solving process.

$$\frac{2}{3}x = 8$$

2. The list of properties you used to solve this equation probably included the use of a multiplicative inverse and the multiplicative identity property. If you didn't specifically list those properties, go back and identify where they might show up in the equation solving process for this particular equation.

Systems of linear equations can be represented with matrix equations that can be solved using the same properties that are used to solve the above equation. First, we need to recognize how a matrix equation can represent a system of linear equations.

3. Write the linear system of equations that is represented by the following matrix equation. (Think about the procedure for multiplying matrices you developed in previous tasks.)

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

4. Using the relationships you noticed in question 3, write the matrix equation that represents the following system of equations.

$$\begin{cases} 2x + 3y = 14 \\ 3x + 4y = 20 \end{cases}$$

5. The rational numbers $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses. What is the multiplicative inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$? Note: The inverse matrix is usually denoted by $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$.

6. The following table lists the steps you may have used to solve $\frac{2}{3}x = 8$ and asks you to apply those same steps to the matrix equation you wrote in question 4. Complete the table using these same steps.

Original equation	$\frac{2}{3}x = 8$	$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \end{bmatrix}$
Multiply both sides of the equation by the multiplicative inverse	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 8$	
The product of multiplicative inverses is the multiplicative identity on the left side of the equation	$1 \cdot x = \frac{3}{2} \cdot 8$	
Perform the indicated multiplication on the right side of the equation	$1 \cdot x = 12$	
Apply the property of the multiplicative identity on the left side of the equation	$x = 12$	

7. What does the last line in the table in question 6 tell you about the system of equations in question 4?

8. Use the process you have just examined to solve the following system of linear equations.

$$\begin{cases} 3x + 5y = -1 \\ 2x + 4y = 4 \end{cases}$$

7.10H Solving Systems with Matrices, Revisited – Teacher Notes

A Solidify Understanding Task

Purpose: Students have previously solved systems using matrices and row reduction—a process associated with the elimination method for solving systems. The purpose of this task is to give students an alternative method for solving systems using the inverse of the coefficient matrix. This method can be generalized to solving larger systems of linear equations using technology to produce the inverse matrix. You may wish to augment this task with a few additional problems in which students solve a larger system of linear equations using inverse matrices produced by technology. This will help them gain an appreciation for the strategy being developed in this task.

Core Standards Focus:

UT Honors Standard: Solve systems of linear equations using matrices.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Related Standards: A.REI.6

Launch (Whole Class):
Give students a few minutes individually to work on questions 1 and 2. While students may solve the equation in question 1 by multiplying both sides of the equation by 3 and then dividing both sides of the equation by 2, question 2 encourages them to multiply both sides of the equation by the multiplicative inverse of $\frac{2}{3}$ to get $1x$ on the left side of the equation and then to simplify $1x$ to x using the multiplicative identity property. Look for a student who has approached the problem in this way to present.

Work with students on using a matrix equation to represent a system of equations, as suggested in questions 3 and 4. Contrast this matrix representation with the augmented matrix representation of a system used in the *Systems* module. You may want to review solving a system by row reduction and then point out that in this task students will develop an alternative method for solving systems based on the arithmetic of matrices. Set students to work on the remainder of the task.

Explore (Small Group):
Since matrix multiplication is not commutative, when students multiply both sides of the matrix equation by the inverse matrix, they will have to place the inverse matrix to the left of the matrices given on each side of the equation. Watch for students who are having difficulty multiplying matrices on the right side of the equation because of this issue. Ask students why this is an issue

with the matrix equation, but not with the equation that contains the rational number coefficient and the multiplication by $\frac{3}{2}$ on both sides of the equation. Likewise, rewriting $1x$ as x is a subtle and over-looked idea when solving the equation containing rational numbers, but it becomes a more apparent issue when working with the matrix equation and recognizing that $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$ can be re-written simply as $\begin{bmatrix} x \\ y \end{bmatrix}$ because of the multiplicative identity property.

Discuss (Whole Class):
Have a student present their work on question 8, showing how they solved the system using a matrix equation and an inverse matrix. While we have not developed a process for finding the inverse of a 3×3 matrix, it may help students appreciate the value of solving systems using inverse matrices to consider solving a larger system using this strategy versus using row reduction, or substitution or elimination. Here is a 3×3 system to consider.

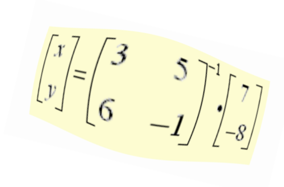
$$\begin{cases} 4x - 2y + z = 3 \\ 2x + y - z = 1 \\ 3x - y + 2z = 7 \end{cases}$$

The inverse of the coefficient matrix can be obtained using technology. Have students confirm that the matrix produced by technology is indeed the inverse matrix by having them carry out the multiplication of the matrix and its inverse by hand. Once the system has been solved using the inverse matrix, have students solve the system using another method, such as row reduction of the augmented matrix, or substitution or elimination.

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.10H

Name: Connecting Algebra and Geometry | 7.10H

Ready, Set, Go!

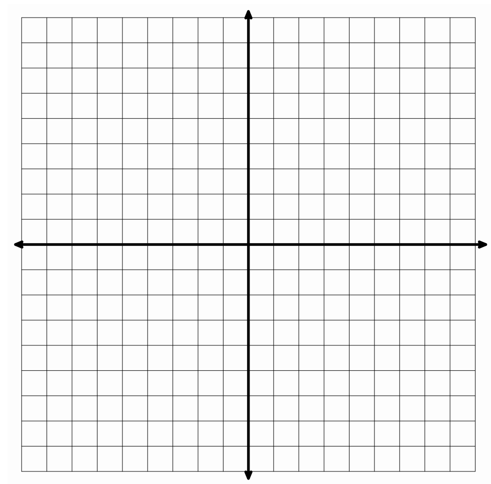


Ready

Topic: Reflections and rotations

1. The following three points form the vertices of a triangle: (3, 2), (6, 1), (4, 3)

a. Plot these three points on the coordinate grid and then connect them to form a triangle.



b. Reflect the original triangle over the *y*-axis and record the coordinates of the vertices here:

c. Reflect the original triangle over the *x*-axis and record the coordinates of the vertices here:

d. Rotate the original triangle 90° counter-clockwise about the origin and record the coordinates of the vertices here:

e. Rotate the original triangle 180° about the origin and record the coordinates of the vertices here.

Set

Topic: Solving systems using inverse matrices

Two of the following systems have unique solutions (that is, the lines intersect at a single point).

2. Use the determinant of a 2 × 2 matrix to decide which systems have unique solutions, and which one does not.

a. $\begin{cases} 8x - 2y = -2 \\ 4x + y = 5 \end{cases}$

b. $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = -5 \end{cases}$

c. $\begin{cases} 4x + 2y = 0 \\ 3x + y = 2 \end{cases}$

3. For each of the systems in #2 which have a unique solution, find the solution to the system by solving a matrix equation using an *inverse matrix*.

a. b. c.

Go

Topic: Properties of arithmetic

Match each example on the left with the name of a property of arithmetic on the right. Not all answers will be used.

____ 4. $2(x + 3y) = 2x + 6y$

____ 5. $(2x + 3y) + 4y = 2x + (3y + 4y)$

____ 6. $2x + 3y = 3y + 2x$

____ 7. $2(3y) = (2 \cdot 3)y = 6y$

____ 8. $\frac{2}{3} \cdot \frac{3}{2} x = 1x$

____ 9. $x + -x = 0$

____ 10. $xy = yx$

a. multiplicative inverses

b. additive inverses

c. multiplicative identity

d. additive identity

e. commutative property of addition

f. commutative property of multiplication

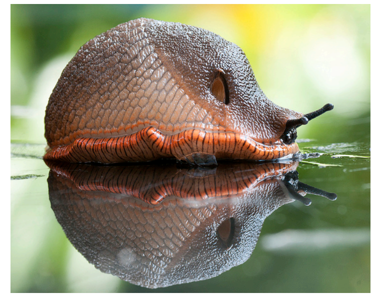
g. associative property of addition

h. associative property of multiplication

i. distributive property of addition over multiplication

7.11H Transformations with Matrices

A Solidify Understanding Task



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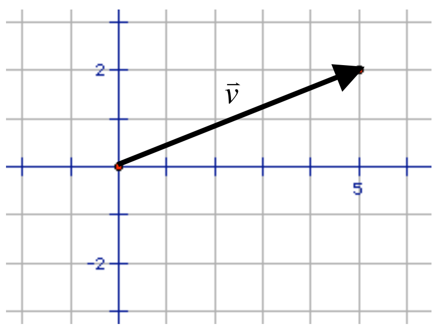
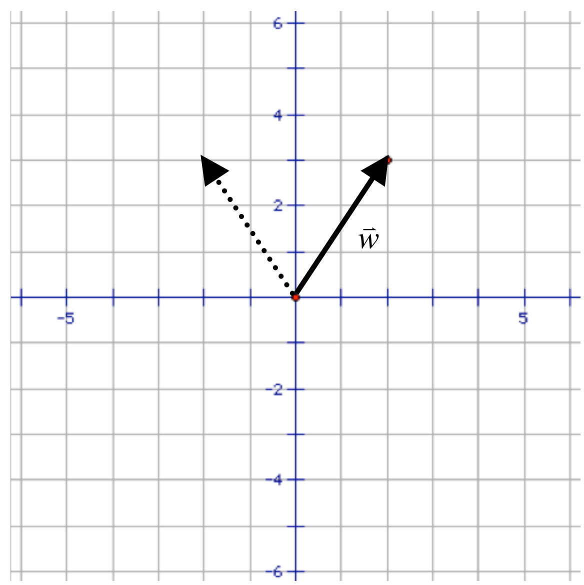
Various notations are used to denote vectors: bold-faced type, ***v***; a variable written with a harpoon over it, $\vec{v}$; or listing the horizontal and vertical components of the vector, $\langle v_x, v_y \rangle$. In this task we will represent vectors by listing their horizontal and vertical components in a matrix with a single column, $\begin{bmatrix} v_x \\ v_y \end{bmatrix}$.

1. Represent the vector labeled $\vec{v}$ in the diagram below as a matrix with one column.



Matrix multiplication can be used to transform vectors and images in a plane.

Suppose we want to reflect $\vec{w}$ over the y-axis. We can represent $\vec{w}$ with the matrix $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and the reflected vector with the matrix $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$.



2. Find the 2×2 matrix that we can multiply the matrix representing the original vector by in order to obtain the matrix that represents the reflected vector. That is, find *a, b, c* and *d* such that $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

3. Find the matrix that will reflect $\vec{w}$ over the *x*-axis.

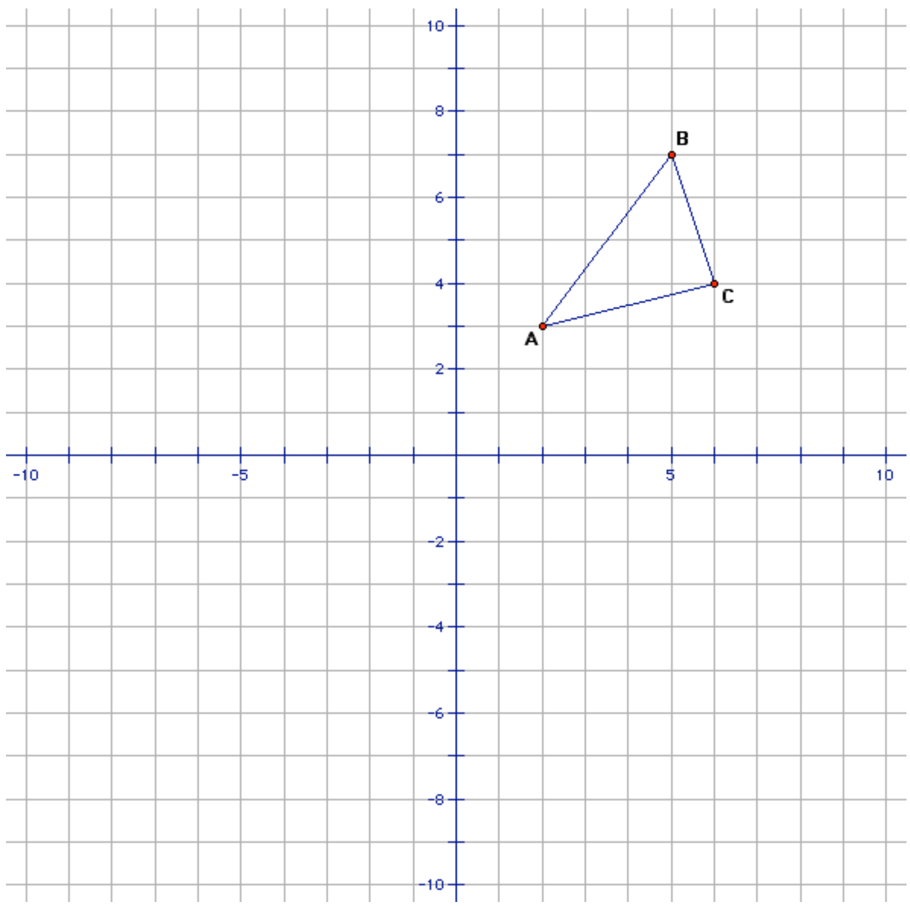
4. Find the matrix that will rotate $\vec{w}$ 90° counterclockwise about the origin.

5. Find the matrix that will rotate $\vec{w}$ 180° counterclockwise about the origin.

6. Find the matrix that will rotate $\vec{w}$ 270° counterclockwise about the origin.

7. Is there another way to obtain a rotation of 270° counterclockwise about the origin other than using the matrix found in question 6? If so, how?

We can represent polygons in the plane by listing the coordinates of its vertices as columns of a matrix. For example, the triangle below can be represented by the matrix $\begin{bmatrix} 2 & 5 & 6 \\ 3 & 7 & 4 \end{bmatrix}$.



8. Multiply this matrix, which represents the vertices of ΔABC, by the matrix found in question 2. Interpret the product matrix as representing the coordinates of the vertices of another triangle in the plane. Plot these points and sketch the triangle. How is this new triangle related to the original triangle?

9. How might you find the coordinates of the triangle that is formed after ΔABC is rotated 90° counterclockwise about the origin using matrix multiplication? Find the coordinates of the rotated triangle.

10. How might you find the coordinates of the triangle that is formed after ΔABC is reflected over the x-axis using matrix multiplication? Find the coordinates of the reflected triangle.

7.11H Transformations with Matrices – Teacher Notes

A Solidify Understanding Task

Purpose: In this task students will examine the use of matrices to produce transformations. Vectors will be represented as matrices with a single column. Students will determine matrices that transform a vector in one of the following ways when the vector matrix is multiplied by the appropriate transformation matrix.

- the vector is reflected over the y-axis
- the vector is reflected over the x-axis
- the vector is rotated 90° about the origin
- the vector is rotated 180° about the origin

Students will use these same matrices to transform polygons by multiplying a matrix that contains the coordinates of the vertices of the original polygon by the appropriate transformation matrix.

Core Standards Focus:

N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

N.VM.12 Work with 2×2 matrices as transformations of the plane.

Related Standards: G.CO.2, G.CO.5

Launch (Whole Class): [questions 3-7]

Review with your students the information about notation for representing vectors, as given on the first page of the task. Make sure students understand how a vector can be represented as a matrix with one column. You may want to review the row-by-column procedure for matrix multiplication that was developed in a previous module. Also remind students of the definitions of rotations and reflections and point out that in this task the *x*- and *y*-axes will serve as lines of reflection, and that rotations will be about the origin. Once students are comfortable with the matrix representation of a vector, and have reviewed the appropriate skills and concepts used in this task, present the issue of finding a matrix that can be used as a factor along with the matrix representing a vector, so that the resulting product matrix represents the vector after it has been reflected about the *y*-axis. Give students a few minutes to work individually to find this matrix, and then share their work—both their final result as well as their strategy for finding this matrix (see potential strategies in the explore). Once strategies have been presented for finding the transformation matrix required in question 2, set students about the work of finding the other transformation matrices required on questions 3-6.

Explore (Small Group): [questions 3-7]
Two strategies that potentially will emerge for finding the transformation matrices include a guess-and-check strategy and a strategy built on writing and solving equations by inspection. For example, on question 2 students might use trial and error to find values for *a, b, c* and *d*—using such reasoning as, "I want to multiply 2 by -1 to change its sign, but then I don't want to add anything to it, so the other partial product needs to be 0." Or, students may be more systematic and multiply the two matrices on the left of the equal sign together and set the resulting expressions equal to the elements in the product matrix on the right. This will lead them to the following two equations, which can be solved by inspection: $2a + 3b = -2$ $2c + 3d = 3$

Students who finish questions 3-6 before other students can work on question 7.

Discuss (Whole Class): [questions 3-7]
Make a list of the matrices students found for each of the required transformations, as follows:

Desired transformation	**Transformation matrix**
Reflect over y-axis	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflect over x-axis	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Rotate 90° counterclockwise about the origin	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
Rotate 180° counterclockwise about the origin	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
Rotate 270° counterclockwise about the origin	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Discuss question 7. This question is intended to help students recognize that they can multiply two transformation matrices together to get a new matrix that represents the combined effect of the two individual transformations. For example, multiplying the matrix that rotates a vector 90° counterclockwise about the origin by the matrix that rotates a vector 180° counterclockwise about the origin produces the matrix that rotates a vector 270° counterclockwise about the origin. Likewise, using the matrix that rotates a vector 90° counterclockwise about the origin as a factor three times also produces the matrix that rotates a vector 270° counterclockwise about the origin, and using that matrix as a factor four times produces the identity matrix, which would return the vector to its original position after a rotation of 360°. Multiplying the matrix that reflects a vector over the x-axis by the matrix that reflects a vector over the y-axis produces the matrix that rotates a vector 180° counterclockwise about the origin.

Launch (Whole Class): [questions 8-10]
Use the given diagram on the third page of the task to show how vertices of a polygon can be represented by the columns of a matrix. Then set students to work on questions 8-10.

Explore (Small Group): [questions 8-10]
As students work on questions 8-10 they should observe that the same matrices we listed previously can be used to reflect or rotate a collection of points that represent the vertices of a polygon.

Discuss (Whole Class): [questions 8-10]
Ask students to describe how a single transformation matrix can be used to reflect or rotate a complete set of points representing the pre-image of the transformation.

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.11H

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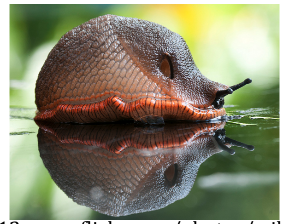

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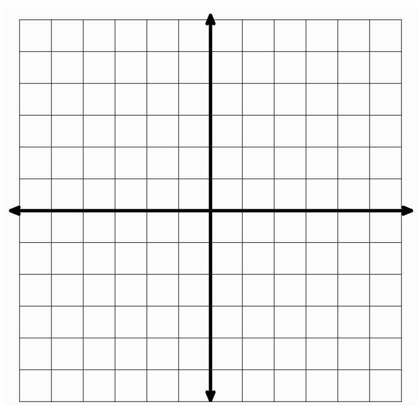
Ready, Set, Go!

Ready

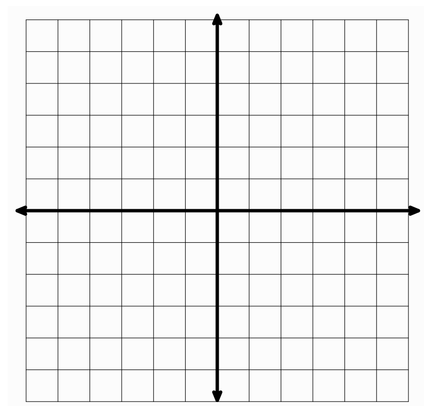
Topic: Adding vectors

Given vectors $\vec{v}:\langle -2,\ 4\rangle$ **and** $\vec{w}:\langle 5,-2\rangle$**, find the following using the parallelogram rule:**

1. $\vec{v}+\vec{w}=$



2. $\vec{v}-\vec{w}=$



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Set

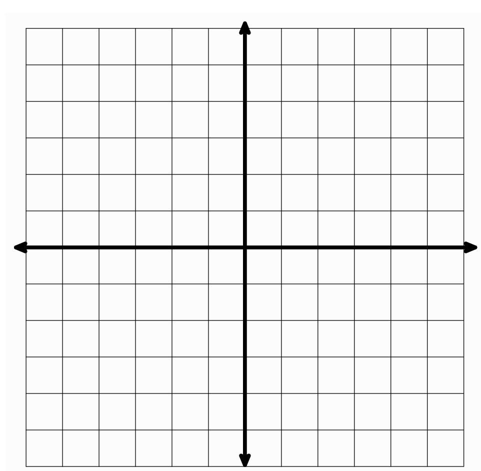
Topic: Matrices and transformations of the plane

3. List the coordinates of the four vertices of the parallelogram you drew in question 1 as a matrix. The x-values will be in the left column, and the y-values will be in the right column.

$$\begin{matrix} & \begin{matrix} x & \quad & y \end{matrix} \\ \begin{matrix} \text{Point 1} \\ \text{Point 2} \\ \text{Point 3} \\ \text{Point 4} \end{matrix} & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{matrix}$$

4. Multiply the matrix you wrote in question 3 by the following matrix: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

5. Plot the original parallelogram. Then, using the ordered pairs from your answer in question 4, using the points from the matrix in number 4 on the following coordinate grid. Connect those 4 points. What transformation occurred between your original parallelogram and the new one?



Name: Connecting Algebra and Geometry | 7.11H

Go

Topic: Transformations of functions

Function $f(x)$ is defined by the following table below:

x	2	4	6	8	10	12	14	16
$f(x)$	-8	-3	2	7	12	17	22	27
$g(x)$								
$h(x)$								

6. Write an equation for $f(x)$.

7a. Fill in the values for $g(x)$ assuming that $g(x) = f(x) + 3$

b. Write an equation for $g(x)$.

8a. Fill in the values for $h(x)$ assuming that $h(x) = 2f(x)$

b. Write an equation for $h(x)$.

7.12H Plane Geometry
A Practice Understanding Task

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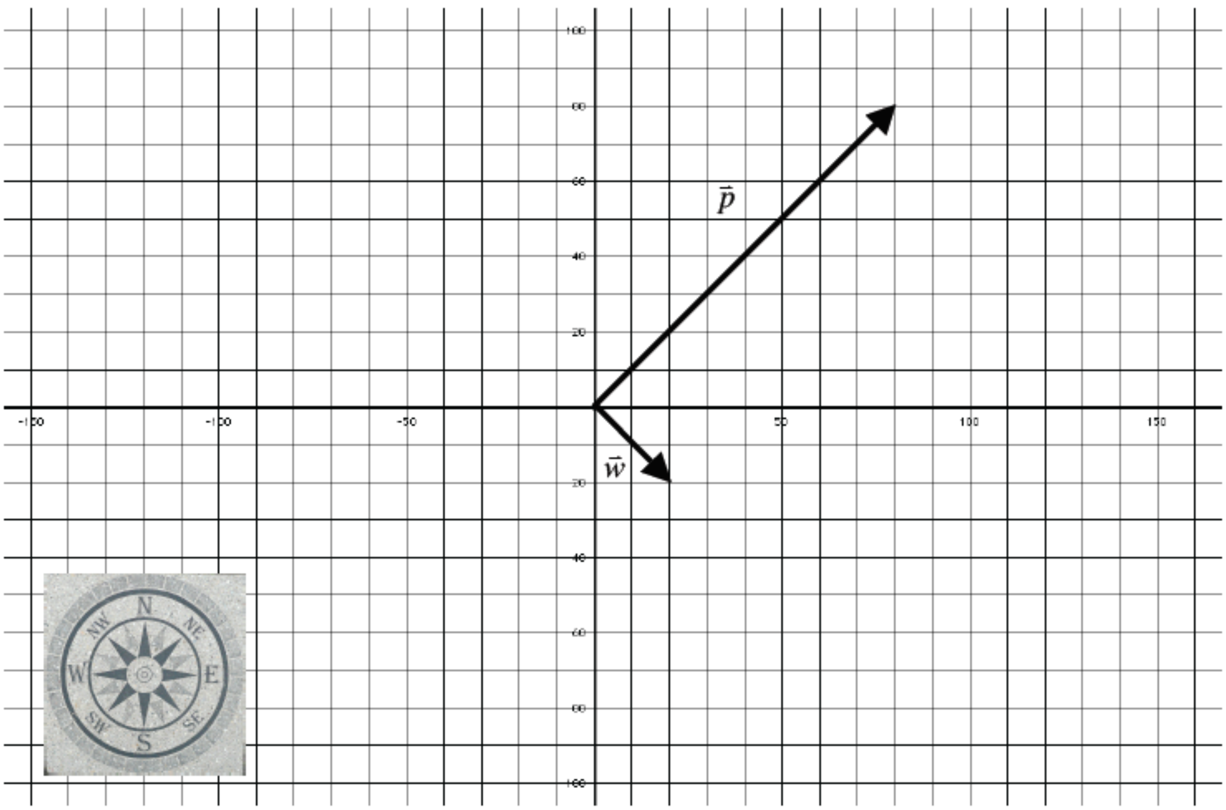
Jon's father is a pilot and he is using vector diagrams to explain some principles of flight to Jon. His father has drawn the following diagram to represent a plane that is being blown off course by a strong wind. The plane is heading northeast as represented by $\vec{p}$ and the wind is blowing towards the southeast as represented by $\vec{w}$.

1. Based on this diagram, what is the plane's speed and what is the wind's speed? (The vector diagram represents the speed of the plane in still air.)



compass rose: www.flickr.com/photos/64167416@N03/7022634029/

2. Use this diagram to find the ground speed of the plane, which will result from a combination of the plane's speed and the wind's speed. Also, indicate on the diagram the direction of motion of the plane relative to the ground.

3. Jon drew a parallelogram to determine the ground speed and direction of the plane. If you have not already done so, draw Jon's parallelogram and explain how it represents the original problem situation as well as the answers to the questions asked in problem 2.

4. Write a matrix equation that will reflect the parallelogram you drew in problem 3 over the *y*-axis. Use the solution to the matrix equation to draw the resulting parallelogram.

5. Prove that the resultant figure of the reflection performed in problem 4 is a parallelogram. That is, explain how you know opposite sides of the resulting quadrilateral are parallel.

6. Find the area of the parallelogram drawn in problem 3. Explain your method for determining the area.

7.12H Plane Geometry – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to practice using matrices to represent vectors, and matrix operations to represent transformations of the plane. In addition, students will practice using the distance formula to determine the magnitude of a vector, and interpret the meaning of the magnitude and orientation of a vector in terms of a real-world context. The properties of a parallelogram will be exploited through the use of the parallelogram rule for adding vectors, through finding the area of a parallelogram using the determinant of a 2×2 matrix, and by observing that parallelism is preserved under rigid-motion transformations.

Core Standards Focus:

N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

N.VM.12 Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Related Standards: N.VM.1, N.VM.11

Launch (Whole Class):
Read the initial context with students and verify that they can explain the relationship of the vector diagram to the context. Then set them to work on the six problems.

Explore (Small Group):
Monitor students to observe how they approach each problem. Use the following hints to prompt students who are struggling with particular problems.

Question 1: The wind's speed and the plane's speed are represented by the length of the vectors. How can we find the length of the vectors (i.,e, the distance from the point at the tail to the point at the head of the vector)?

Question 2: The ground speed and actual direction of motion of the plane is represented by the sum of the vectors representing the wind's speed and the plane's speed. Note that the magnitude of the sum of the two vectors is not the sum of the magnitudes of the individual speeds.

Question 3: Why does the diagonal of a parallelogram represent the sum of the vectors?

Question 4: The coordinates of the parallelogram's vertices can by represented by the columns of a matrix. Remind students that we have already found the elements of matrices that reflect images across the x- or y-axes, as well as matrices that rotate images 90°, 180° or 270° about the origin.

Question 5: Given the coordinates of the reflected image based on the matrix multiplication in question 4, how might we determine that opposite sides of this image quadrilateral are parallel?

Question 6: We have found a relationship between the determinant of a matrix and the area of a parallelogram. How might we apply that relationship to this situation? Note: Because this parallelogram is also a rectangle, students might also find the area of the parallelogram using the lengths of the sides, as found in question 1. It would be good to present both approaches to the whole class to confirm that the determinant does find the area of the parallelogram correctly.

Discuss (Whole Class):
Since this is a practice task, you will need to determine what issues have come up in the individual work of the students that might benefit from a whole class discussion. This practice task includes a variety of procedural work including finding distance between points, verifying that line segments are parallel or perpendicular, adding vectors, and multiplying matrices where one factor represents a transformation and the other factor represents the vertices of an image. Make sure that students are confident with carrying out the procedural work, while also interpreting the meaning of the work in terms of the context of the airplane or the context of the geometry of the plane.

Aligned Ready, Set, Go: Connecting Algebra and Geometry 7.12H

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Ready, Set, Go!

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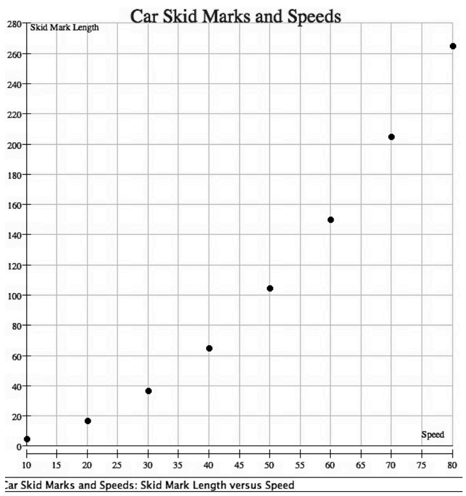
Ready

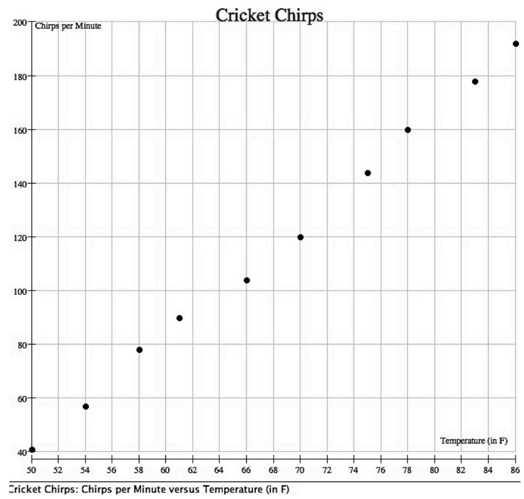
Topic: Scatterplots and trend lines

Examine each of the scatterplots shown below. If possible, make a statement about relationships between the two quantities depicted in the scatterplot.

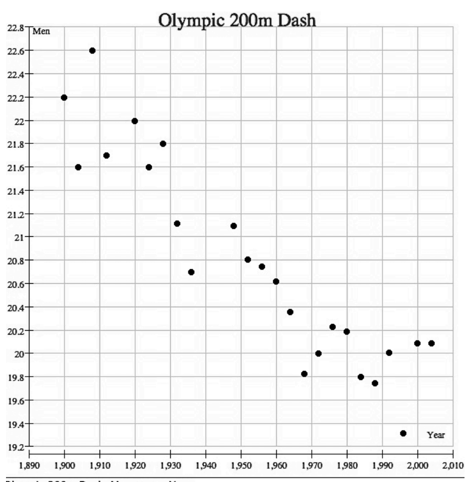
1.



2.



3.



4. For each scatterplot, write the equation of a trend line that you think best fits the data.

a. Trend line #1

b. Trend line #2

c. Trend line #3

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Set

Topic: Applications of vectors

Given: $\vec{u}:\langle -5,1\rangle$, $\vec{v}:\langle 3,5\rangle$, $\vec{w}:\langle 4,-3\rangle$. **Each of these three vectors represents a force pulling on an object—such as in a three-way tug of war—with force exerted in each direction being measured in pounds.**

5. Find the magnitude of each vector. That is, how many pounds of force are being exerted on the object by each tug? (Round to the nearest hundredth)

a. $\|\vec{u}\| =$

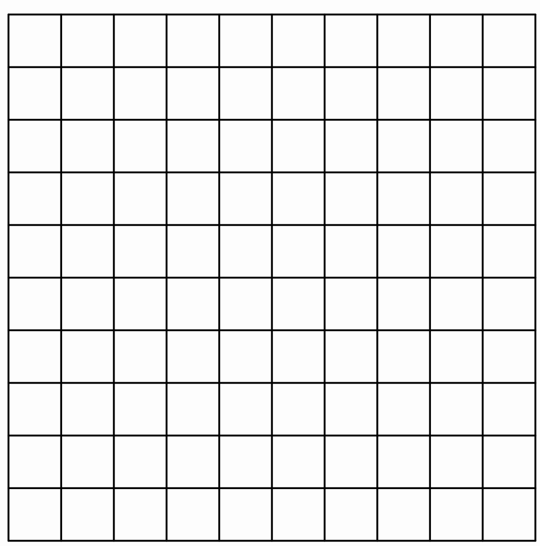
b. $\|\vec{v}\| =$

c. $\|\vec{w}\| =$

6. Find the magnitude of the sum of the three forces on the object.

$\|\vec{u}+\vec{v}+\vec{w}\| =$

7. Draw a vector diagram showing the resultant direction and magnitude of the motion resulting from this three-way tug of war.



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Go

Topic: Solving systems

Given: $\begin{cases} 4x - 4y = 7 \\ 6x - 8y = 9 \end{cases}$

8. Solve the given system in each of the following ways.

a. By substitution

b. By elimination

c. Using matrix row reduction

d. Using an inverse matrix