Secondary Mathematics I: An Integrated Approach
Module 4 Honors
Linear and Exponential Functions

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 4H – Linear and Exponential Functions

4.1 Classroom Task: Connecting the Dots: Piggies and Pools – A Develop Understanding Task
Introducing continuous linear and exponential functions **(F.IF.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.1

4.2 Classroom Task: Sorting Out the Change – A Solidify Understanding Task
Defining linear and exponential functions based upon the pattern of change (**F.LE.1, F.LE.2**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.2

4.3 Classroom Task: Where's My Change – A Practice Understanding Task
Identifying rates of change in linear and exponential functions (**F.LE.1, F.LE.2**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.3

4.4 Classroom Task: Linear, Exponential or Neither – A Practice Understanding Task
Distinguishing between linear and exponential functions using various representations (**F.LE.3, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.4

4.5 Classroom Task: Getting Down to Business – A Solidify Understanding Task
Comparing the growth of linear and exponential functions (**F.LE.2, F.LE.3, F.LE.5, F.IF.7**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.5

4.6 Classroom Task: Growing, Growing, Gone – A Solidify Understanding Task
Comparing linear and exponential models of population **(F.BF.1, F.BF.2, F.LE.1, F.LE.2, F.LE.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.6

4.6H Classroom Task: I Can See—Can't You? – A Solidify Understanding Task
Using secant lines to find the average rate of change **(F.IF.6)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.6H

4.7 Classroom Task: Making My Point – A Solidify Understanding Task
Interpreting equations that model linear and exponential functions (**A.SSE.1, A.CED.2, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.7

4.8 Classroom Task: Efficiency Experts – A Solidify Understanding Task
Evaluating the use of various forms of linear and exponential equations (**A.SSE.1, A.SSE.3, A.CED.2, F.LE.5**)
Ready, Set, Go Homework: Linear and Exponential Functions 4.8

4.9 Classroom Task: Up a Little, Down a Little – A Solidify Understanding Task
*Understanding and interpreting formulas for exponential growth and decay (**A.SSE.1, A.CED.2, F.LE.5, F.IF.7**)*
Ready, Set, Go Homework: Linear and Exponential Functions 4.9

4.10 Classroom Task: X Marks the Spot – A Practice Understanding Task
Solving exponential and linear equations (**A.REI.3)**
Ready, Set, Go Homework: Linear and Exponential Functions 4.10

4.1 Connecting the Dots: Piggies and Pools

A Develop Understanding Task

www.flickr.com/photos/teegardin

1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Create a mathematical model for the number of pennies in the piggy bank on day n.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute. Create a mathematical model for the number of gallons of water in the pool at t minutes.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. Create a mathematical model of the amount of money I will have in the account after m months.

4. At the end of the summer, I decide to drain the swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that it was draining at a rate of 3% every minute. Create a mathematical model of the gallons of water in the pool at t minutes.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?

4.1 Connecting the Dots: Piggies and Pools – Teacher Notes

A Develop Understanding Task

Special Note to Teachers: Problem number three uses the ideas of compound interest, but in an informal way. Students are expected to draw upon their past work with geometric sequences to create representations that they are familiar with. The formula for compound interest will be developed later in the module.

Purpose: This task builds upon students' experiences with arithmetic and geometric sequences to extend to the broader class of linear and exponential functions with continuous domains. The term "domain" should be introduced and used throughout the whole group discussion. Students are given both a discrete and a continuous linear function, and a discrete and a continuous exponential function. They are asked to compare these types of functions using various representations.

New Vocabulary:
Domain
Discrete function
Continuous function

Core Standards Focus:

F-IF3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F-BF1: Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps from a calculation from a context.

F-LE1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relations, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Launch (Whole Class):

Begin the lesson by helping students to read the four problems and understand the contexts. Since students are already comfortable with arithmetic and geometric sequences and their representations, these questions should be quite familiar, with no need for the teacher to offer a suggested path for solving them. Remind students that their mathematical models should include tables, graphs, and equations.

Explore (Small Group):

As students begin working, circulate among the groups to see that students understand the problems. Problems 1 and 2 are fairly straightforward, but there are possible interpretations that could lead to productive discussions in questions 3 and 4. In problem 3, they will need to consider that the account pays 3% on the principal and whatever interest is in the account from previous months. In problem 4, they will have to consider how to deal with the 3%. Watch for students that may try subtracting 3% of the original amount each time (using linear thinking), rather than taking 3% of the existing amount of water in the pool at the given time.

Select student work that makes use of tables, graphs and explicit equations. Listen for students that are noticing that the graphs of #1 and #3 should be unconnected points, while #2 and #4 will be connected.

Discuss (Whole Class):

Start the discussion by asking students to present a table, graph, and equation for problem #1. Be sure that the graph is unconnected points. Ask students what they know about the relationship described in #1. They should know it to be an arithmetic sequence.

Next, present a table, graph, and equation for problem #2. This graph should be a solid line. (If no student has a solid line graph, use a graph that is otherwise correct and ask students to consider if it would be possible to have points in between the ones that have been marked, based upon the current context. Once they have discussed that the water is filling continuously, fill in the rest of the line.)

Now ask students to compare the two functions. Create a chart of similarities and differences. Students should notice that they both have a constant rate of change, both are increasing or have a positive slope. They may not have noticed differences, so this is the time to highlight the difference between a continuous context (water filling) and a discrete context where pennies are added a few at a time, with no change in between. Start with how this difference shows in the graph, then proceed to the table. Often, students choose only whole number or integer inputs for their tables. If this is the case, ask them if using some fractions or decimal numbers for inputs would make sense in each of the contexts. Introduce the idea that the inputs for a function are the domain. The input on problem #1 is the number of days. Since money is only put in once a day, then it doesn't make sense to have inputs like ½ or 3.5. That makes the domain the set of whole numbers. (Assuming she started with 5 pennies on day 0). Discuss the domain of the function generated by problem #2. Students should recognize that it is the time, and the water in the pool is increasing continuously as time goes on. They should also recognize that the time measurement can't be negative, so the domain in this case is real numbers greater than or equal to 0, assuming that 0 is the time that they started filling the pool.

Proceed with the discussion of #3 and #4 in a similar fashion. Again emphasize that the domain of #3 is whole numbers. Tell students that sequences have whole number domains. Functions that are not discrete are not sequences, therefore, we do not use the terms arithmetic or geometric sequences even though they may exhibit similar growth patterns. More work will be done in the next two tasks to define linear and exponential functions by their patterns of growth, so the emphasis is this task needs to be on the difference between the terms discrete and continuous.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.1***

Ready, Set, Go!

www.flickr.com/photos/teegardin

Ready

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1. 4 , -20 , 100 , -500 , . . .

2. 3 , 5 , 8 , 12 , . . .

3. 64 , 48 , 36 , 27 , . . .

4. 1.5 , 0.75 , 0 , -0.75 , . . .

5. 40 , 10 , $\frac{5}{2}$, $\frac{5}{8}$, ...

6. 1, 11, 111, 1111, . . .

7. -3.6, -5.4, -8.1, -12.15, . . .

8. -64, -47, -30, -13, . . .

9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

Name: Linear and Exponential Functions | 4.1

Set

Topic: Discrete and continuous relationships

Identify whether the following statements represent a *discrete* or a *continuous* relationship.

10. The hair on your head grows ½ inch per month.

11. For every ton of paper that is recycled, 17 trees are saved.

12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily.

13. The average person laughs 15 times per day.

14. The city of Buenos Aires adds 6,000 tons of trash to its landfills every day.

15. During the Great Depression, stock market prices fell 75%.

Go

Topic: Slopes of lines

Determine the slope of the line that passes through the following points.

16. (-15 , 9) , (-10 , 4)

17. (0.5 , 4) , (3 , 3.5)

18. (50 , 85) , (60 , 80)

19.

x	y
-5	-20
-4	-17
-3	-14

20.

x	y
-1	-1
0	½
1	2

21.

x	y
-5	33
0	30
5	27

Need Help? Check out these related videos and internet sites:

Discrete vs. continuous: http://www.mathsisfun.com/data/data-discrete-continuous.html

Arithmetic and geometric sequences: http://home.windstream.net/okrebs/page131.html

Slope: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/slope-of-a-line

Linear relationships: http://www.mathsteacher.com.au/year7/ch15_linear/04_modelling/linear.htm

4.2 Sorting Out the Change

A Solidify Understanding Task

© 2012 www.flickr.com/photos/ 401(K) 2012

A. Identify the pattern of change in each of the relations and sort each relation into the following categories:
   - Equal differences over equal intervals
   - Equal factors over equal intervals
   - Neither

B. Be prepared to describe the pattern of change and to tell how you found it.

1.

x	$f(x)$
-30	-57
-25	-47
-20	-37
-15	-27
-10	-17
-5	-7
0	3

Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____

2.

$f(0) = -3,\ f(n+1) = \frac{5}{3} f(n)$

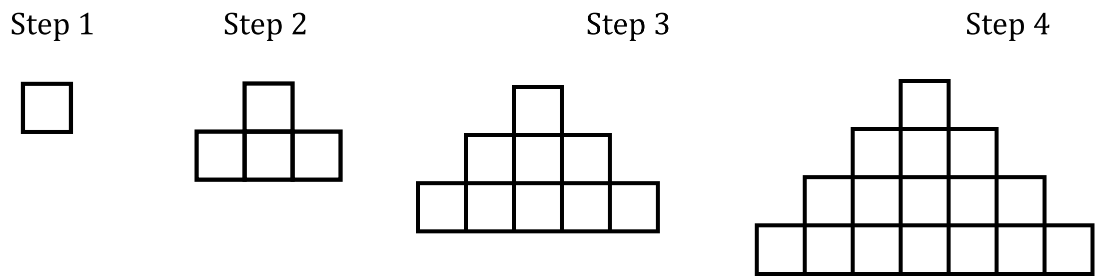
Type of pattern of change ______________________________

How I found the pattern of change:

____

____

____



3. The pattern of change in the perimeter of the figures from one step to the next.

Type of pattern of change ______________________________

How I found the pattern of change:

____
____
____

4. The pattern of change in the area of the figures from one step to the next.

Type of pattern of change ______________________________

How I found the pattern of change:

____
____
____

5. $y = ax - 3$

Type of pattern of change ______________________________

How I found the pattern of change:

____
____
____

6.

x	$f(x)$
-10	7
-5	7
-0	7
5	7
10	7
15	7
20	7

Type of pattern of change ______________________

How I found the pattern of change:

____

____

____

---

7. The height from the ground of a person on a ferris wheel that is rotating at a constant speed.

Type of pattern of change ______________________

How I found the pattern of change:

____

____

____

---

8. $y = x$

Type of pattern of change ______________________

How I found the pattern of change:

____

____

____

9.

x	$f(x)$
0	-5
4	-2
-4	-8
8	1
-8	-11

Type of pattern of change ______________________

How I found the pattern of change:

____

____

____

---

10. The algae population in a pond increases by 3% each year until it depletes its food supply and then maintains a constant population.

Type of pattern of change ______________________

How I found the pattern of change:

____

____

____

11.

x	$f(x)$
-5	238
-4	76
-3	22
-2	4
-1	-2
0	-4

Type of pattern of change ______________________

How I found the pattern of change:

____

____

____

12. The change in the height of the ball from one bounce to the next if the ball is dropped from a height of 8 feet and the ball bounces to 80% of its previous height with each bounce.

Type of pattern of change ______________________

How I found the pattern of change:

____
____
____

13.

x	y
3	-5
3	-10
3	-20
3	15
3	35
3	50

Type of pattern of change ______________________

How I found the pattern of change:

____
____
____

4.2 Sorting Out the Change – Teacher Notes
A Solidify Understanding Task

Purpose: The purpose of this task is to define linear and exponential functions based on their patterns of growth. In the past module, students identified arithmetic sequences by a constant difference between consecutive terms. That idea is extended in this task to identify linear functions as those in which one quantity changes at a constant rate per unit interval relative to the other. In the sequences module students identified geometric sequence as having a constant ratio between consecutive terms. In this task, they extend the idea to identify exponential functions as those that grow or decay by equal factors over equal intervals. Students will be challenged with several novel situations in this task, including tables that are out of order or with irregular intervals, a constant function, and story contexts that are neither linear nor exponential.

New Vocabulary:
Linear function
Exponential function

Core Standards Focus:

F-LE1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relations, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Launch (Whole Class):

Before distributing the task, ask students how they were able to identify arithmetic and geometric sequences in the last module. They should be able to answer that they looked for a constant difference between consecutive terms to identify arithmetic sequences and a constant ratio between consecutive terms to identify geometric sequences. Tell them that in this exercise they will be looking for something similar, but a little broader. The first category is equal differences over equal interval. The graph of a line might be given as an example. Identify two equal intervals of some size other than 1. Then demonstrate that the change in y is equal in both intervals. From there, ask students what equal factors over equal intervals might look like. A possible example might be a table like this:

x	y
5	100
10	150
15	225
20	337.5
25	506.25

Each line of the table represents an interval of 5. There is no constant difference between each line of the table, but the ratio between y values from one line to the next is 1.5.

Explore (Small Group):

Monitor students as they are working. The problems are designed to stretch student thinking about the representations that they worked with previously. When students are stuck on a particular problem, you may suggest that they create a different representation so they can either support their claim or to be able to identify the type of growth that is exhibited. Listen to discussions and identify problems that are generating statements that get to the heart of what it means to be a linear function or an exponential function, based on the pattern of growth.

Discuss (Whole Class):

Start the discussion with an interesting problem selected during the small group discussion that is identified to have equal growth over equal intervals. Have a group show how they were able to determine the pattern of growth. Ask the class what other problems seemed to fit into the category of "equal differences over equal intervals". If there is disagreement on a particular problem, let students justify their answer and other students to dispute the claim until they have arrived at consensus on the correct category. At this point, tell the class that this category of functions is called "linear". They are defined by a pattern of growth characterized by equal differences over equal intervals. The category includes all the sequences that were previously described as arithmetic, but is extended to continuous functions with constant change over equal intervals. Since students have worked with linear functions previously in module 1 and 2, they may talk about a constant rate of change or the slope as a way to describe linear functions.

Conduct the next part of the discussion in a similar fashion, focusing on the functions that show a growth pattern of equal factors over equal intervals. After a focused discussion of one of the problems and agreement on the problems that fall into the category, tell the class that this category of functions is called "exponential". Exponential functions are defined by a pattern of change that is equal factors over equal intervals. Geometric sequences are part of the larger category of exponential functions which also includes the continuous functions that exhibit equal factors over equal intervals.

End the discussion by focusing on why problem #4 is neither linear nor exponential. Students should be clear that there are many “regular” patterns of growth that are neither linear nor exponential.

Note: Be sure to have students keep their work, because it will be used in the next task.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.2***

Ready, Set, Go!

© 2012 www.flickr.com/photos/ 401(K) 2012

Ready

Topic: Rates of change in linear models

Say which situation has the greatest rate of change

1. The amount of stretch in a short bungee cord stretches 6 inches when stretched by a 3 pound weight. A slinky stretches 3 feet when stretched by a 1 pound weight.

2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.

3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.

4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

Set

Topic: linear rates of change

Determine the rate of change in each table below.

5.

x	y
-3	-13
-1	-5
0	-1
3	11

6.

x	y
-4	-4
0	4
2	8
6	16

7.

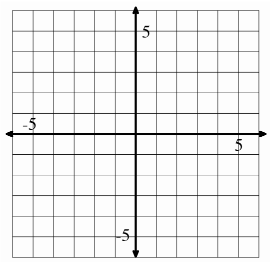
x	y
-10	-14
5	-8
25	0
50	10

Go

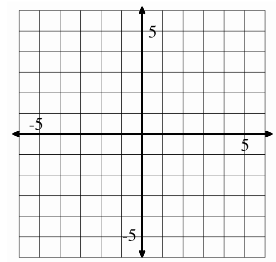
Topic: Graphing linear equations in slope-intercept form.

Graph the following equations

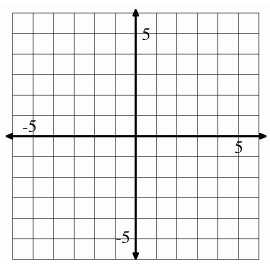
8. $y = 3x - 1$



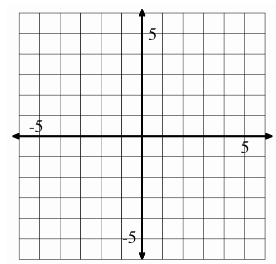
9. $y = -5x + 4$



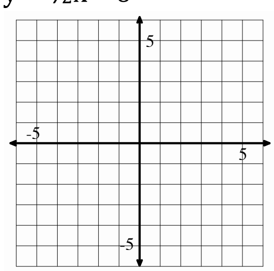
10. $y = x$



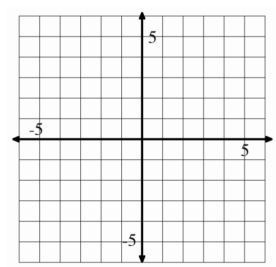
11. $y = -4$



12. $y = \frac{1}{2}x - 6$



13. $x = 3$



Need Help? Check out these related videos:

http://www.algebra-class.com/rate-of-change.html

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphs-using-slope-intercept-form

4.3 Where's My Change?
A Practice Understanding Task

Look through the problems that you worked with in the "Sorting Out the Change" task.

Choose two problems from your linear category (equal differences over equal intervals) and two problems from your exponential category (equal factors over equal intervals).

Add as many representations as you can to the problem you selected. For instance, if you choose problem #1 which is a table, you should try to represent the function with a graph, an explicit equation, a recursive equation, and a story context.

Identify the rate of change in the function. If the function is linear, identify the constant rate of change. If the function is exponential, identify the factor of change.

How does the rate of change appear in each of your representations?

4.3 Where's My Change – Teacher Notes

A Practice Understanding Task

Purpose:
The purpose of this task is for students to practice recognizing the patterns of growth in the various representations of linear and exponential functions. Students are asked to create tables, graphs, story contexts, and equations for linear and exponential functions so that they can articulate how the pattern of change is shown in each of the representations. They are also asked to calculate the rate of change for a linear function and the change factor for an exponential function.

Core Standards Focus:

F-LE1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relations, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

New Vocabulary:
Change factor

Launch (Whole Class):

Remind students of the definitions of linear and exponential functions that they worked on previously. For this task you may allow students to choose the linear and exponential problems that they decide to develop representations for or you may decide to assign different problems to each group so that there are more possibilities available for the whole group discussion. Tell students that they will need to find the rate of change of the linear functions and the "change factor" for the exponential functions. Tell them that since exponential functions don't have a constant rate of change, we identify the constant factor over the equal interval. It is called the change factor.

Explore (Small Group):

Monitor students as they work. Because the focus of the discussion will be on how the pattern of growth is shown in the various representations, be alert for useful comments and discussions.

Discuss (Whole Class):

Start the whole group discussion with one of the linear functions. Have a group describe how they built each of the representations from the one that was given and then how they found the rate of change. Then ask the class how they could use each of the representations to find the rate of change and identify that the function is linear. At this point, develop a chart like this:

Linear functions: Equal differences over equal intervals	
Representation:	Identify the rate of change by:
Tables	
Graphs	
Equations	

Continue the discussion in a similar fashion with one of the exponential functions. Create a second chart like this:

Exponential functions: Equal factors over equal intervals	
Representation:	Identify the change factors by:
Tables	
Graphs	
Equations	

If time permits, you may wish to have other groups show their problems to support and/or refine the charts. This part of the discussion could focus on developing the idea that the rate of change is positive for increasing linear functions and negative for decreasing linear functions. If the first linear function that was discussed was increasing, then ask for a decreasing one to be presented (or vice versa). Then ask students how they can predict from the rate of change whether the function is increasing or decreasing. Ask if the same is true for exponential functions. Then have a group show an exponential function that is decreasing if the first one discussed was increasing. Ask students how the change factor of an exponential function determines whether the function is increasing or decreasing.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.3***

Name: Linear and Exponential Functions | 4.3

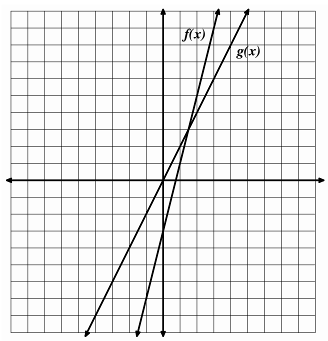
Ready, Set, Go!

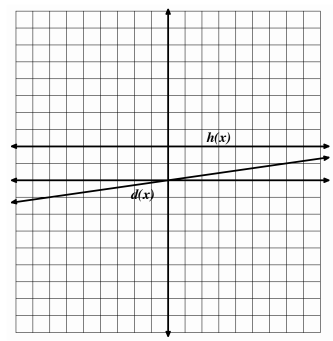
© 2012 www.flickr.com/photos/JennuineCaptures

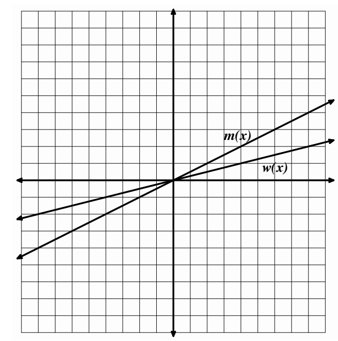
Ready

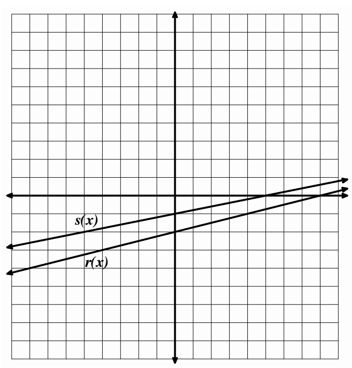
Topic: Recognizing the greater rate of change when comparing 2 linear functions or 2 exponential functions.

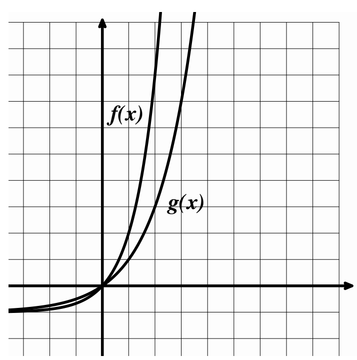
Decide which function is growing faster

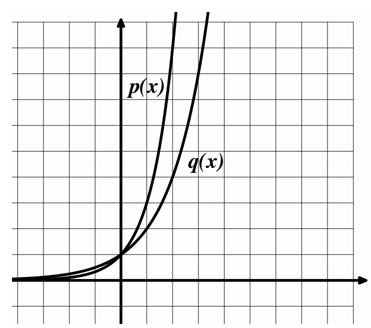
1.


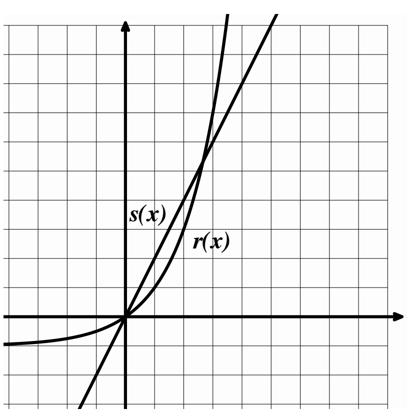
2.


3.


4.


5.


6.




7a. Examine the graph at the left from 0 to 1.
Which graph do you think is growing faster?

b. Now look at the graph from 2 to 3.
Which graph is growing faster in this interval?

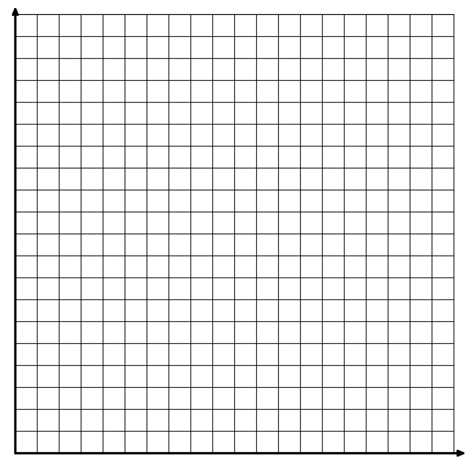
Set

Topic: Representations of linear and exponential functions.

In each of the following problems, you are given one of the representations of a function. Complete the remaining 3 representations. Identify the rate of change for the relation.

9. **Equation**:

Graph



Table

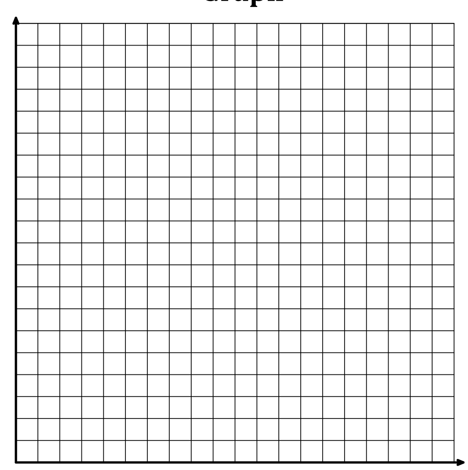
Rides	Cost

Create a context

You and your friends go to the state fair. It costs $5 to get into the fair and $3 each time you go on a ride.

10. **Equation**:

Graph



Table

Time	Amount
1	18
2	53
3	162
4	486
5	1458
6	4374

Create a context

Go

Topic: Recursive and explicit equations of geometric sequences.

Write the recursive and explicit equations for each geometric sequence.

10. Marissa has saved $1000 in a jar. She plans to withdraw half of what's remaining in the jar at the end of each month.

11.

Time (Days)	Number of Bacteria
1	10
2	100
3	1000
4	10000

12.

Folds in paper	Number of rectangles
0	1
1	2
2	4
3	8

13. 1024, 256, 64, 16, . . .

14. 3, 9, 27, 81, . . .

Need Help? Check out these related videos:

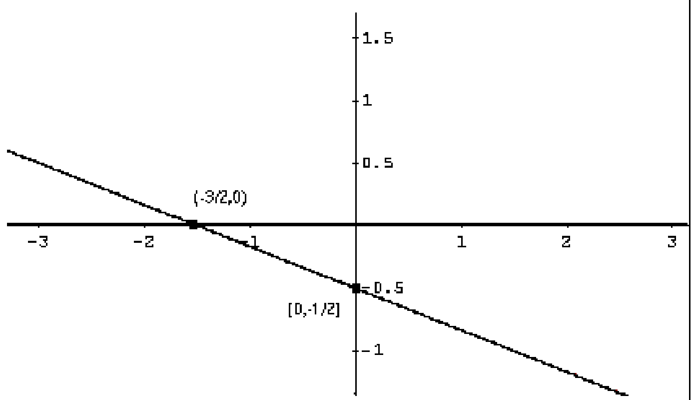
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/identifying-exponential-models

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/linear--quadratic--and-exponential-models

4.4 Linear, Exponential or Neither?
A Solidify Understanding Task

For each representation of a function, decide if the function is linear, exponential, or neither. **Give at least 2 reasons for your answer.**

1.



2.

Tennis Tournament

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

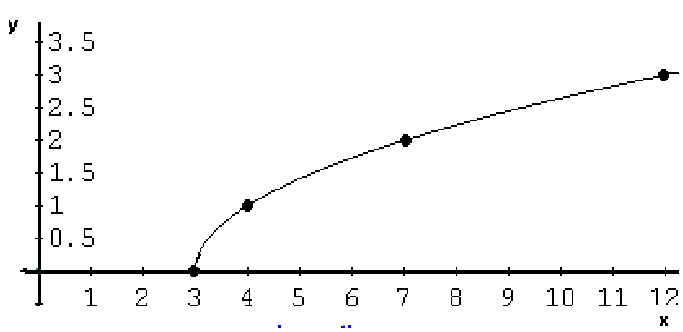
There are 4 players remaining after 5 rounds.

3.

$$y = 4x$$

4.
This function is decreasing at a constant rate

5.



6.
A person's height as a function of a person's age (from age 0 to 100)

7.

$$-3x = 4y + 7$$

8.

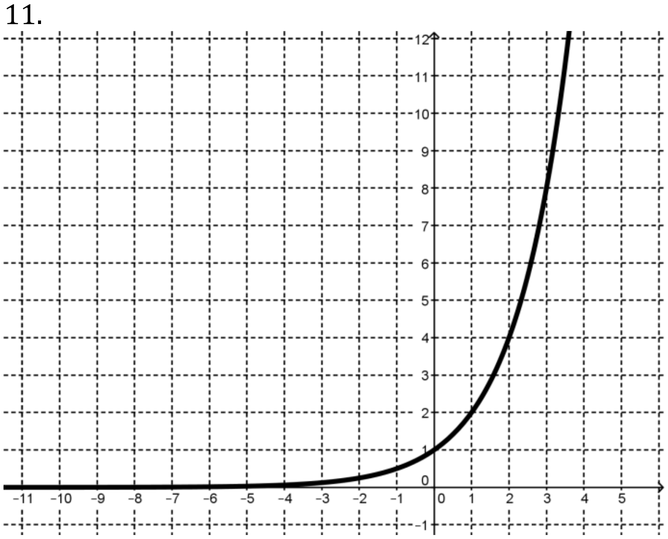
x	*y*
-2	23
0	5
2	-13
4	-31
6	-49

9.

Height in Inches	Shoe Size
62	6
74	13
70	9
67	11
53	4
58	7

10.
The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

11.



12.
The time it takes you to get to work as a function the speed at which you drive

13. $y = 7x^2$	14. Each point on the graph is exactly 1/3 of the previous point.
15. $f(1) = 7, f(2) = 7, f(n) = f(n-1) + f(n-2)$	16. $f(0) = 1, f(n+1) = \frac{2}{3}f(n)$

4.4 Linear, Exponential, or Neither – Teacher Notes
A Practice Understanding Task

Purpose:
The purpose of this task is to develop fluency in determining the type of function using various representations. The task also provides opportunities for discussion of features of the functions based upon the representation given.

Core Standards Focus:

F-LE1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relations, or two input-output pairs (include reading these from a table).

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Launch (Whole Class):

Refer the class to the linear and exponential charts made in the previous task. In this task they will be looking at a number of functions, some linear, some exponential, some neither. They need to identify what kind of function is shown in each problem and provide two reasons for their answers. One reason may be fairly easy, based upon the chart, the second one will require them to stretch a little.

Explore (Small Group):

During the small group work, listen for problems that are generating controversy. If students feel that a particular problem is too vague, ask them what information would be necessary for them to decide and why that information is important. If there are groups that finish early, you may ask them to go back through the problems and think about everything they know about the function from the information that is given.

Discuss (Whole Class):

Start the discussion by going through each problem and asking a group to say how they categorized it and why. After each problem, ask if there was any disagreement or if another group could add another reason to support the category. If there is disagreement, ask students to present their arguments more formally and add at least one representation to support their claim.

If time permits, you may choose to some problems and ask students to tell everything they know about the function from what is given. For instance, you may choose #2:

Tennis Tournament

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

There are 4 players remaining after 5 rounds.

Some possible answers would be:

- It is an exponential function.
- It is a geometric sequence.
- It is a discrete function.
- The tournament has one more round.
- The change factor is ½ .
- The function is decreasing.
- Shape of the graph, etc.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.4***

Ready, Set, Go!

©2012 www.flickr.com/photos/scenesfromamemory

Ready

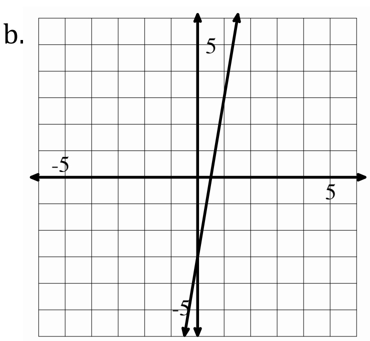
Topic: Comparing rates of change in both linear and exponential situations.

Identify whether situation "a" or situation "b" has the greater rate of change.

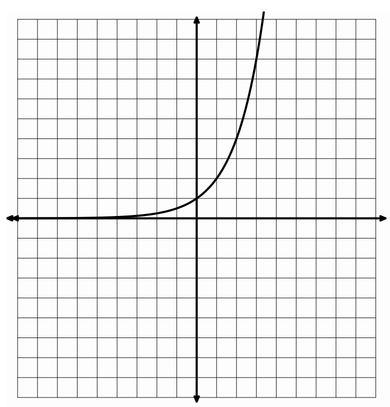
1. a.

x	y
-10	-48
-9	-43
-8	-38
-7	-33

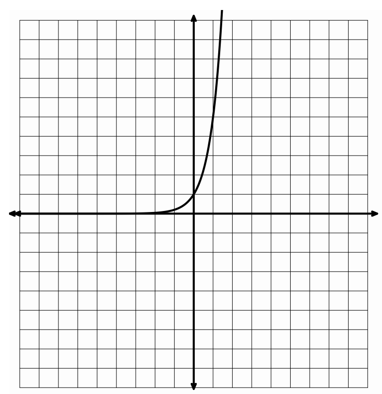
b.



2. a.



b.



3. a. Lee has $25 withheld each week from his salary to pay for his subway pass.

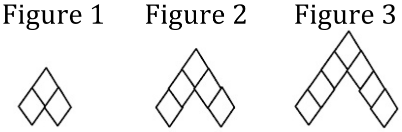
b. Jose owes his brother $50. He has promised to pay half of what he owes each week until the debt is paid.

4. a.

x	6	10	14	18
y	13	15	17	19

b. The number of rhombi in each shape.

Figure 1 Figure 2 Figure 3



5. a. $y = 2(5)^x$

b. In the children's book, *The Magic Pot,* every time you put one object into the pot, two of the same object come out. Imagine that you have 5 magic pots.

Set

Topic: Recognizing linear and exponential functions.

For each representation of a function, decide if the function is linear, exponential, or neither.

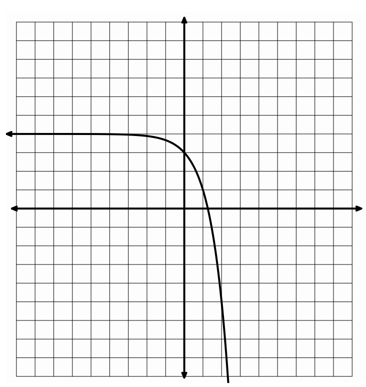
6. The population of a town is decreasing at a rate of 1.5% per year.

7. Joan earns a salary of $30,000 per year plus a 4.25% commission on sales.

8. 3x +4y = -3

9. The number of gifts received each day of "The 12 Days of Christmas" as a function of the day. ("On the 4th day of Christmas my true love gave to me, 4 calling birds, 3 French hens, 2 turtledoves, and a partridge in a pear tree.")

10.



11.

Side of a square	Area of a square
1 inch	1 in^2
2 inches	4 in^2
3 inches	9 in^2
4 inches	16 in^2

Name: Linear and Exponential Functions | 4.4

Go

Topic: Geometric means

For each geometric sequence below, find the missing terms in the sequence.

12.

x	1	2	3	4	5
y	2				162

13.

x	1	2	3	4	5
y	1/9			-3	

14.

x	1	2	3	4	5
y	10				0.625

15.

x	1	2	3	4	5
y	g				gz^4

16.

x	1	2	3	4	5
y	-3				-243

Need Help? Check out these related videos and internet sites:

Sequences http://www.youtube.com/watch?v=THV2Wsf8hro

4.5 Getting Down to Business

A Solidify Understanding Task

©2012 www.flickr.com/photos/pretpriemac

Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

a. Express the projected net incomes in these two business plans as recursive formulas.

b. Write an explicit equation for the net income as a function of time for each company's business plan.

c. Compare your answers in a and b. How are the two representations similar? How do they differ? What relationships are highlighted in each representation?

d. Explain why if both companies are able to meet their net income growth goals, the net income of Computafest will eventually be larger than that of Calcu-rama. In what year will the net income of Computafest be larger than that of Calcu-rama?

4.5 Getting Down to Business – Teacher Notes

A Solidify Understanding Task

Special Note: Use of technology tools such as graphing calculators is recommended for this task.

Purpose:

The purpose of this task is to compare the rates of growth of an exponential and a linear function. The task provides an opportunity to look at the growth of an exponential and a linear function for large values of *x*, showing that increasing exponential functions become much larger as *x* increases. This task is a good opportunity to model functions using technology and to discuss how to set appropriate viewing windows for functions. The task also revisits comparisons between explicit and recursive equations, leading to a discussion of whether this particular situation should be modeled using discrete or continuous functions.

Core Standards Focus:

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

F.IF Analyze functions using different representations

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

★For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot2^n$.

Launch (Whole Class):

Start with a short discussion of the context to be sure that students understand the problem situation. As part of this discussion, clarify the choice of units and scale. Students may choose to use 5 million with million being the unit or as 5,000,000 in their equations. (They will probably find it easier to use millions as a unit, but they will need to interpret the scale on their graphs and be consistent in their equations.) Be sure that students understand terms like "net income" so that they know what the problem is asking. When students understand the problem, set them to work on the task, starting with parts a, b, and c.

Explore (Small Group):

Monitor students as they work on the task. Be prepared to redirect students that may not think of one function as linear, based on the constant growth, and the other as exponential based on the 15% growth factor. Be sure that students have discussed their answers to "c" before returning to the whole group discussion. The discussion for "d" will follow later.

Discuss and Re-launch (Whole Class):

Have a group that has written the explicit and recursive equations correctly present their work. Ask the class which represents a linear function and which is exponential and how can they tell in both recursive and explicit form. Ask how the growth pattern is shown in the equations. Finally ask if the functions should be modeled as discrete function or exponential. The assumption in writing a recursive formula is that the function is a sequence with discrete terms. Ask why the companies might choose to model it either way. They may choose a continuous model because they feel that the net income is increasing on a steady basis across the year, so it makes sense to fill in all the points on the graph and use an explicit formula. They may choose a discrete model because there are fluctuations in income during the year, with the net income increasing. If they can't predict the fluctuations, they may choose to use a discrete function, modeling with just one point each year.

Once students have discussed the equations, ask students to focus on the explicit equations and complete part "d" of the task. Encourage the use of technology, either graphing calculators or computers with programs such as NCTM Core Tools.

When students have completed their work, ask a group to present their tables showing the projected net income of the two businesses. Ask how they could find where the net income of the two businesses would be the same using their tables. Then have a group present their graphs and demonstrate how to find the year where Computafest exceeds the net income of Calcu-rama. (You may ask how to use the equations to find where the net incomes will be equal, but students will not be able to find an analytic solution to the equation.)

Conclude the task with a discussion of the end behavior of the two functions. How much will each company be making in 10 years, 20 years, etc.? Trace the graphs and look at the difference between the net incomes over time. Ask why an exponential function becomes so much larger than a linear function over time.

Aligned Ready, Set, Go: *Linear and Exponential Functions 4.5*

Ready, Set, Go!

©2012 www.flickr.com/photos/pretpriemac

Ready

Topic: Comparing arithmetic and geometric sequences

The first and 5th terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

1.

Arithmetic	4				324
Geometric	4				324

2.

Arithmetic	3				48
Geometric	3				48

3.

Arithmetic	-6250				-10
Geometric	-6250				-10

4.

Arithmetic	-12				-0.75
Geometric	-12				-0.75

5.

Arithmetic	-1377				-17
Geometric	-1377				-17

Name: Linear and Exponential Functions | 4.5

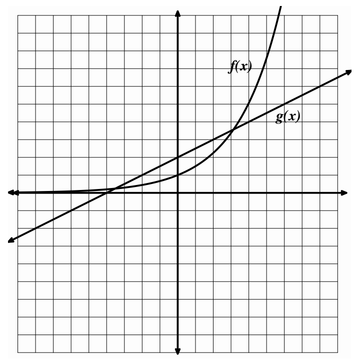
Set

Topic: comparing the rates of change of linear and exponential functions.

Compare the rates of change of each pair of functions by identifying the interval where it appears that *f(x)* is changing faster and the interval where it appears that *g(x)* is changing faster. Verify your conclusions by making a table of values for each equation and exploring the rates of change in your tables.

6. $f(x) = (1.5)^x$

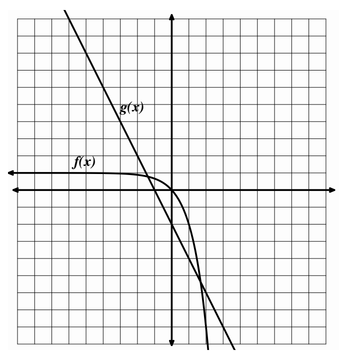
   $g(x) = \frac{1}{2}x + 2$



x	$f(x)$	$g(x)$

7. $f(x) = -3^x + 1$

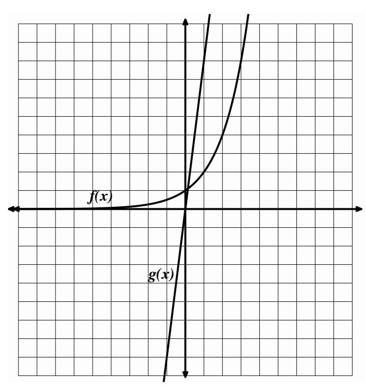
   $g(x) = -2x - 2$



x	$f(x)$	$g(x)$

8. $f(x) = 2^x$

   $g(x) = 8x$



x	$f(x)$	$g(x)$

Go

Topic: Writing explicit equations for linear and exponential models.

Write the explicit equation for the tables and graphs below.

9.

x	$f(x)$
2	-4
3	-11
4	-18
5	-25

10.

x	$f(x)$
-1	2/5
0	2
1	10
2	50

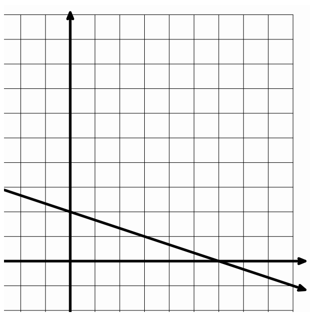
11.

x	$f(x)$
2	-24
3	-48
4	-96
5	-192

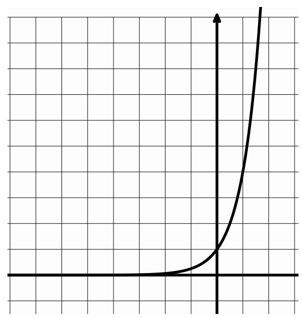
12.

x	$f(x)$
-4	81
-3	27
-2	9
-1	3

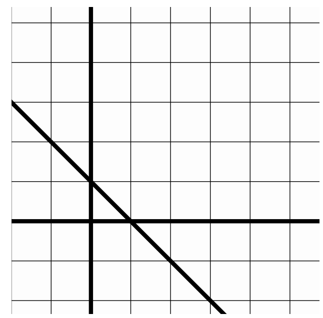
13.



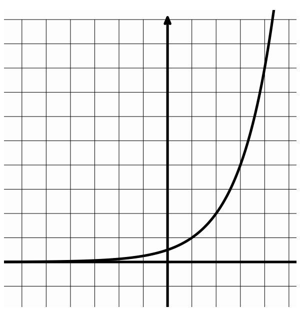
14.



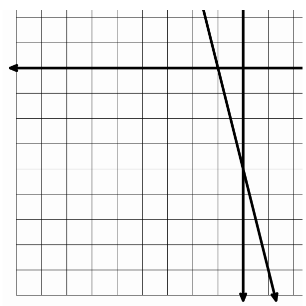
15.



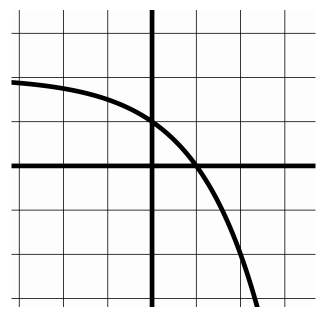
16.



17.



18.



Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/equations-of-sequence-patterns

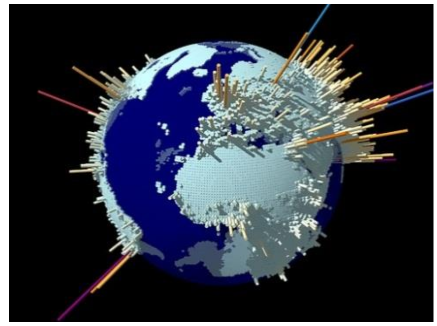
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/geometric-sequences--introduction

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/exponential-growth-functions

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/exponential-decay-functions?v=AXAMVxaxjDg

4.6 Growing, Growing, Gone

A Solidify Understanding Task



©www.flickr.com/photos/arenamontanus

1. The U.S. population in 1910 was 92 million people. In 1990 the population was 250 million. Create both a linear and an exponential model of the population from 1910 to 2030, with projected data points at least every 20 years, starting in 1910.

2. The actual U.S. population data (in millions) was:
   1930: 122.8
   1950: 152.3
   1970: 204.9
   Which model provides a better forecast of the U.S. population for the year 2030? Explain your answer.

4.6 Growing, Growing, Gone – Teacher Notes
A Solidify Understanding Task

Special Note: Use of technology tools such as graphing calculators is recommended for this task.

Purpose:

The purpose of this task is for students to use their understanding of linear and exponential patterns of growth to model the growth of a population. Students are given two data points and asked to create both an exponential and a linear model containing the points. Students may draw upon their experience with arithmetic and geometric means to develop new points in the model. The task provides opportunities to create tables, equations, and graphs and use those representations to argue which model is the best fit for the data.

Core Standards Focus:

F.BF Build a function that models a relationship between two quantities

F. BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.IF Analyze functions using different representations

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

★For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot 2^n$.

Related Standards:

F.BF.1 Write a function that describes a relationship between two quantities.
NQ.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Launch (Whole Class):

Start the task by discussing the growth of the earth's population. Ask students if they have thought about what a mathematical model for population might look like. Many students will suppose that population is growing fast, and may say that they have heard that it is "growing exponentially". Ask students to predict what they think a graph of the population vs time would look like.

Explore (Small Group):

Distribute the task and monitor students as they are working. One of the issues that students will need to address is to select the units on the time. Some groups may choose to think about the year 1910 as year 0, others may choose to call it 1910 or 10. This may lead to an interesting conversation later, and some differences in writing equations. Watch that students are being consistent in their units and ask questions to help support them in keeping units consistent in their work.

Once the units of time are determined, the problem of creating a linear model is much like the work that students have done in finding arithmetic means. When they begin to work on the exponential model, they will probably find difficulty if they try to write and solve equations to get the points in their model between 1910 and 2030. Encourage them to use other strategies, including developing a table. This requires students to guess at the growth factor for the model, and build a table based on that growth factor. Technology such as graphing calculators or computers will provide important tools for this task. The following table shows a possible guess and check strategy for the exponential model developed using a spreadsheet. The change factors used in each column are shown in the top row. Two different ways of labeling the years are shown in the first two columns. Cells were rounded to reflect the precision of the given information in the problem.

			5%	10%	20%	25%	27%	28%	28.5%
1910	0	92	92	92	92	92	92	92	92
1930	1		97	101	110	115	117	118	118
1950	2		101	111	132	144	148	151	152
1970	3		107	122	159	180	188	193	195
1990	4		112	135	191	225	239	247	251
2010	5		117	148	229	281	304	316	322
2030	6	230	123	163	275	351	386	405	414
			Too low	Too low	Too low	Too low	Too low	Too low	Just Right

Look for students who have modeled the situations with tables, equations and graphs for the discussion.

Discuss (Whole Class):

Before presenting either of the two models, get the class to come to consensus about which time scale to use consistently through the discussion. It may not be an issue because all of the groups chose the same strategy, but if not, it will be important to reconcile the two scales and the differences that they might create in equations before starting the discussion of the two models.

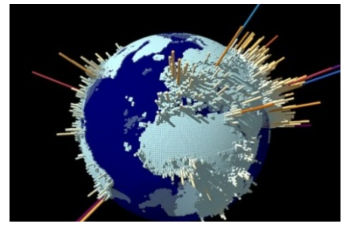
Start the discussion with a table of the linear model. Ask the presenter how they developed the table, expecting an answer that is related to repeatedly finding the arithmetic mean between two terms. Continue with a presentation of both the graph of the linear model and the equation. You can expect some student to say that they started with a graph of the line between the beginning and end points and constructed the table using the points on the graph. Ask the class how that strategy is related to the strategy of building a table by “averaging” the terms.

Next, have a group present a table of their exponential model. If the whole class hasn’t found the change factor, ask the group to show how they found the change factor, probably with successive approximations, as shown in the table above. It may also be useful to ask them to show how they used the technology to generate the table, with particular focus on how the formula they used relates to the definition of an exponential function. Continue with a graph and an equation of the exponential model.

If it hasn't already been done, use the technology to show the graph of the two models together in the same viewing window. Discuss how to find a viewing window that makes sense with the equations that were written. The two models are not dramatically different in many viewing windows, so you may need to model how to adjust the viewing window to get the graphs to appear distinct. Next, turn the discussion to the question of which model seems to model the actual data. Use the technology to place the given points on the same graph as the two mathematical models and ask students to justify arguments for the two models. Although students may make reasonable arguments for either model based on selecting particular points of focus, it will be important to tell students that population growth is often modeled with an exponential function because it is the kind of growth that is based upon the population that exists at a given time. In simple terms, if animals in a particular population can reproduce at a certain rate per year, the number of animals in the population the next year depends on how large the population was to start with. The more animals there are to reproduce, the more animals there will be in the future. The principle is the same idea as compound interest and many other situations that are modeled with exponential functions. Of course, the exponential pattern of growth will continue only until the population depletes the resources it needs to survive.

Aligned Ready, Set, Go: *Linear and Exponential Functions 4.6*

Ready, Set, Go!

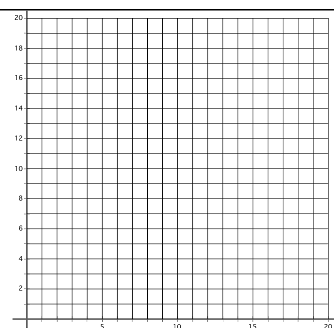
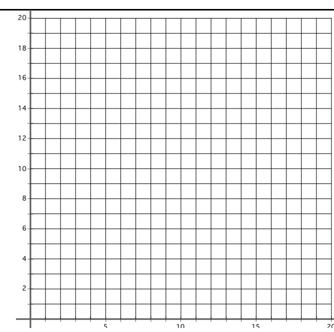


©www.flickr.com/photos/arenamontanus

Ready

Topic: Comparing Linear and Exponential Models

Compare different characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 4 + 3x$	$y = 4(3^x)$
1. Type of growth		
2. What kind of sequence corresponds to each model?		
3. Make a table of values	x | y	x | y
4. Find the rate of change		
5. Graph each equation. Compare the graphs. What is the same? What is different?		
6. Find the y-intercept for each function.		

7. Find the y-intercepts for the following equations

a) $y = 3x$

b) $y = 3^x$

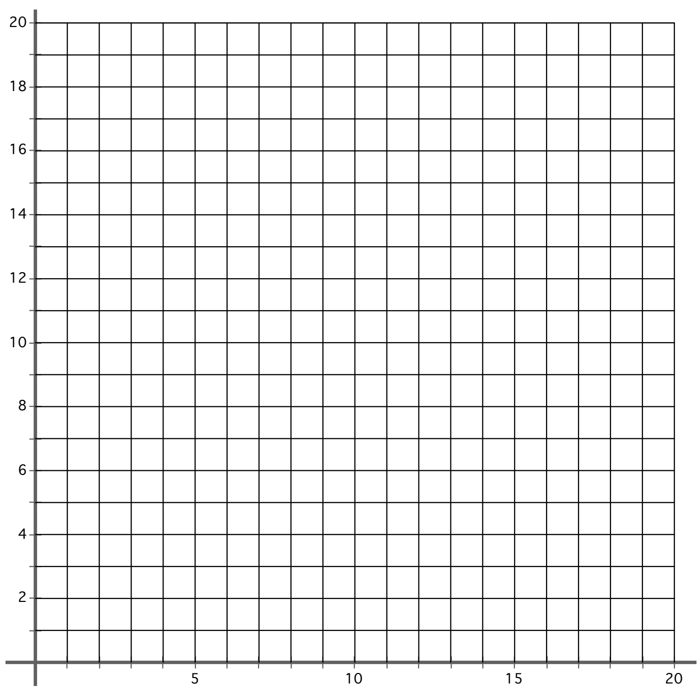
8. Explain how you can find the y-intercept of a linear equation and how that is different from finding the y-intercept of a geometric equation.

Set

Topic: Finding patterns

Use the picture below to answer questions 9-12

9. Graph.



10. Table

Stage	# of small triangles
1	
2	
3	
4	
5	
⋮	
10	

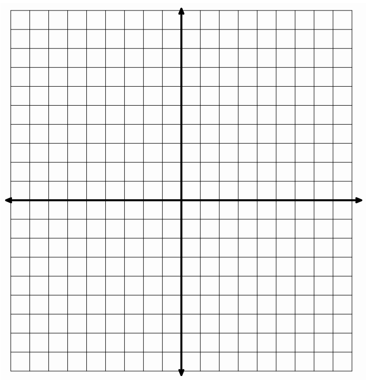
11. Write an explicit function to describe the pattern

Go

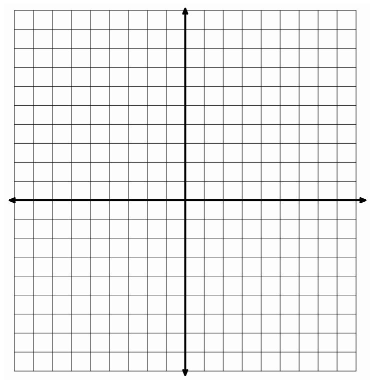
Topic: Solving systems through graphing.

Find the solution of the systems of equations by graphing.

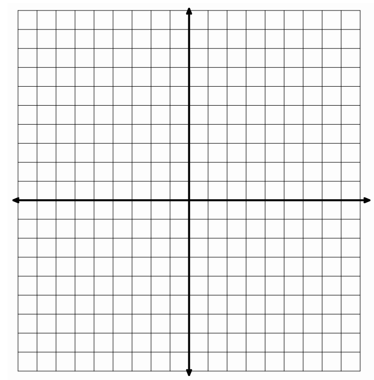
12. $\begin{cases} y = -x \\ y = 3x - 4 \end{cases}$



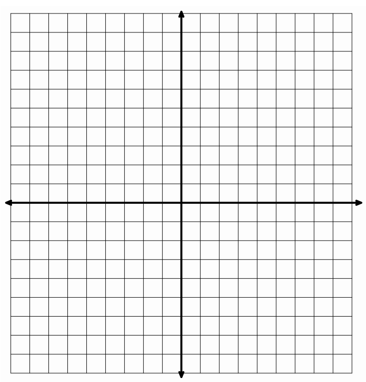
13. $\begin{cases} 2x + y = -6 \\ y = 6 \end{cases}$



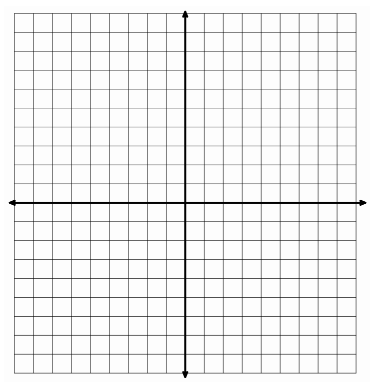
14. $\begin{cases} y = 2x - 2 \\ x + 3y = 15 \end{cases}$



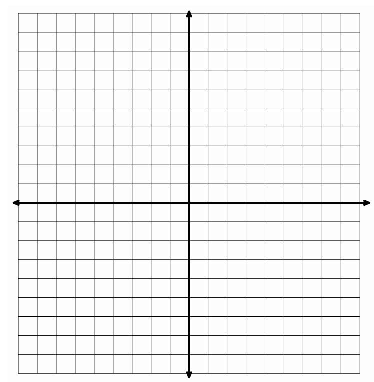
15. $\begin{cases} y + 3 = 6x - 2 \\ y - 2x + 1 = 4(x - 1) \end{cases}$



16. $\begin{cases} y = -(x - 4) \\ y - 2x - 1 = 0 \end{cases}$



17. $\begin{cases} y = 3(x - 2) \\ y + x - 2 = 4(x - 1) \end{cases}$



Need Help? Check out these related videos:

Comparing Linear and exponential functions:

http://www.khanacademy.org/math/algebra/algebra-functions/v/recognizing-linear-functions

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/identifying-exponential-models

4.6H I Can See—Can't You?

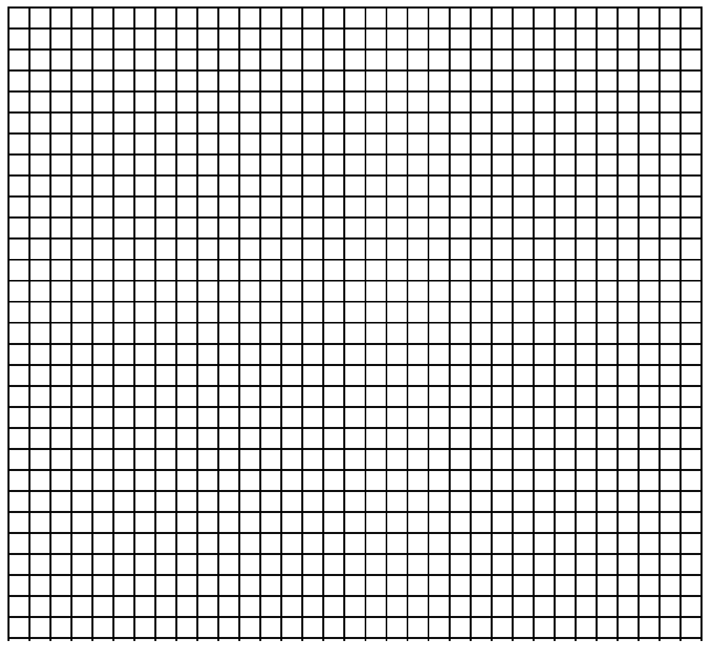
A Solidify Understanding Task

Kwan's parents bought a home for $50,000 in 1997 just as real estate values in the area started to rise quickly. Each year, their house was worth more until they sold the home in 2007 for $309,587.

1. Model the growth of the home's value from 1997 to 2007 with both a linear and an exponential equation. Graph the two models below.

Linear model:

Exponential model:



2. What was the change in the home's value from 1997 to 2007?

The average rate of change is defined as the change in y (or *f(x)*) divided by the change in x.

3. What was the average rate of change of the linear function from 1997 to 2007?

4. What is the average rate of change of the exponential function in the interval from 1997 to 2007?

5. How do the average rates of change from 1997 to 2007 compare for the two functions? Explain.

6. What was the average rate of change of the linear function from 1997 to 2002?

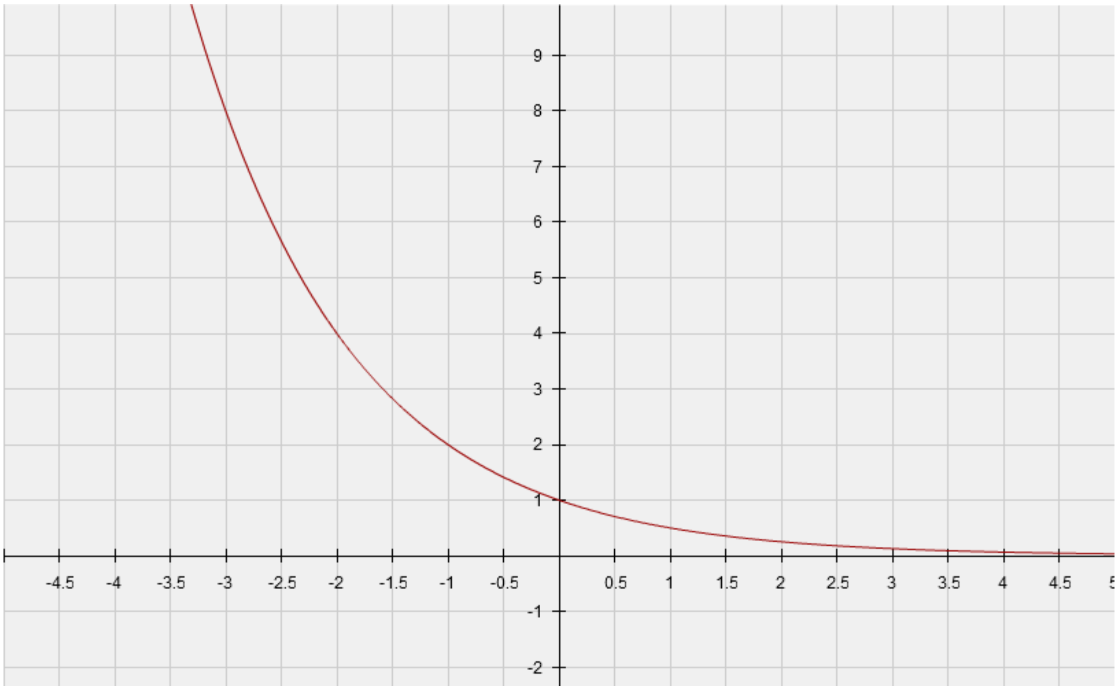
7. What is the average rate of change of the exponential function in the interval from 1997 to 2002?

8. How do the average rates of change from 1997 to 2002 compare for the two functions? Explain.

9. How can you use the equation of the exponential function to find the average rate of change over a given interval?

   How does this process compare to finding the slope of the line through the endpoints of the interval?

Consider the following graph:



10. What is the equation of the graph shown?

11. What is the average rate of change of this function on the interval from x = -3 to x = 1?

12. What is the average rate of change of this function in the interval from x = -3 to x = 0?

13. What is the average rate of change of this function in the interval from x = -3 to x = -1?

14. What is the average rate of change of this function in the interval from x = -3 to x = -1.5?

15. Draw the line through the point at the beginning and end of each of the intervals in 11, 12, 13 and 14. What is the slope of each of these lines?

16. Which of these average rates of change best represents the change at the point (-2, 4)? Explain your answer.

17. How does the average rate of change compare to the change factor for an exponential function? What is described by each of these quantities?

I Can See—Can't You? – Teacher Notes
A Solidify Understanding Task

Purpose:
The purpose of this task is to introduce students to the idea of the average rate of change of a function in a given interval. Students will compare the average rate of change in a given interval for both a linear and an exponential function. This will develop the idea of the slope of the secant line through the endpoints of the interval is equal to the average rate of change of the function in the interval. Students will calculate average rates of change in an increasingly smaller interval and be asked to consider how they could model the instantaneous rate of change at a given point. This task introduces many ideas that are typically taught in a Calculus course, in the context of linear and exponential functions.

Core Standards Focus:

F.IF Interpret functions that arise in applications in terms of the context

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Utah Honors: Represent average rate of change as the slope of the secant line.

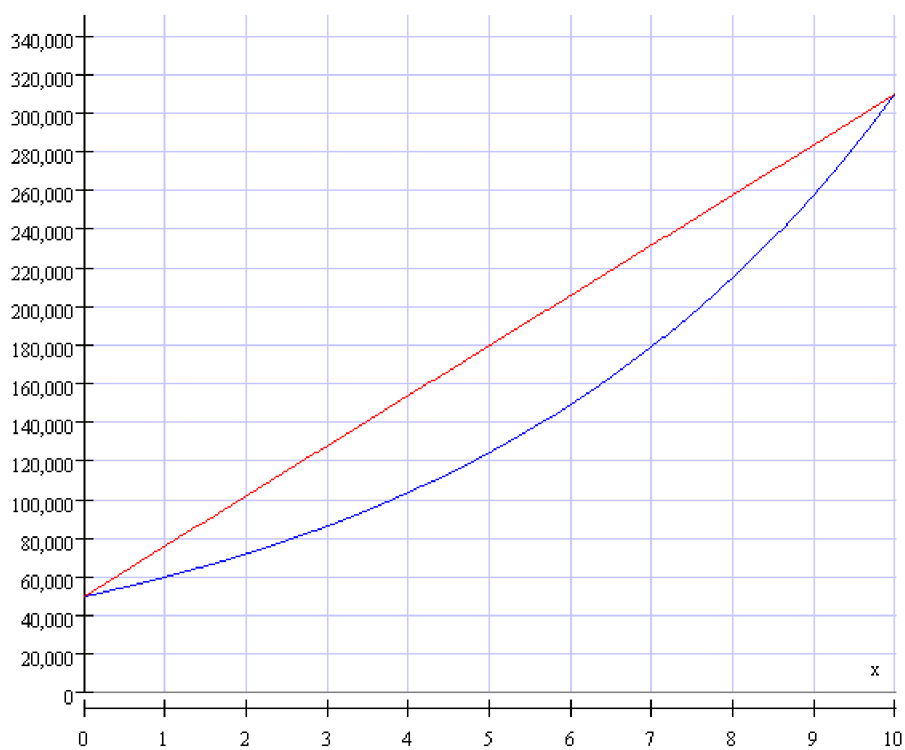
Vocabulary:
Average rate of change
Secant line
Tangent line

Launch 1 (Whole Group)
Before starting, be sure that students have graphing calculators or other technology tools available. This task begins like Growing, Growing, Gone, so the idea of modeling two points with both a linear and exponential model is already familiar. Begin this lesson by providing necessary background information about home prices and reading the problem situation. Ask students to develop the linear and exponential models for the situation and calculate the change, problems 1 and 2.

Explore 1 (Small Group)
Most students will probably create their exponential model using the same strategies that they used in Growing, Growing, Gone. Many will guess at a change factor at create a table like this one, which shows 20% growth each year (the right change factor for this data).

	Year	Value
1997	0	50,000
1998	1	60000
1999	2	72000
2000	3	86400
2001	4	103680
2002	5	124416
2003	6	149299.2
2004	7	179159
2005	8	214990.8
2006	9	257989
2007	10	309586.8

Ask students are working on the graph, you may need to help them to consider a useful viewing window so that they can see all the graphs that relate to this context. An example would be:



Discuss (Whole Class):

Once students have created equations and graphs, project a graph of the equations. Ask students how the calculated the change in the interval and how it appears on the graph. Students should describe subtracting the y-values at the beginning and end of the interval. Next, ask students how they could find the average rate of change of each of the linear functions over the interval from $x = 0$ to $x = 10$. The word "average" may throw them off a bit, but remind them how they have found the rate of change on a linear function previously. Then ask how they might calculate average rate of change for the exponential function. Students will probably draw upon their previous experience with averages and think about ways to add up numbers and divide, like when they have calculated means. Remind them that in this case they don't have any rates to add up, so this method will not work. Help students to think about the idea that the average rate of change will again be the change in y values, divided by the change in x.

Explore (Small Group): Let students work on the remaining problems. Monitor students as they work, especially in the beginning as they discuss how to find the change in y, divided by the change in x. Listen for students describe using the graph and the equation. If students are struggling, the graph will probably be the easiest way to get them started, but they will need to consider how to use the equations by problem 9. After they have used both the equation and the graph to find the average rate of change, then encourage them to use both strategies in problems 11-14. In #14 the graph will not be accurate enough, so they should use the equation and a calculator.

Watch for students that have articulated a process for using the equation to find average rate of change and for students that have created a formula (even if the notation isn't quite right) for using the equation so that these strategies can be shared in the whole group discussion. Also listen for students who are recognizing that the formula or process that they are working on is the same as finding the slope of the secant line (this term will be defined in the whole group discussion.)

Discuss (Whole Group): Start the whole group discussion with questions 3 and 4. Have students show both how they used the graph and the equation for each. Students should be noticing that these processes are exactly the same. If this is not coming up, quickly go through questions 6 and 7. Turn attention to question 9. Have students give a step-by-step process for using the equation to find the average rate of change. Then have a student write a formula for the process. If no student has a formula, walk them through the steps of developing:

Average rate of change of a function over the interval [a,b]: $\frac{f(b)-f(a)}{b-a}$

After presenting the formula, ask students to work on questions, ask students about question 10. Tell them that the line that intersects a function through the endpoints of the interval is called a **secant line**. The average rate of change on an interval is the same as the slope of the secant line in that interval.

Finally turn to the remaining problems. Project the given graph for the discussion and graph the secant lines for questions 11, 12, 13, and 14. Ask students to give the slopes of the secant lines, the average rates of change over each interval. Finally ask how they might find the rate of change right at the point (-2, 4). This would be called the instantaneous rate of change because there is no interval. The four secant lines that they have drawn will suggest that diminishing the size of the interval brings us closer to the instantaneous rate of change. Ask what the line might look like at that point, helping students to visualize the tangent line at that point. Finding the slope of the tangent line and the instantaneous rate of change will be a Calculus topic. This task is designed as an early introduction to the idea.

Aligned Ready, Set, Go: *Linear and Exponential Functions 6H*

Ready, Set, Go!

© 2012 www.flickr.com/photos/azadam

Ready

Topic: Finding an appropriate viewing window.

When viewing the secant line of an exponential function on a calculator, you want a window that shows the two points on the curve that are being connected. Since exponential functions get very large or small in just a few steps, you may want to change the scale as well as the dimensions of the window. Don't be afraid to experiment until you are satisfied with what you see.

The graphs below depict an exponential function and a secant line. The equations are given. Identify the dimensions of the viewing window. Include the scale for both the x and y values. Check your answer by matching your calculator screen to the one displayed here.

1. $Y_1 = 4(0.2)^x$ and $Y_2 = -1.92x + 4$

WINDOW

a. X min = ______________________

b. X max = ______________________

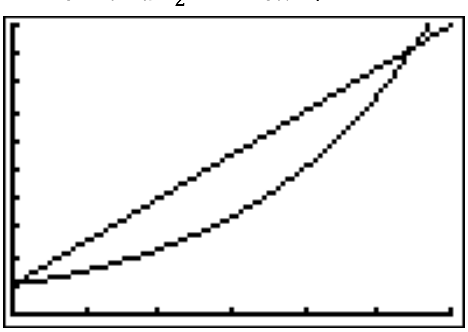
c. X scl = ______________________

d. Y min = ______________________

e. Y max = ______________________

f. Y scl = ______________________

2. $Y_1 = 1.5^X$ and $Y_2 = 1.5x + 1$



WINDOW

a. X min = ______________________

b. X max = ______________________

c. X scl = ______________________

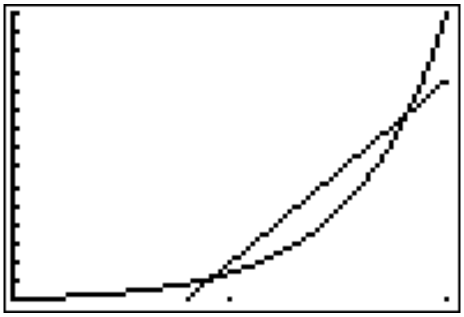
d. Y min = ______________________

e. Y max = ______________________

f. Y scl = ______________________

Name: Linear and Exponential Functions | 4.6H

3. $Y_1 = 150\,(10)^X$ and $Y_2 = 9500x - 7500$



WINDOW

a. X min = ______________________

b. X max = ______________________

c. X scl = ______________________

d. Y min = ______________________

e. Y max = ______________________

f. Y scl = ______________________

Set

Topic: Using slope to compare change in linear and exponential models.

The tables below show the values for a linear model and an exponential model. Use the slope formula between *each set of 2 points* to calculate the rate of change.

Example: Find the slope between the points (30 , 1) and (630 , 2) then between (630 , 2) and (1230 , 3). Do the same between each pair of points in the table for the exponential model.

4a. Linear Model

x	*y*
1	30
2	630
3	1230
4	1830
5	2430

b. Exponential Model

x	*y*
1	30
2	90
3	270
4	810
5	2430

5. Compare the change between each pair of points in the linear model to the change between each pair of points in the exponential model. Describe your observations and conclusions.

6. Find the average of the 4 rates of change of the exponential model. How does the average of the rates of change of the exponential model compare to the rates of change of the linear model?

7. Without using a graphing calculator, make a rough sketch on the same set of axes of what you think the linear model and the exponential model would look like.

8. How did your observations in #5 influence your sketch?

9. Explain how a table of 5 consecutive values can begin and end with the same y-values and be so different in the middle 3 values. How does this idea connect to the meaning of a secant line?

Go

Topic: Developing proficiency on a calculator by using the slope formula

Use your calculator and the slope formula to find the slope of the line that passes through the 2 points.

10. A (-10, 17) , B (10, 97)

11. P (57, 5287) , Q (170, 4948)

12. R (6.055, 23.1825) , S (5.275, 12.0675)

13. G (0.0012, 0.125) , H (2.5012, 6.375)

Need Assistance? Check out these additional resources:

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

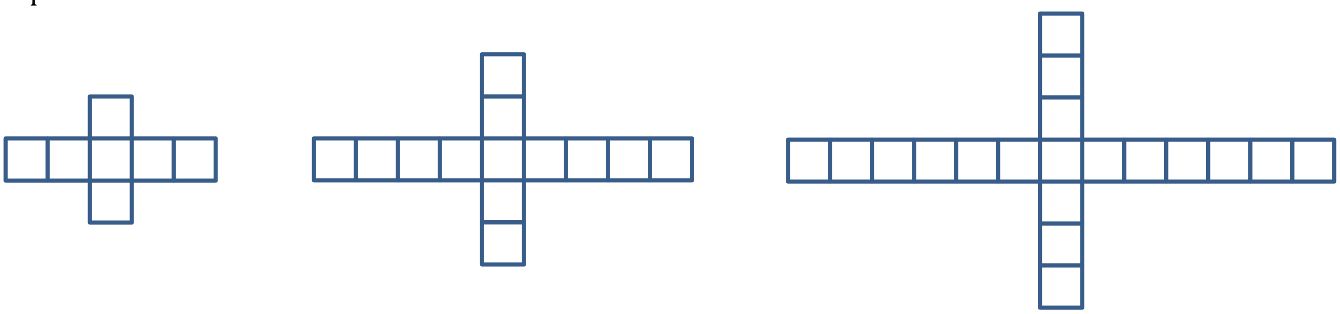
Selecting a viewing window:

http://www.online.math.uh.edu/GraphCalc/Graping-and-Windows-Examples/Graping-and-Windows-Examples.htm

4.7 Making My *Point*

A Solidify Understanding Task

Zac and Sione were working on predicting the number of quilt blocks in this pattern:



When they compared their results, they had an interesting discussion:

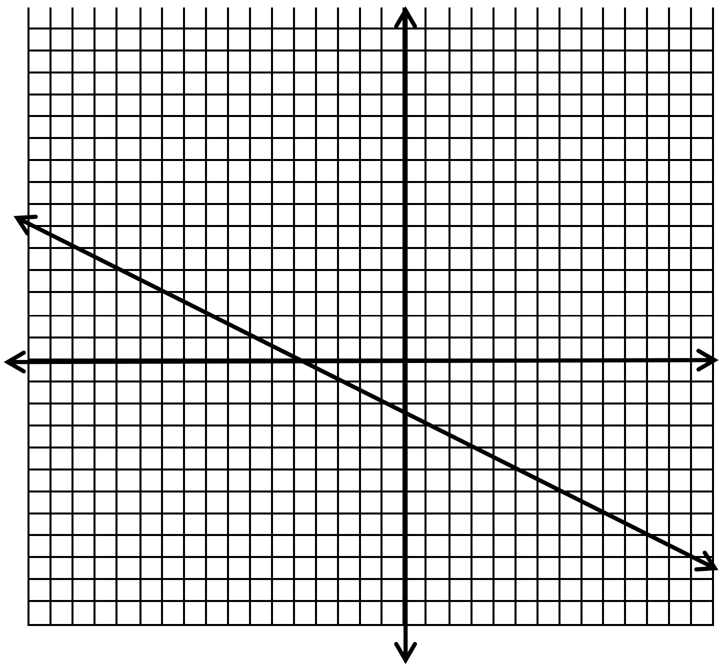
Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at n = 0.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at n = 0 there were 7 blocks and at n = 1 there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

0	1	2	n
7	13	19	$6(n\text{-}1) + 7$

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.

The next problem Zac and Sione worked on was to write the equation of the line shown on the graph below.



When they were finished, here is the conversation they had about how they got their equations:

Sione: It was hard for me to tell where the graph crossed the y axis, so I found two points that I could read easily, (-9, 2) and (-15, 5). I figured out that the slope was $-\frac{1}{2}$ and made a table and checked it against the graph. Here's my table:

x	-15	-13	-11	-9	n
$f(x)$	5	4	3	2	$-\frac{1}{2}(n+9)+2$

I was surprised to notice that the pattern was to start with the n, add 9, multiply by the slope and then add 2.

I got the equation: $f(x) = -\frac{1}{2}(x+9)+2$.

Zac: Hey—I think I did something similar, but I used the points, (7,-6) and (9,-7).

I ended up with the equation: $f(x) = -\frac{1}{2}(x - 9) - 7$. One of us must be wrong because yours says that you add 9 to the *n* and mine says that you subtract 9. How can we both be right?

2. What do you say? Can they both be right?

Zac: My equation made me wonder if there was something special about the point (9,- 7) since it seemed to appear in my equation $f(x) = -\frac{1}{2}(x - 9) - 7$ when I looked at the number pattern. Now I'm noticing something interesting—the same thing seems to happen with your equation, $f(x) = -\frac{1}{2}(x + 9) + 2$ and the point (-9, 2)

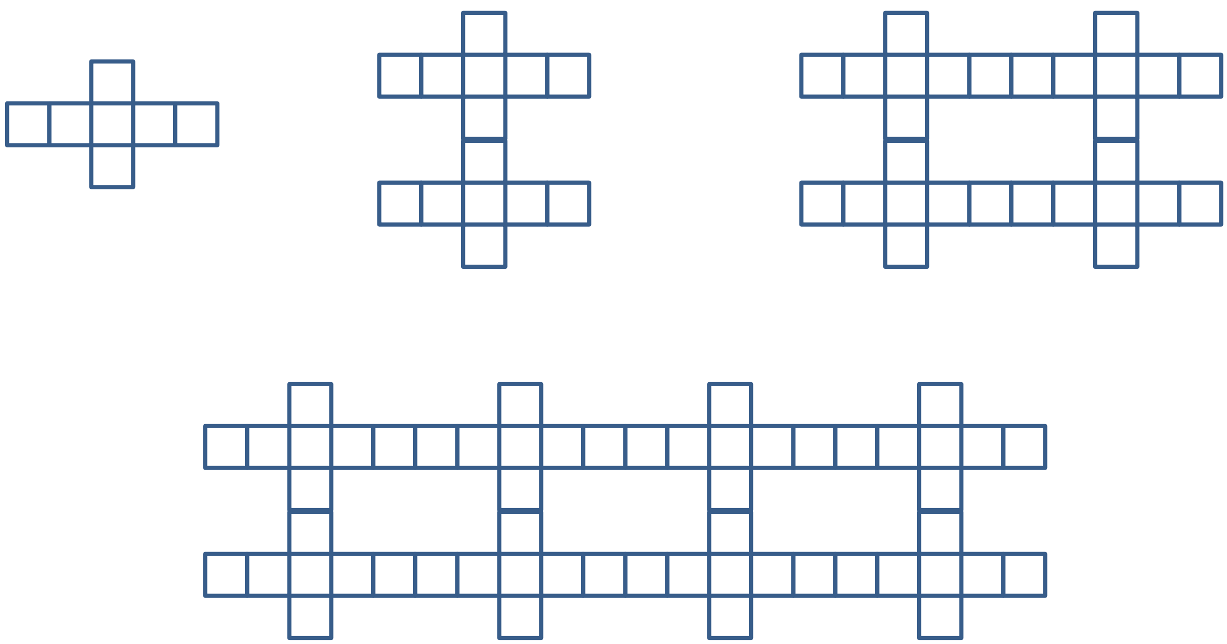
3. Describe the pattern that Zac is noticing.

4. Find another point on the line given above and write the equation that would come from Zac's pattern.

5. What would the pattern look like with the point (*a, b*) if you knew that the slope of the line was *m*?

6. Could you use this pattern to write the equation of any linear function? Why or why not?

Zac and Sione went back to work on an extension of the quilt problem they were working on before. Now they have this pattern:



Zac: This one works a lot like the last quilt pattern to me. The only difference is that the pattern is doubling, so I knew it was exponential. I thought that it starts with 7 blocks and doubles, so the equation must be $f(x) = 7(2)^x$.

Sione: I don't know about that. I agree that it is an exponential function—just look at that growth pattern. But, I made this table:

x	1	2	3	n
f(x)	7	14	28	$7(2)^{n-1}$

I used the numbers in the table and got this equation: $f(x) = 7(2)^{x-1}$.

This seems just like all that stuff we were doing with the lines, but I think that the graphs of these two equations would be different. There is something definitely wrong here.

7. What is different about the thinking that Zac and Sione used to come to different equations?

8. How are their results similar to their results on the linear quilt pattern above? How are they different?

Zac: I know! Let's try doing the same thing with your exponential function as the linear function. What if we took the point (1, 7) and wrote the equation this way:

$$f(x) = 2^{(x-1)} + 7$$

See what I did? I did the subtract 1 thing with the *x* and then added on the 7 from the *y* value of the point. I'll bet this is a really good shortcut trick.

9. What do you think of Zac's equation and his strategy? Does it always work? Why or why not?

4.7 Making My *Point*– Teacher Notes
A Solidify Understanding Task

Purpose:

This is the first task in a series that focuses on understanding and using the notation for linear and exponential functions. The task involves students in thinking about a context where students have selected the index in two different ways, thus getting two different, but equivalent equations. The idea is extended so that students can see the relationship expressed in point/slope form of the equation of the line. The task also explores related ideas with exponential equations and asks to students to test to see if similar reasoning works with exponential functions.

Core Standards Focus:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
  a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Launch (Whole Class):

At this point, students should be quite familiar with working with geometric situations such as those in this task. Start the lesson by telling students that Zac and Sione have worked the problem and come up with two different answers, which they are trying to resolve with sound reasoning. Students need to figure out how Zac and Sione have arrived at different equations and who is right through each of the scenarios in the task.

Explore (Small Group):

Monitor students as they work through the task to see that they understand each scenario. For problems #1 and #7, watch for students that have labeled the figures to match the equations; either starting with n = 0 or n = 1. For problems 2-5, watch to see that students are noticing patterns in how the numbers are used in the equation and making sense of the tables.

Discuss (Whole Class):

Be prepared for the whole group discussion by having large versions of the figures in #1 and #7 ready to be used. Ask a student to explain the difference between Zac and Sione's equations and why they both make sense as models for the figures. Ask a student to show whether or not the two equations are equivalent.

Move to the next scenario, asking for verbal descriptions of the pattern they noticed in #3. Ask for a student to give some examples of equations that they wrote for #4 using the pattern. Ask, "Are the equations equivalent? How do you know?" Ask for students to give their answer for #4. If there are differences in equations among the groups, discuss the differences. Finally, ask students for reasons why this relationship should hold for any linear function. After discussing their reasons, offer that this pattern is often used as a formula for writing equations and graphing lines and is called point/slope form of the equation of a line. You may wish to show them that this form can be derived from the slope formula:

$$m = \frac{y-y_1}{x-x_1}$$

With a little rearranging:

$$m(x - x_1) = y - y_1$$

$$y = m(x - x_1) + y_1$$

Move the discussion to exponential functions. What is the difference between the two models that Zac and Sione created for problem #7? Discuss again the way the figures were labeled in the problems. Are the two equations produced equivalent? Are they both reasonable models for the figures? Ask students to examine the graphs of the two equations and interpret each in relation to the pattern of figures. Finally, graph the equation in #9. Ask students how it compares to the two other graphs. Is this a reasonable model for the figures? Why or why not?

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.7***

Name: Linear and Exponential Functions | 4.7

Ready, Set, Go!

Ready

Topic: Writing equations of lines.

Write the equation of a line in slope-intercept form: y = mx + b, using the given information.

1. m = -7, b = 4
2. m = 3/8, b = -3
3. m = 16, b = -1/5

Write the equation of the line in point-slope form: $y - y_1 = m(x - x_1)$, using the given information.

4. m = 9, (0. -7)
5. m = 2/3, (-6, 1)
6. m = -5, (4, 11)

7. (2,-5) (-3, 10)
8. (0, -9) (3, 0)
9. (-4, 8) (3, 1)

Name: Linear and Exponential Functions 4.7

Set

Topic: Graphing linear and exponential functions

Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.

10. The beginning value of the function is 5 and its value is 3 units smaller at each stage. Equation:	11. The beginning value is 16 and its value is ¼ smaller at each stage. Equation:
12. The beginning value is 1 and its value is 10 times as big at each stage. Equation:	13. The beginning value is -8 and its value is 2 units larger at each stage. Equation:

Name: Linear and Exponential Functions 4.7

Go

Topic: Slope-Intercept Form

Rewrite the equations in slope-intercept form.

14. $2y + 10 = 6x + 12$

15. $5x + y = 7x + 4$

16. $(y - 13) = \frac{1}{2}(8x - 14)$

17. $(y + 11) = -7(x - 2)$

18. $(y - 5) = 3(x + 2)$

19. $3(2x - y) = 9x + 12$

20. $y - 2 = 1/5\ (10x - 25)$

21. $y + 13 = -1(x + 3)$

22. $y + 1 = \frac{3}{4}(x + 3)$

Need Help? Check out these related videos:

Equations in slope-intercept form:
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/linear-equations-in-slope-intercept-form

Equations in point-slope form:
http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/linear-equations-in-point-slope-form

4.8 Efficiency Experts

A Solidify Understanding Task

In our work so far, we have worked with linear and exponential equations in many forms. Some of the forms of equations and their names are:

Equation	**Name**
$y = \frac{1}{2}x + 1$	Slope Intercept Form $y = mx + b$, where m is the slope and b is the y-intercept
$y - 3 = \frac{1}{2}(x - 4)$	Point Slope Form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) the coordinates of a point on the line
$x - 2y = -2$	Standard Form $ax + by = c$
$f(0) = 1, f(n) = f(n-1) + \frac{1}{2}$	Recursion Formula $f(n) = f(n-1) + \text{D}$, Given an initial value *f(a)* D = constant difference in consecutive terms

1. Verify that the four equations above are equivalent.

2. Explain how you know that the four equations are linear.

You have been appointed as a mathematics efficiency expert. Your job is to compare these four forms of equations for different uses and decide which form is most efficient and effective for each use. The investigation will be conducted in four parts with a report to be written at the end.

Linear Investigation Part A: Which form best tells the story?

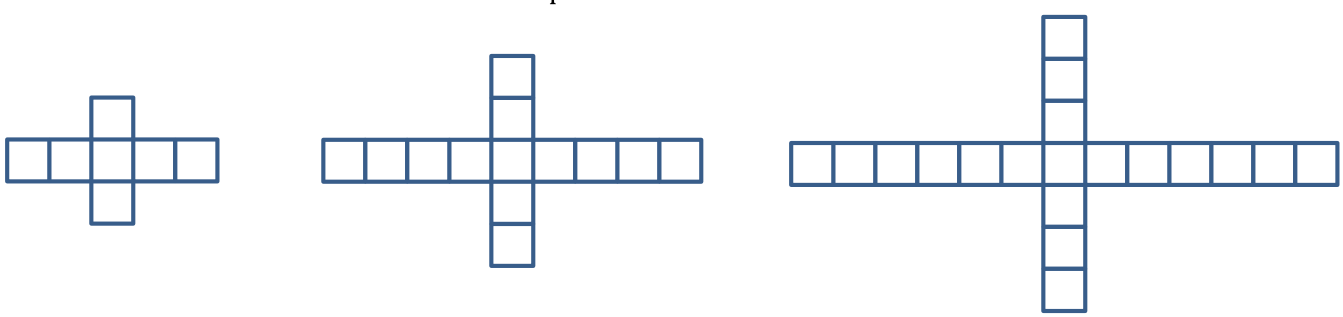
1. In his job selling vacuums, Joe makes $500 each month plus $20 for each vacuum he sells. Which equation best describes Joe's monthly income?

   $20x - y = 500$ $\qquad$ $y = 20x + 500$

2. The Tree Hugger Granola Company makes trail mix with candies and nuts. The cost of candies for a trail mix is $2 per pound and the cost of the nuts is $1.50 per pound. The total cost of a batch of trail mix is $540. Which equation best models the quantities of candies and nuts in the mix?

   $2x + 1.5\,y = 540$ $\qquad$ $y = \frac{4}{3}x + 360$

3. Grandma Billings is working on a quilt with blocks in the following pattern. Which equation best models the number of blocks in each pattern?



   $f(n) = 6n + 1$ $\qquad$ $f(1) = 7, f(n) = f(n-1) + 6$

4. What is the important difference between the type of situations that can be modeled with a recursion formula and the other equation forms?

Linear Investigation Part B: Which is the best form for writing equations?

1. Write the equation of the line with a slope of -2 through the point (-2, 5)

2. Write the equation of the line through the points (1, -2) and (4, 1)

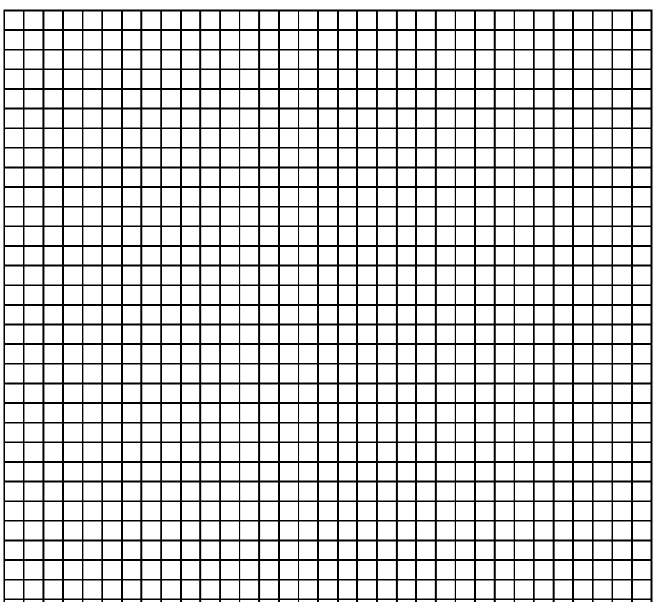
3. Write the equation of the arithmetic sequence that starts with -7 and each term decreases by 3.

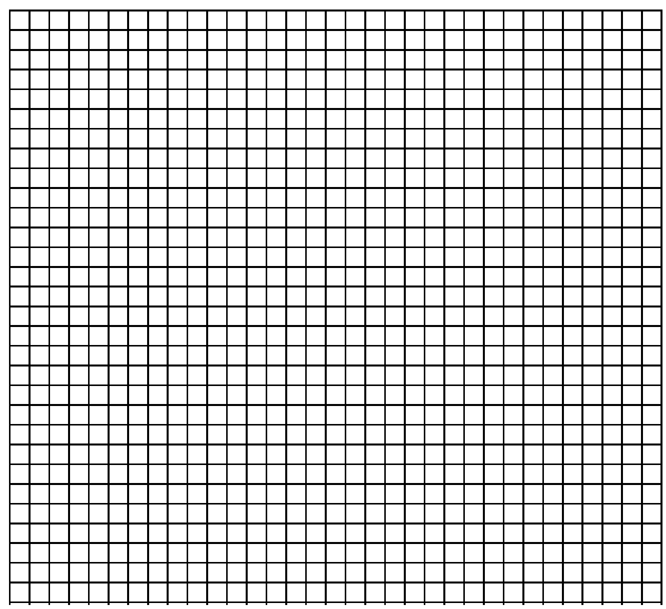
Linear Investigation Part C: Which is the best form for graphing?

Graph the following equations:

1. $y = \frac{3}{4}x + 5$

2. $3x - 5y = 15$





3. $f(n) = -4(n+6)+5$

4. $f(0) = -2, f(n) = f(n-1) - 5$

Linear Investigation Part D: What about tables?

1. Create a table for each equation.

   a. $-x + 2y = 6$

   b. $y = -\frac{1}{2}x - 4$

2. Write an equation for the relation described in this table.

x	*y*
12	-3
9	-8
6	-13
3	-18
0	-23

<u>Your Efficiency Analysis Report, Part 1</u>

Using the results of your investigation, describe the best uses for each form of an equation of a line, with sections for standard form, slope intercept form, point slope form and recursion formulas. Be sure to include a discussion of tables, graphs, and story contexts as part of your report.

Investigating Exponential Forms

During the course of the year, we have also worked with forms of exponential equations, with a few more coming before the end of the module. The forms of exponential equations that we have seen so far:

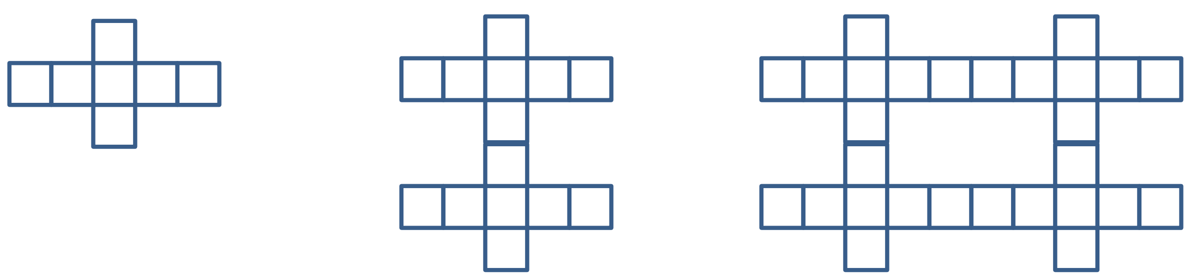
Equation	Name
$y = 10(3)^x$	Explicit Form $y = a(b)^x$
$f(0) = 10, f(n+1) = 3f(n)$	Recursion Formula $f(n+1) = Rf(n)$ Given an initial value f(a) R = constant ratio between consecutive terms

Test out the efficiency of these two exponential equation types for these tasks.

Exponential Investigation Part A: Which form tells the story best?

1. Grandma Billings has started piecing her quilt together and has created the following growth pattern:

Block 1 Block 2 Block 3



Which equation best models the number of squares in each block?

$f(n) = 7(2)^{n-1}$ $f(1) = 7, f(n) = 2f(n-1)$

2. The population of the resort town of Java Hot Springs in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Which equation best models the number of people in Java Hot Springs, with t = the number of years from 2003?

   $f(t) = 35{,}000(1.024)^t$ $\qquad$ $f(0) = 35{,}000, f(t) = 1.024 \cdot f(t-1)$

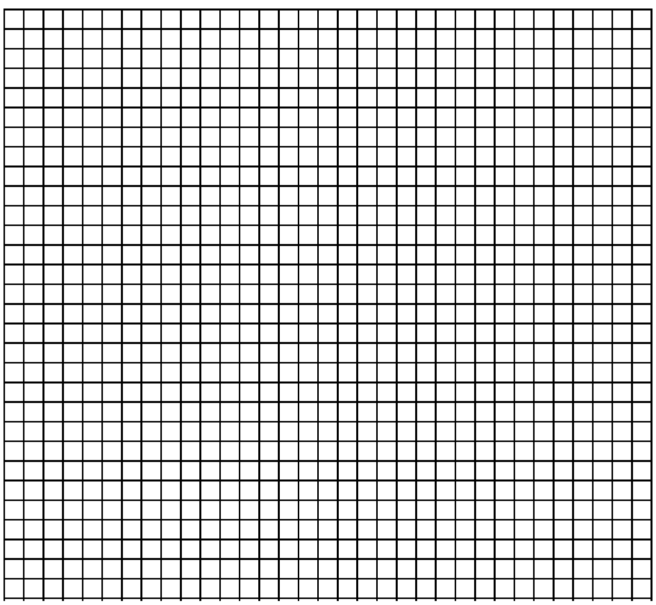
3. How would you have to change the definition of t in the recursive formula to model the situation?

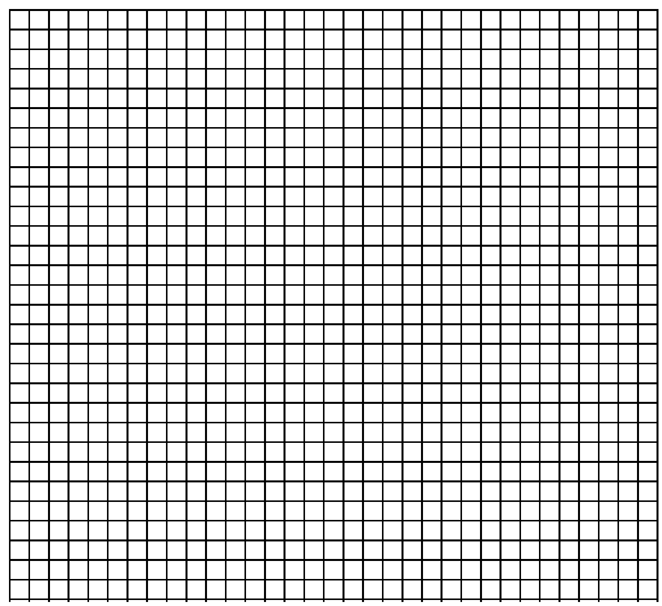
Exponential Investigation Part B: Which is the best form for graphing?

Graph each equation:

1. $y = 2(\,1.8)^x$

2. $f(0) = 5, f(n) = \; 0.6 \cdot f(n-1)$





<u>Your Efficiency Analysis Report, Part 2</u>

Using the results of your investigation, describe the best uses for each form of an exponential equation, with sections for standard form and recursion formulas. Be sure to include a discussion of tables, graphs, and story contexts as part of your report.

4.8 Efficiency Experts – Teacher Notes
A Solidify Understanding Task

Purpose:
The purpose of this task is for students to identify efficient procedures for modeling situations, graphing, writing equations, and making tables for linear and exponential functions. Various forms of the equations of linear functions are named and tested for their efficiency for different mathematical purposes.

Core Standards Focus:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.3 Write expressions in equivalent forms to solve problems
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Launch (Whole Class):

Distribute the handouts to the class. Start the discussion by describing each of the various forms of equations of linear functions and how the examples given illustrate the form. For each form, ask students to name a task that they have done in previous modules in which they used that particular form of equation. After going over each form, ask students to do question #1 and #2. It is important for students to understand that the various forms may tell a different story or be useful for different purposes, but they all give equivalent equations. When discussing #2, be sure that students recognize how to see the constant rate of growth in the various forms of the equations.

Tell students that the task will be giving them a number of different examples that are devised to help them think about which of the forms are most efficient for the various purposes. They should keep track of their work and the pros and cons of the equation type for each purpose, since they will be writing a report on their findings. Ask students to work the problems up to the beginning of the exponential investigation.

Note: The task is designed to have students work together to complete their exploration of the equations in parts 1 and 2, and then write the report on their results individually.

Explore Part 1(Small Group):

Most of the situations should be fairly familiar for students, but be sure they are considering how the various forms fit the problem, not just solving the problem. Listen for comments that are comparing how using the form makes the problem easier, or in the case of the story contexts how the various forms model the situation differently.

Discussion Part 1 and Re-launch (Whole Class):

Prepare the following table for the discussion:

Forms of Equations of Linear Functions

Form of Equation	Type of Contexts	Writing Equations	Graphing	Tables
Slope Intercept Form				
Point Slope Form				
Standard Form				
Recursion Formula				

Start by discussing the types of contexts that are best modeled with each equation form. Ask groups to describe their conclusions, and why they believe that the equation is a good model for each context. Be sure that they are describing the parameters in the equation—how they identify the rate of change, intercepts, and other relationships that may occur in the story context.

Continue through the discussion recording student's evaluation of each type of linear equation. Ask students to do go back to working in their groups to complete the exploration of the uses of exponential equations.

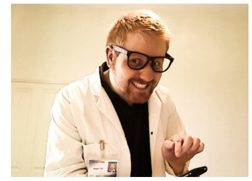
Discussion Part 2:

When students have completed their investigation, facilitate a discussion of the advantages of explicit exponential equations versus recursion formulas. What exponential contexts are best modeled by each type of equation? What limitations exist for each type? What relationships are highlighted in each type of equation? During the discussion be sure that students have discussed the idea that recursion formulas describe sequences (discrete linear or exponential functions) well, but are not used for continuous functions.

At the end of the discussion, assign students to write their individual reports of the appropriate and efficient use of each of the linear and exponential equation forms.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.8***

Ready, Set, Go!



Ready

Topic: Simple interest

*When a person borrows money, the lender usually charges "rent" on the money. This "rent" is called interest. Simple interest is a percent "**r**" of the original amount borrowed "**p**" multiplied by the time "**t**", usually in years. The formula for calculating the interest is* **i = prt**.

Calculate the simple interest owed on the following loans.

1. $p = \$1000$ $r = 11\%$ $t = 2$ years $i =$ ____________

2. $p = \$6500$ $r = 12.5\%$ $t = 5$ years $i =$ ____________

3. $p = \$20{,}000$ $r = 8.5\%$ $t = 6$ years $i =$ ____________

4. $p = \$700$ $r = 20\%$ $t = 6$ months $i =$ ____________

*Juanita borrowed $1,000 and agreed to pay 15% interest for 5 years. Juanita did not have to make any payments until the end of the 5 years, but then she had to pay back the amount borrowed "**P**" plus all of the interest "**i**" for the 5 years "**t**." Below is a chart that shows how much money Juanita owed the lender at the end of each year of the loan.*

End of year	Interest owed for the year	Total Amount owed to the lender to pay back the loan.
1	$1000 X .15 = $150	A = Principal + interest = $1150
2	$1000 X .15 = $150	A = P + i + i = $1300
3	$1000 X .15 = $150	A = P + i + i + i = $1450
4	$1000 X .15 = $150	A = P + i + i + i + i = $1600
5	$1000 X .15 = $150	A = P + i + i + i + i + i = $1750

5. Look for the pattern you see in the chart above for the amount (A) owed to the lender. Write an function that best describes A with respect to time (in years).

6. At the end of year 5, the interest was calculated at 15% of the original loan of $1000. But by that time Juanita owed $1600 (before the interest was added.) What percent of $1600 is $150?

7. Consider if the lender charged 15% of the amount owed instead of 15% of the amount of the original loan. Make a fourth column on the chart and calculate the interest owed each year if the lender required 15% of the amount owed at the end of each year. Note that the interest owed at the end of the first year would still be $150. Fill in the 4th column.

Name: Linear and Exponential Functions | 4.8

Set

Topic: The 4 forms of a linear equation

8. Below are the 4 forms of the same linear equation. For each equation, do the following
   (a) Circle the rate of change
   (b) Name the point that describes the y-intercept
   (c) Name the point that describes the x-intercept

Slope-intercept	Point-slope	Standard	Recursive formula	(b)	(c)
8. y = 3x – 2	y – 13 = 3(x – 5)	3x – y = 2	$f(0) = -2$ $f(n) = f(n-1) + 3$		
9. y = ¼ x +7	y – 5 = ¼ (x + 8)	x – 4y = -28	$f(0) = 7$ $f(n) = f(n-1) + ¼$		
10. y = -⅔x + 3	y + 1 = -⅔(x – 6)	2x + 3y = 9	$f(0) = 3$ $f(n) = f(n-1) - ⅔$		

Go

Topic: Solving multi-step equations

Solve the following equations

11. $12 + 6x - 4 = 5 + 2(3x - 1)$

12. $5(2x + 4) = 3(x + 5) - 19$

13. $7 - 3(4x + 2) = 6(2x + 3) - 17$

14. $2(x + 1) = 6(x - 3)$

15. What does it mean when you have solved an equation?

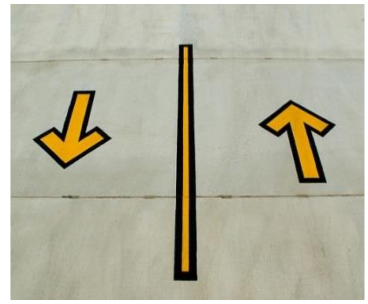
16. Explain how a linear equation can have more than one solution.

Need Help? Check out these related videos:

Solving equations: http://www.purplemath.com/modules/solvelin4.htm

Interest: http://www.khanacademy.org/finance-economics/core-finance/v/introduction-to-interest

4.9 Up a Little, Down a Little
A Solidify Understanding Task



One of the most common applications of exponential growth is compound interest. For example, Mama Bigbucks puts $20,000 in a bank savings account that pays 3% interest compounded annually. "Compounded annually" means that at the end of the first year, the bank pays Mama 3% of $20,000, so they add $600 to the account. Mama leaves her original money ($20000) and the interest ($600) in the account for a year. At the end of the second year the bank will pay interest on the entire amount, $20600. Since the bank is paying interest on a previous interest amount, this is called "compound interest".

Model the amount of money in Mama Bigbucks' bank account after t years.

Use your model to find the amount of money that Mama has in her account after 20 years.

A formula that is often used for calculating the amount of money in an account that is compounded annually is:

$$A = P(1+r)^t$$

Where:
A = amount of money in the account after t years
P = principal, the original amount of the investment
r = the annual interest rate
t = the time in years

Apply this formula to Mama's bank account and compare the result to the model that you created.

Based upon the work that you did in creating your model, explain the (1 + r) part of the formula.

Another common application of exponential functions is depreciation. When the value of something you buy goes down a certain percent each year, it is called depreciation. For example, Mama Bigbucks buys a car for $20,000 and it depreciates at a rate of 3% per year. At the end of the first year, the car loses 3% of its original value, so it is now worth $19,400.

Model the value of Mama's car after *t* years.

Use your model to find how many years will it take for Mama's car to be worth less than $500?

How is the situation of Mama's car similar to Mama's bank account?

What differences do you see in the two situations?

Consider your model for the value of Mama's car and develop a general formula for depreciation.

4.9 Up a Little, Down a Little – Teacher Notes
A Solidify Understanding Task

Purpose:
The purpose of this task is for students to connect their understanding of exponential functions with standard formulas for compound interest and depreciation. Students will consider how a percent change is written in a formula in both an exponential growth and decay situation. Students will develop a formula for depreciation.

Core Standards Focus:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

F.IF Analyze functions using different representations

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

★For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot 2^n$.

Launch (Whole Class):

Start the lesson by explaining the general concept of compound interest (without giving a formula) and ensuring that students understand the example given at the beginning of the task. Ask students to work in their groups to model the amount of money in Mama's bank account and complete the questions about compound interest on the task. Encourage the use of technology for the task.

Explore (Small Group):

Monitor students as they work, supporting them in using technology to create tables, graphs, and equations for the model. Be sure that every group has written an equation on their own before using the formula.

Discuss Part 1and Re-launch(Whole Class):

Using a graphing calculator or other technology, graph the equation that students have written. Use the graph to find approximately how much money will be in the account after 20 years and then ask a student to show how they used the equation to find the amount. Ask students what they notice about how Mama's money grows when they look at the graph. Be sure that they discuss how money grows slowly at the beginning and then the rate of growth increases dramatically as the money remains in the account over time. Emphasize the importance of this principal in saving money. Ask students to predict how the curve would change using different interest rates. Use the formula to write and graph equations that model 5% interest and 10% interest. Display all three graphs together and compare the amount of money in the account over time with the various rates. How much difference does the rate make if the money is kept in the account for 5 years, 10 years, or 20 years?

Next, describe the meaning of the term "depreciation". Demonstrate the idea by working through the example given about Mama's car. Be sure that students understand that depreciation reduces the value of the item, so that it is typically modeled with a decreasing exponential function. Once the basic idea of depreciation is understood, let students work on the rest of the task.

Discuss Part 2 (Whole Group):

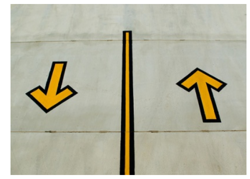
Start the discussion by using technology to display the graph of the equation that students wrote to model the depreciation of Mama's car. When does the car depreciate the fastest? According to the model, will the car ever be worth nothing? Discuss the similarity and differences that students notice between the compound interest and the depreciation scenarios.

Turn the focus of the discussion to the formula that students have written for depreciation. Is it the same as the compound interest formula? Why do the two formulas generate such different curves? What is the meaning of the (1+r) or (1-r) part of the formulas? Where does the one come from?

Aligned Ready, Set, Go: *Linear and Exponential Functions 4.9*

Name: Linear and Exponential Functions 4.9

Ready, Set, Go!



Ready

Topic: Evaluating equations

Fill out the table of values for the given equations.

1. $y = 17x - 28$

x	y
-3	
1	
4	
5	

2. $y = -8x - 3$

x	y
-10	
-6	
2	
9	

3. $y = \frac{1}{2}x + 15$

x	y
-26	
-14	
-1	
9	

4. $y = 6^x$

x	y
-3	
-1	
1	
2	
5	

5. $y = 10^x$

x	y
-3	
-1	
0	
2	
6	

6. $y = \left(\frac{1}{5}\right)^x$

x	y
-4	
-2	
0	
3	
5	

Name: Linear and Exponential Functions | 4.9

Set

Topic: Evaluate using the formulas for simple interest or compound interest.

Given the formula for simple interest: $i = Prt$**, calculate the simple interest paid.**

(Remember, i = interest, P = the principal, r = the interest rate per year as a decimal, t = time in years)

7. Find the simple interest you will pay on a 5 year loan of $7,000 at 11% per year.

8. How much interest will you pay in 2 years on a loan of $1500 at 4.5% per year?

Use $i = Prt$ **to complete the table. All interest rates are annual.**

	i =	P ×	r ×	t
9.		$11,275	12%	3 years
10.	$1428	$5100	4%	
11.	$93.75	$1250		6 months
12.	$54		8%	9 months

Given the formula for compound interest: $A = P(1+r)^t$, write a compound interest function to model each situation. Then calculate the balance after the given number of years.

(Remember: *A = the balance after t years, P = the principal, t =the time in years, r = the annual interest rate expressed as a decimal)*

13. $22,000 invested at a rate of 3.5% compounded annually for 6 years.

14. $4300 invested at a rate of 2.8% compounded annually for 15 years.

15. Suppose that when you are 15 years old, a magic genie gives you the choice of investing $10,000 at a rate of 7% or $5,000 at a rate of 12%. Either choice will be compounded annually. The money will be yours when you are 65 years old. Which investment would be the best? Justify your answer.

Name: Linear and Exponential Functions | 4.9

Go

Topic: Using order of operations when evaluating equations

Evaluate the equations for the given values of the variables.

16. $pq \div 6 + 10$; when $p = 7$ and $q = -3$

17. $m + n(m - n)$; when $m = 2$, and $n = 6$

18. $(b - 1)^2 + ba^2$; when $a = 5$, and $b = 3$

19. $y(x - (9 - 4y))$; when $x = 4$, and $y = -5$

20. $x - (x - (x - y^3))$; when $x = 7$, and $y = 2$

21. $an^4 + a(n - 7)^2 + 2n$; when $a = -2$, and $n = 4$

Need Help? Check out these related videos:

http://www.basic-mathematics.com/simple-vs-compound-interest.html

http://www.khanacademy.org/finance-economics/core-finance/v/introduction-to-interest

4.10 *X* Marks the Spot

A Practice Understanding Task

Table Puzzles

1. Use the tables to find the missing values of x:

a.

x	$y = 0.7x - 3$
-2	-4.4
-10	10
	-8.6
4	-0.2
	1.2

b.

x	$y = -\frac{2}{3}x + 4$
10	$-10\frac{2}{3}$
-3	6
5	$\frac{2}{3}$
	0
	10

c. What equations could be written, in terms of *x* only, for each of the rows that are missing the *x* in the two tables above?

d.

x	$y = 3^x$
5	243
	81
-3	$\frac{1}{27}$
	$\frac{1}{3}$
2	9

e.

x	$y = \left(\frac{1}{2}\right)^x$
-5	32
	8
	1
2	$\frac{1}{4}$
	$\frac{1}{16}$

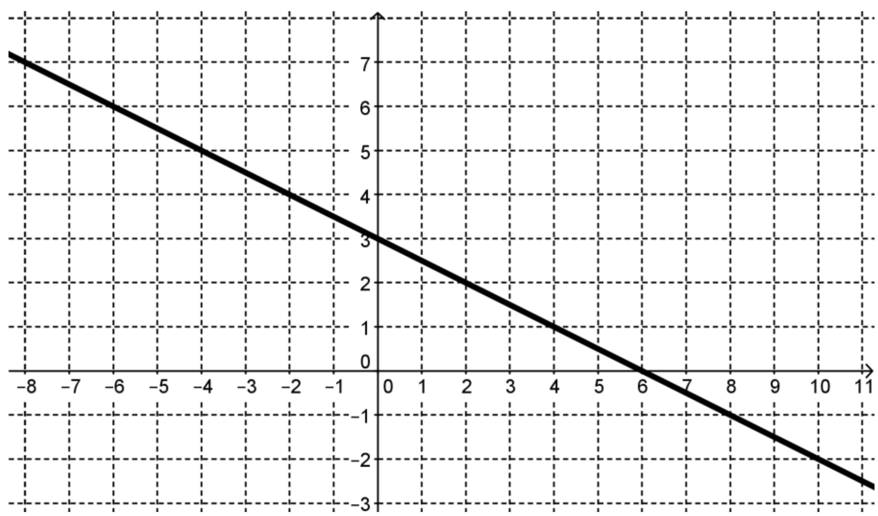
f. What equations could be written, in terms of *x* only, for each of the rows that are missing the *x* in the two tables above?

2. What strategy did you use to find the solutions to equations generated by the tables that contained linear functions?

3. What strategy did you use to find the solutions to equations generated by the tables that contained exponential functions?

Graph Puzzles

4. The graph of y= $-\frac{1}{2}$x+3 is given below. Use the graph to solve the equations for *x* and label the solutions.



a. $5 = -\frac{1}{2}x + 3$

x = _____

Label the solution with an A on the graph.

b. $-\frac{1}{2}x + 3 = 1$

x = _____

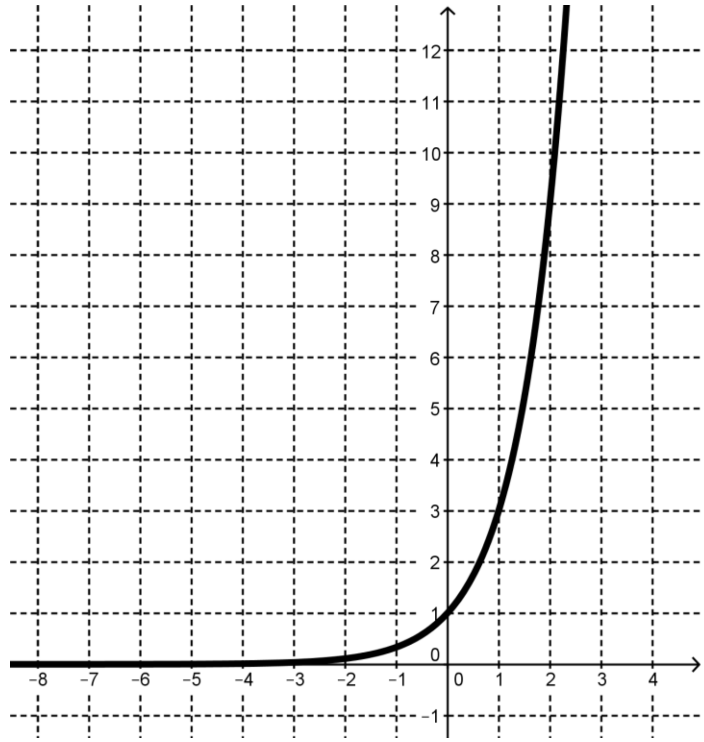
Label the solution with a B on the graph.

c. $-0.5x + 3 = -1$

x = _____

Label the solution with a C on the graph.

5. The graph of y= 3^x is given below. Use the graph to solve the equations for *x* and label the solutions.



a. $3^x = \frac{1}{9}$

x = _____

Label the solution with an A on the graph.

b. $3^x = 9$

x = _____

Label the solution with a B on the graph.

c. $3\sqrt{3} = 3^x$

x = _____

Label the solution with a C on the graph.

d. $1 = 3^x$

x = _____

Label the solution with a D on the graph.

e. $6 = 3^x$

x = _____

Label the solution with an E on the graph.

6. How does the graph help to find solutions for *x*?

Equation Puzzles:

Solve each equation for *x:*

7. 5^x=125

8. $7 = -6x + 9$

9. $10^x = 10{,}000$

10. $2.5 - 0.9x = 1.3$

11. $6^x = \frac{1}{36}$

12. $\left(\frac{1}{4}\right)^x = 16$

4.10 *X* Marks the Spot – Teacher Notes

A Practice Understanding Task

Purpose:
The purpose of this task is to build fluency with understanding for solving exponential equations such as $2^x = 32$. The task uses tables and graphs to help students make connections between the work they have done with exponential functions and the solution to exponential equations. Most of the equations will yield exact solutions, although a few will rely on the use of the graph to estimate a solution.

Core Standards Focus:

A.REI Solve equations and inequalities in one variable.*

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.*

Launch (Whole Class):

Remind students that they are very familiar with constructing tables for linear and exponential functions. In previous tasks, they have selected values for *x* and calculated the value of *y* based upon an equation or other representation. They have also constructed graphs based upon having an equation or a set of *x* and *y* values. In this task they will be using tables and graphs to work in reverse, finding the *x* value for a given *y*.

Explore (Small Group):

Monitor students as they work and listen to their strategies for finding the missing values of *x*. As they are working on the table puzzles, encourage them to consider writing equations as a way to track their strategies. In the graph puzzles, they will find that they can only get approximate answers on a few equations. Encourage them to use the graph to estimate a value and to interpret the solution in the equation. The purpose of the tables and graphs is to help students draw upon their thinking from previous tasks to solve the equations. Remind students to connect the ideas as they work on the equation puzzles.

Discuss (Whole Class):

Start the discussion with a student that has written and solved an equation for the fourth row in table b. The equation written should be:

$$\mathbf{0 = -\frac{2}{3}x + 4}$$

Ask the student to describe how they wrote the equation and then their strategies for solving it. Ask the class where this point would be on the graph. Remind students that the point on the graph where $y = 0$ will be the x intercept on the graph. Ask students what the graph of the function would be, and they should be able to describe a line with a slope of -2/3 and y-intercept of (0, 4).

Move the discussion to the graph of y= 3^x. Ask students to describe how they used the graph to find the solution to "a". Ask students how they could check the solution in the equation. Does the solution they found with the graph make sense? Ask students how they used the graph to solve "c". Students will have approximate answers. (An exact answers is possible, but students have not yet learned rational exponents.) Ask students how they could check their solution. Use a calculator to demonstrate, checking whatever approximate solution was given. You may wish to try other approximate solutions.

Finally, ask students to show solutions to as many of the equation puzzles that time will allow. End the discussion by asking how solving exponential equations differs from solving linear equations.

Aligned Ready, Set, Go: ***Linear and Exponential Functions 4.10***

Name: Linear and Exponential Functions | 4.10

Ready, Set, Go!

Ready

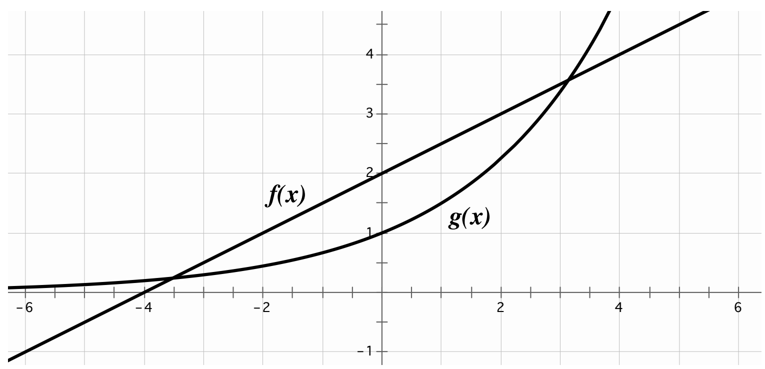
1. Give an example of a discrete function.

2. Give an example of a continuous function.

3. The first and 5th terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

Arithmetic	-6250				-10
Geometric	-6250				-10

4. Compare the rate of change in the pair of functions in the graph by identifying the interval where it appears that *f (x)* is changing faster and the interval where it appears that *g (x)* is changing faster. Verify your conclusions by making a table of values for each function and exploring the rates of change in your tables.



5. Identify the following sequences as linear, exponential, or neither.

a. -23, -6. 11, 28, . . .

b. 49, 36, 25, 16, . . .

c. 5125, 1025, 205, 41, . . .

d. 2, 6, 24, 120, . . .

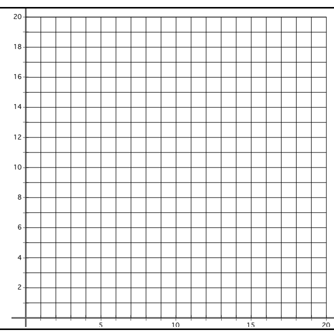
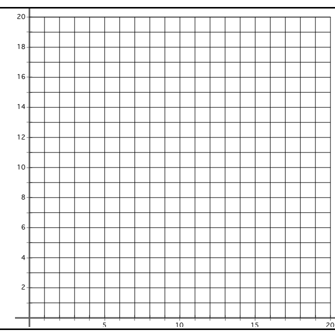
e. 0.12, 0.36, 1.08, 3.24, . . .

f. 21, 24.5, 28, 31.5, . . .

Name: Linear and Exponential Functions 4.10

Set

Describe the defining characteristics of each type of function by filling in the cells of each table as completely as possible.

	$y = 6 + 5x$	$y = 6(5^x)$
6. Type of growth		
7. What kind of sequence corresponds to each model?		
8. Make a table of values	x | y	x | y
9. Find the rate of change		
10. Graph each equation. Compare the graphs. What is the same? What is different?		
11. Find the y-intercept for each function.		
12. Write the recursive form of each equation.		

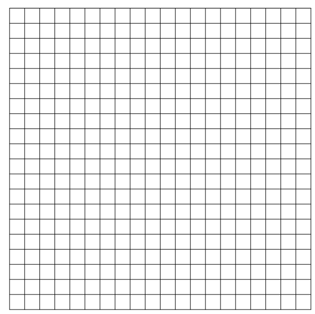
Name: Linear and Exponential Functions | 4.10

There were 2 girls in my grandmother's family, my mother and my aunt. They each had 3 daughters. My two sisters, 3 cousins, and I each had 3 daughters. Each one of our 3 daughters have had 3 daughters...

13. If the pattern of each girl having 3 daughters continues for 2 more generations (my mom and aunt being the 1st generation, I want to know about the 5th generation), how many daughters will be born then?

14. Write the explicit equation for this pattern.

15. Create a table and a graph describing this pattern. Is this situation discrete or continuous?



Go

Solve the following equations.

16. $5x + 3 = 2(x - 6)$

17. $6x - 12x + 10 = 2(-3x - 6)$

18. $13x - 12x + \frac{1}{2} = x + \frac{3}{6}$

Write the equation of the line in slope-intercept form given the following information. (P and Q are points on the line)

19. $f(0) = 6,\ f(n) = f(n-1) + \frac{1}{4}$

20. m = -3, P : (-5, 8)

21. $14x - 2y + 9 = 0$

22. P : (17, -4) , Q : (-5, -26)

23. $y - 9 = \frac{1}{2}(x + 6)$

24. P : (11, 8) , Q : (-1, 8)

Name: Linear and Exponential Functions 4.10

Recall the following formulas: Simple interest **$i = prt$** Compound interest **$A=P(1+r)^t$**

Using the formulas for simple interest or compound interest, calculate the following.

25. The simple interest on a loan of $12,000 at an interest rate of 17% for 6 years.

26. The simple interest on a loan of $20,000 at an interest rate of 11% for 5 years.

27. The amount owed on a loan of $20,000, at 11%, compounded annually for 5 years.

28. Compare the interest paid in #26 to the interest paid in #27. Which kind of interest do you want if you have to take out a loan?

29. The amount in your savings account at the end of 30 years, if you began with $2500 and earned an interest rate of 7% compounded annually.