# Secondary One Mathematics: An Integrated Approach
# Module 2 Honors
# Systems

By

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# Module 2 – Systems of Equations and Inequalities

**Classroom Task:** Pet Sitters- A Develop Understanding Task
*An introduction to representing constraints with systems of inequalities* (**A.CED.3**)
**Ready, Set, Go Homework:** Systems 2.1

**Classroom Task:** Too Big or Not Too Big, That is the Question - A Solidify Understanding Task
*Writing and graphing linear inequalities in two variables* (**A.CED.2, A.REI.12**)
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**Honors Classroom Task:** To Market with Matrices – A Solidify Understanding Task
*An introduction to solving systems of linear equations using matrices* (**UT Honors Standard**)
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**Honors Classroom Task:** Solving Systems with Matrices – A Practice Understanding Task
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**Ready, Set, Go Homework:** Systems 2.14H

# 2.1 Pet Sitters
## *A Develop Understanding Task*



The Martinez twins, Carlos and Clarita, are trying to find a way to make money during summer vacation. When they overhear their aunt complaining about how difficult it is to find someone to care for her pets while she will be away on a trip, Carlos and Clarita know they have found the perfect solution. Not only do they have a large, unused storage shed on their property where they can house animals, they also have a spacious fenced backyard where the pets can play.

Carlos and Clarita are making a list of some of the issues they need to consider as part of their business plan to care for cats and dogs while their owners are on vacation.

- *Space*: Cat pens will require 6 $ft^2$ of space, while dog runs require 24 $ft^2$. Carlos and Clarita have up to 360 $ft^2$ available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Of course, Carlos and Clarita want to make as much money as possible from their business, so they are trying to determine how many of each type of pet they should plan to accommodate. They plan to charge $8 per day for boarding each cat and $20 per day for each dog.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for? Their dad has suggested the same number of each, perhaps 12 cats and 12 dogs. Carlos thinks they should plan for more dogs, since they can charge more. Clarita thinks they should plan for more cats since they take less space and time, and therefore they can board more.

What do you think? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

# 2.1 Pet Sitters – Teacher Notes

*A Develop Understanding Task*

**Purpose**: As students work with the context of making recommendations for how many dogs and cats Carlos and Clarita should plan to accommodate, they will surface many ideas, strategies and representations related to solving systems of equations and inequalities. For example, they will explore the notion of *constraints* since in this task the number of each type of pet that can be accommodated is limited by space and money, but many different combinations of dogs and cats are possible. They may consider the notion of a *system of equations* since each constraint (space, start-up costs) allows for a different set of possibilities—a particular combination of dogs and cats may satisfy one constraint but not another—so both constraints must be considered simultaneously. Finally, they may surface the notion of *a system of inequalities* since Carlos and Clarita don't have to use up all of the available space or money, implying that each constraint may be represented by an inequality.

**Core Standards Focus**:

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**Related Standards: N.Q.2, A.REI.12**

**Launch (Whole Class):**

After reading and discussing the "Pet Sitters" scenario, challenge students to come up with a combination of dogs and cats that would yield the highest daily income. Give students a few minutes to work independently to find the income for a particular combination of dogs and cats. After a few minutes have students compare their daily income with others and then work with a partner to improve their initial guesses

**Explore (Small Group):**

For students who don't know where to begin, have them determine the daily income if Carlos and Clarita follow their dad's advice of boarding "the same number of each, perhaps 12 cats and 12 dogs." After calculating the income, ask if Carlos and Clarita have enough space and money to accommodate 12 cats and 12 dogs each day.

It is anticipated that many students will use a guess and check strategy: pick a particular combination of dogs and cats, check that the particular combination satisfies both constraints, and then calculate the income. After finding a combination that works, students may consider what happens as the number of dogs, cats or both is increased.

Students who focus on just one constraint and find combinations of cats and dogs that satisfy that constraint have naturally simplified the larger problem into a more manageable task. They will gain valuable insight into the mathematical work of this module, and need not be pressed at this time to consider both constraints, or the bigger issue of maximizing the profit. Press for the work you feel your students can handle, and use this task to assess what ideas, strategies and representations will be available for future work in this module. For example, a student who visualizes the space constraint by drawing possible layouts of how the shed might be occupied with cat pens and dog runs is noticing that there are many such combinations that use up the total available space.

Watch for students who focus on the following ideas, since they underpin essential strategies in future tasks: (This list is sequenced from most likely to less likely to occur; don't be concerned if not all of these ideas are present in your class, since future tasks will elicit each of these ways of thinking about the system of constraints.)

- Students who calculate the intercepts of the constraints; that is, students who consider how they might use up all of the money or all of the space, by boarding just cats or just dogs.
- Students who make note of an "exchange rate" between cats and dogs in terms of either of the constraints. For example, four cats use the same space as one dog.
- Students who create charts to keep track of the combinations of dogs and cats they have tried. Such charts will probably include columns to track the number of dogs, the number of cats, and the money earned. Students will also have to keep track of whether a particular combination of dogs and cats satisfies each of the constraints.
- Students who plot combinations on a coordinate grid to keep track of the combinations they have tested.
- Students who try to write equations or inequalities to represent the constraints.

**Discuss (Whole Class):**

Begin with a combination of dogs and cats that worked, a second combination that worked and resulted in a higher profit, followed by a combination that didn't work because it didn't satisfy one or more of the constraints. Collect combinations that work using a table or coordinate grid, as suggested by student work. Have students share their ideas about how they can improve the daily income while satisfying the constraints, connecting their ideas to the organizational representations of tables and/or points plotted on a coordinate grid.

**Aligned Ready, Set, Go: Systems 2.1**

## Ready, Set, Go!



### Ready

Topic: Determine if given value is a solution and solve systems of equations

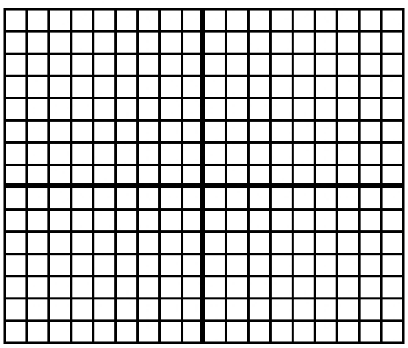
**Substitute the given points into the equations to determine which ordered pair satisfies the system of linear equations, then graph both equations and label the point of intersection.**

1. $y = 3x - 2 \ and \ y = \ x$

   a. $(0, -2)$

   b. $(2, 2)$

   c. $(1, 1)$

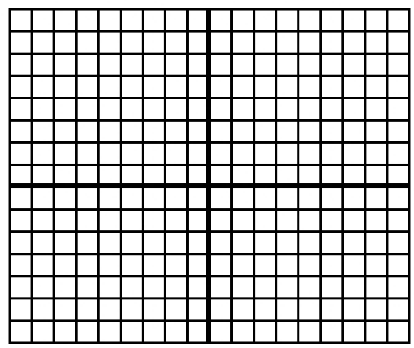


2. $y = 2x + 3 \ and \ y = \ x + 5$

   a. (2, 7)

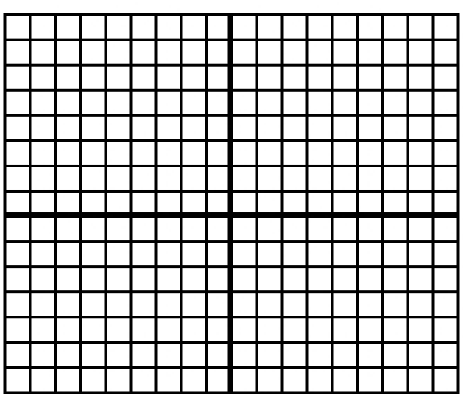
   b. (-7, 11)

   c. (0, 5)

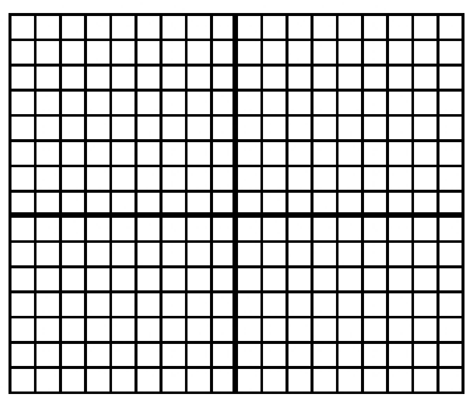


**Solve the following systems by graphing. Check the solution by evaluating both equations at the point of intersection.**

3. $y = x + 3 \ and \ y = -2x + 3$



4. $y = 3x - 8 \ and \ y = -x$



Name: Systems 2.1

**Set** Topic: Determining possible solutions

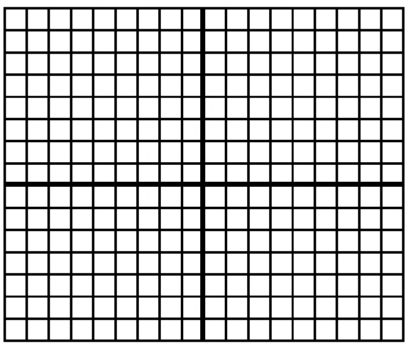
5. A theater wants to take in at least $2000 for a certain matinee. Children's tickets cost $5 each and adult tickets cost $10 each. The theater can seat up to 350 people. Find five combinations of children and adult tickets that will make their goal.

**Go**

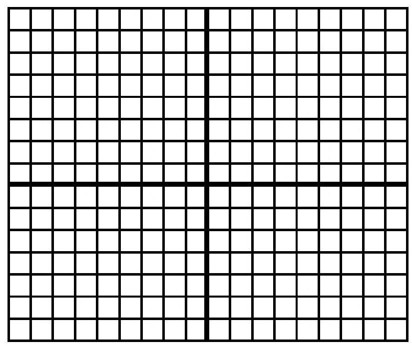
Topic: graphing linear equations and determining if a given value is a solution

**Graph each equation below, then determine if the point (3,5) is a solution to the equation. Name two additional points that are solutions to the equation and show these points on the graph.**

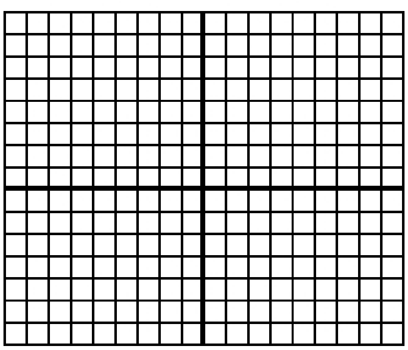
6. $y = 2x - 1$



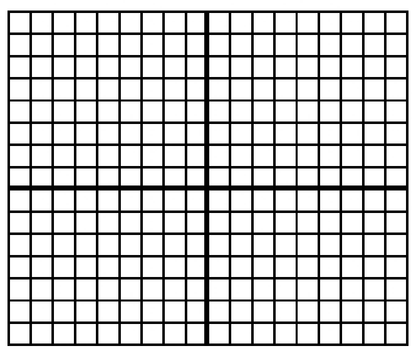
7. $y = \frac{1}{3}x + 2$



8. $y = -3x + 5$



9. $y = \frac{-3}{5}x + 4$



Need help? Check out this related video:

https://www.youtube.com/watch?v=vo-CXaCf1I4

# 2.2 Too Big or Not Too Big, That is the Question

*A Solidify Understanding Task*



As Carlos is considering the amount of money available for purchasing cat pens and dog runs (see below) he realizes that his father's suggestion of boarding "the same number of each, perhaps 12 cats and 12 dogs" is too big. Why?

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

1. Find at least 5 more combinations of cats and dogs that would be "too big" based on this *Start-up Cost* **constraint.** Plot each of these combinations as points on a coordinate grid using the same color for each point.

2. Find at least 5 combinations of cats and dogs that would not be "too big" based on this *Start-up Cost* constraint. Plot each of these combinations as points on a coordinate grid using a different color for the points than you used in #1.

3. Find at least 5 combinations of cats and dogs that would be "just right" based on this *Start-up Cost* constraint. That is, find combinations of cat pens and dog runs that would cost exactly $1280. Plot each of these combinations as points on a coordinate grid using a third color.

4. What do you notice about these three different collections of points?

5. Write an equation for the line that passes through the points representing combinations of cat pens and dog runs that cost exactly $1280. What does the slope of this line represent?

Carlos and Clarita don't have to spend all of their money on cat pens and dog runs, unless it will help them maximize their profit.

6. Shade all of the points on your coordinate grid that **satisfy** the *Start-up Costs* constraint.

7. Write a mathematical rule to represent the points shaded in #6. That is, write an inequality whose **solution set** is the collection of points that satisfy the *Start-up Costs* constraint.

In addition to *start-up costs,* Carlos needs to consider how much space he has available, base on the following:

- *Space*: Cat pens will require 6 $ft^2$ of space, while dog runs require 24 $ft^2$. Carlos and Clarita have up to 360 $ft^2$ available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

8. Write an inequality to represent the solution set for the *space* constraint. Shade the solution set for this inequality on a different coordinate grid.

# 2.2 Too Big or Not Too Big, That is the Question – Teacher Notes

*A Solidify Understanding Task*

**Special Note to Teachers:** There are two possible ways you may consider implementing this task. The first approach assumes that students will not struggle with writing equations of the boundary line—probably in slope-intercept form—representing the two constraints, and that the new issue for these students is focusing on the half-plane that represents the solution set to the inequality associated with the constraint. The second approach to the task would be to split the work into two parts. The first part ending with students considering how to write the equation of the line representing the points for which the combination of cats and dogs costs exactly $1280 (question 5). Before proceeding to the next part of the task, you could have students work on the next task "Some of One, None of the Other" where they will examine specific strategies for writing the equations of the lines representing the constraints. The students can then return to the remainder of this task to consider the fact that the constraints are actually written using terms that suggest they are inequalities, not equations. If you use the first approach this task, you should still follow it with "Some of One, None of the Other" to solidify the additional ideas about equivalent forms of the boundary line equations that are presented there.

**Purpose**: As students continue to work with the pet sitter context they will examine and extend many ideas, strategies and representations related to solving systems of equations and inequalities. For example, each constraint (space, start-up costs) can be represented by an inequality since Carlos and Clarita don't have to use up all of the available space or money. The solution set of an inequality (in quadrant I) consists of all of the points that satisfy the constraint. The boundary between points that satisfy the constraint and those that do not is a line. The slope of the line represents the "exchange rate" between cat pens and dog runs (e.g., for every 2 fewer dog runs we buy, we can purchase 5 more cat pens), and the *x*- and *y*-intercepts of the line represent purchasing either all cat pens and no dog runs, or all dog runs and no cat pens. A shaded half-plane in the coordinate grid is being used to represent the solution set for an inequality in two-variables. In this case, the points on the boundary line are also included in the solution set, indicated by drawing a solid line for the boundary.

**Core Standards Focus**:

**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality)

**Related Standards: N.Q.2, A.SSE.1a, A.REI.10, A.CED.2, F.LE.1b, F.LE.5**

**Launch (Whole Class):** ***Q1-Q5***
Read through the first four questions on the worksheet and verify that students recognize that they are to identify three different sets of points—those that are too big, too small and just right, relative to the start-up costs constraint—and to use a different color to plot each set of points on a coordinate grid. Discuss the meaning of the word "constraint" (shown in bold type on the handout). Leave graphing decisions—such as which axis to label "cats" and which to label "dogs", or what to use as an appropriate scale for each axis—up to students to make.

**Explore (Small Group):**
Students may have already found some points that work or do not work for the start-up costs from the previous task. Today they will color-code these points and try to identify the boundary that separates points that work from those that do not. Listen for how students are determining which color should be used when plotting points, and that they are producing a collection of points that start to "fill up" the regions on either side of the boundary (e.g., it is best if some, but not all points, are clustered around the boundary—help students see that there are obvious points that lie in either set, as well as points for which they need to carefully check to see if they satisfy the constraint).

Watch for successful strategies for identifying points that lie on the boundary line. For example, students might begin to articulate that they need to find a combination of cats and dogs, such that $32 times the number of cats added to $80 times the number of dogs is exactly $1280. Particularly watch for students who identify combinations that involve either all dogs and no cats, or no cats and all dogs. Some students may notice a ratio between the cost of dog runs and cat pens, and how they might use that ratio to help them find other combinations of dogs and cats that lie on the boundary line.

Suggest that students think about what they know about the boundary line that will help them write its equation.

**Discuss (Whole Class):**
Focus the first part of the discussion on question 4, "What do you notice about the three different collections of points?" Students should be able to describe that the points that lie on the boundary form a line, and that points that lie below the line represent combinations of dogs and cats for which the twins can afford to purchase runs and pens, and that points that lie above the line represent combinations of cats and dogs that would be too expensive. Have students present points that they found which lie on the boundary line and explain how they know their points lie on the boundary.

(Note: if students are not ready to articulate these ideas, you will need to do more specific work before you can focus on question 4. Have students agree on colors to use for each collection of points, how to label the $x$-and $y$-axis, and what scale to use on each axis so that points identified by individual groups can be collected onto a single class graph. First collect several points that worked, then several points that didn't work, and finally, points on the boundary. Using the class graph, move on to a discussion of question 4.)

Move to a discussion of question 5 by asking what students know about the boundary line that will help them write its equation. Students might use a variety of strategies to write the equation of the boundary line. Watch for strategies that use information from the graph (e.g., slope, vertical intercept, or both intercepts) as well as strategies based on the wording of the constraint, leading to an equation of the line in standard form.

As possible, have presentations from students who can articulate the meaning of the slope as the "exchange rate" between cat pens and dog runs. Identify students who can articulate that the $x$- and $y$-intercepts represents situations where there are either no cats or no dogs. Also look for students who have written the equation of the line in standard form from the language of the constraint.

Note: If your students are unable to write the equation of the boundary line at this time, move onto the next task "Some of One, None of the Other" which solidifies ways of writing the equation of the boundary line in both slope-intercept form and standard form. Then return to part 2 of this task.

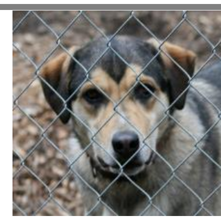
**Launch (Whole Class): *Q6-Q8***
Launch this portion of the task by having students consider the wording in the start-up cost constraint that suggest that Carlos and Clarita may not need to spend all of their money on dog runs and cat pens. Ask why this might be the case. Once students have identified that the amount of space available for runs and pens may affect how much they spend, have them work on the last portion of this task.

**Explore (Small Group):**
Note students use of various representations to indicate *all* of the combinations of cats and dogs that work for each constraint, including the shading of a half-plane (or shading discrete points in the first quadrant below the boundary line, since we can't actually board a partial cat or dog), and the use of inequality notation to denote the solution set algebraically.

**Discuss (Whole Class):**
Note the use of inequality notation instead of an equation to algebraically identify *all* of the combinations of cats and dogs that work, that is, the points that lie below the boundary line. Ask students to describe how they knew whether to use a greater than sign or a less than sign in their inequality. Make a point of defining the boldfaced words "satisfy" and "solution set" in terms of these inequalities. At this point you should distinguish between the solution set to the inequality (the half-plane lying below the boundary line) and the solution set to the constraint (which only includes the lattice points below the boundary line that lie in quadrant 1).

**Aligned Ready, Set, Go: Systems 2.2**

Name: Systems | 2.2





# Ready, Set, Go!

## Ready

Topic: Determining if given values are solutions to an equation

**Identify which of the given points are solutions to the following linear equations.**

1. 3x + 2y = 12
   a. (2, 4)
   b. (3, 2)
   c. (4, 0)
   d. (0, 6)

2. 5x – y = 10
   a. (2, 0)
   b. (3, 0)
   c. (0, -10)
   d. (1, 1)

**Find the value that will make each ordered pair a solution to the given equations.**

3. x + y = 6
   a. (2, __)
   b. (0, __)
   c. (__, 0)

4. 2x + 4y = 8
   a. (2, __)
   b. (0, __)
   c. (__, 0)

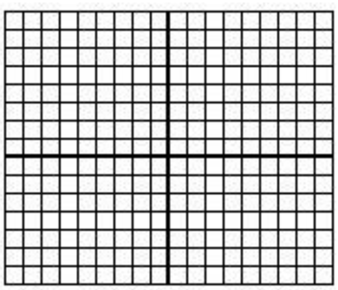
5. 3x – y = 8
   a. (2, __)
   b. (0, __)
   c. (__, 0)

## Set

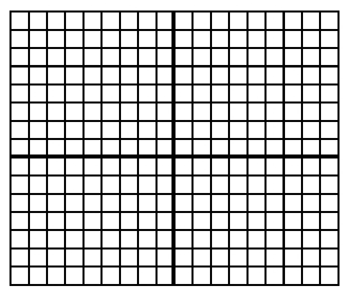
Topic: Graph linear inequalities

**Graph the following inequalities on the coordinate plane. Name one point that is a solution to the inequality and one point that is not a solution. Show algebraically and graphically that your points are correct.**

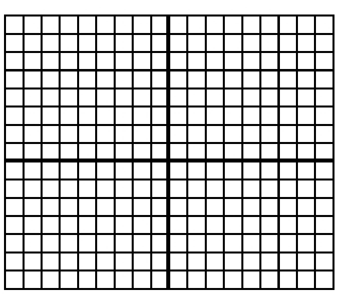
6. $y \leq 3x + 4$



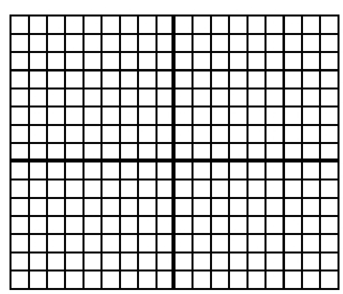
7. $x < 7$



8. $y > \frac{-3}{5}x + 2$



9. $y \geq -6$



**Go** Topic: Solving inequalities

**Follow the directions for each problem below. (Show your work!)**

10. $10 - 3x < 28$ a. Solve for x. Then graph the solution on the number line.

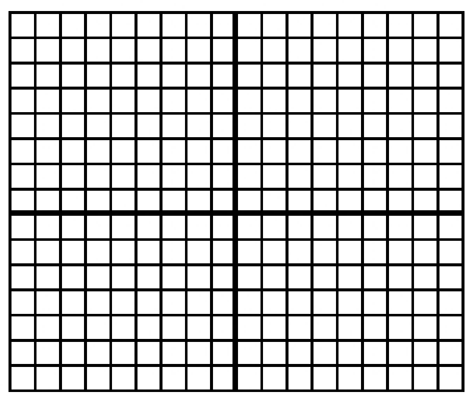
b. Select an x-value from your graph of the solution of the inequality. Replace x in the original inequality $10 - 3x < 28$ with your chosen value. Does the inequality hold true?

c. Select an x-value that is outside of the solution set on your graph. Replace x in the original inequality $10 - 3x < 28$ with your chosen value. Does the inequality still hold true?

11. $4x - 2y \geq 6$

a. Solve for y.

b. Now imagine that your inequality is an equation. In other words, your solution will say y = , instead of y ≥ or y ≤. With the equal sign, it should be the equation of a line. Graph your equation.



c. Find the y – intercept.

d. Find the slope.

e. Select a point that is above the line. ( , )
Replace the x and y – values in the inequality $4x - 2y \geq 6$.
Is the inequality still true?

f. Select a point that is below the line. ( , )
Replace the x and y – values in the inequality $4x - 2y \geq 6$.
Is the inequality still true?

g. Explain which side of the line should be shaded.

h. Decide whether the line should be solid or dotted. Justify your decision.

Need help? Check out these related videos:

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-linear-inequalities-in-two-variables-2



# 2.3 Some of One, None of the Other

*A Solidify Understanding Task*

Carlos and Clarita are comparing strategies for writing equations of the boundary lines for the "Pet Sitter" constraints. They are discussing their work on the *space* constraint.

- *Space*: Cat pens will require 6 $ft^2$ of space, while dog runs require 24 $ft^2$. Carlos and Clarita have up to 360 $ft^2$ available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

Carlos' Method: "I made a table. If I don't have any dogs, then I have room for 60 cats. If I use some of the space for 1 dog, then I can have 56 cats. With 2 dogs, I can boards 52 cats. For each additional dog, I can board 4 fewer cats. From my table I know the $y$-intercept of my line is 60 and the slope is -4, so my equation is $y = -4x + 60$."

Clarita's Method: "I let $x$ represent the number of dogs, and $y$ the number of cats. Since dog runs require 24 $ft^2$, $24x$ represents the amount of space used by dogs. Since cat pens require 6 $ft^2$, $6y$ represents the space used by cats. So my equation is
$24x + 6y = 360$."

1. Since both equations represent the same information, they must be equivalent to each other.

    a. Show the steps you could use to turn Clarita's equation into Carlos' equation. Explain why you can do each step.

    b. Show the steps you could use to turn Carlos' equation into Clarita's. Explain why you can do each step.

2. Use both Carlos' and Clarita's methods to write the equation of the boundary line for the *start-up costs* constraint.

    - *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

3. Show the steps you could use to turn Clarita's *start-up costs* equation into Carlos' equation. Explain why you can do each step.

4. Show the steps you could use to turn Carlos' *start-up costs* equation into Clarita's. Explain why you can do each step.

In addition to writing an equation of the boundary lines, Carlos and Clarita need to graph their lines on a coordinate grid.

Carlos' equations are written in **slope-intercept form**. Clarita's equations are written in **standard form**. Both forms are ways of writing **linear equations**.

Both Carlos and Clarita know they only need to plot two points in order to graph a line.

Carlos' strategy: How might Carlos use his slope-intercept form, $y = -4x + 60$, to plot two points on his line?

Clarita's strategy: How might Clarita use her standard form, $24x + 6y = 360$, to plot two points on her line? (Clarita is really clever, so she looks for the two easiest points she can find.)

# 2.3 Some of One, None of the Other – Teacher Notes
*A Solidify Understanding Task*

**Purpose**: As students examine Carlos' and Clarita's methods for writing and graphing equations to fit the "Pet Sitter" constraints students will solidify the following mathematics:

- The *x*- and *y*-intercepts of a line represent situations where one of the quantities represented by an ordered-pair $(x, y)$ is 0. The slope of a linear equation represents a rate of change, such as the exchange rate between cats and dogs. For example, in the space constraint the slope is -4 since we lose space for 4 cat pens each time we increase the number of dog runs by 1. The standard form of a equation of a line can be written as a linear combination of terms, such as the cost constraint, $80x + 32y = 1280$, where the first term represents the cost of *x* dog runs, the second term represents the cost of *y* cat pens, and the sum or combination of terms gives the total cost. This same relationship can be represented with a linear equation in slope-intercept form, $y = -\frac{5}{2}x + 40$, which captures two important facts about this relationship: we can afford 5 fewer cat pens for every 2 dog runs we purchase, and if we purchase no dog runs we can afford 40 cat pens.

- Using properties of equality, a logical sequence of equivalent equations can be created to move between a linear equation in slope-intercept form, $y = mx + b$, and standard form, $Ax + By = C$. To find the *x*-intercept of a linear equation in either form, let $y = 0$ and solve the equation for *x*. To find the *y*-intercept, let $x = 0$ and solve the equation for *y*. Since two points determine a line, the graph of a line can be obtained by plotting both the *x*-and *y*-intercepts from standard form, or the *y*-intercept and a second point found by counting the ratio of rise to run as indicated by the slope.

**Core Standards Focus**:

**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**Related Standards: A.SSE.1a, A.REI.10, F.LE.1b**

**Launch (Whole Class):**

Have students read both Carlos' and Clarita's methods for writing the equations of the line representing the "Pet Sitters" space constraint. Make sure they understand each method, and can explain details of the methods, such as how Carlos found the slope and *y*-intercept from his table, and how Clarita is finding each of the terms in her linear combination. Clarifying the details of these two strategies will allow students to work independently or collaboratively on the remainder of the task. Point out to students that their work on question 1 is to show how they can move from one form of the equation of the space constraint line to the other.

**Explore (Small Group):**

Starting with Clarita's equation and moving to Carlos' equation is a matter of solving Clarita's equation for *y.* This should not be difficult for students, but going the other direction may prove more difficult until they realize they can just reverse the steps. Questions 2-4 reinforce the work students did previously by applying the same strategies to the cost constraint. If students are having difficulty with the algebra, have them move to the graphical portion of the task. They should be familiar with the strategy of plotting points on a line given the slope and a starting point, such as the *y*-intercept. Press students to find the *x*- and *y*-intercepts of the line in standard form by asking what they know about the coordinates of a point that lies on the *x*-axis or the *y*-axis. Once students find a strategy for finding both intercepts when the equation of the line is in standard form, have them see how these principles could be applied to finding <u>both</u> intercepts when the equation of the line is in slope-intercept form.

As you prepare for the whole group discussion, identify students who can write the linear equation for the start-up cost constraint in both standard form and slope-intercept form, and students who can move algebraically between the two forms. Also identify students who can explain the algorithm for finding *x*- and *y*-intercepts.

**Discuss (Whole Class):**

Begin the discussion by focusing on the last part of the task, Carlos' and Clarita's strategies for plotting two points on the line using either slope-intercept or standard form. Allow the strategy of finding the intercepts by setting one of the variables equal to zero to emerge. Make sure students can apply this strategy to both forms of the line.

Following the graphical discussion about intercepts, return to the algebra of Q2-Q4. Have students present the two forms of the equation for the start-up cost constraint, and show they are equivalent by moving back and forth between the two forms algebraically. Then have students find the *x*- and *y*-intercepts of the graph of this line, plot the graph using these two points, and verify that the slope of the line is the same as that predicted by the slope-intercept equation.

**Aligned Ready, Set, Go: Systems 2.3**

Name: Systems | 2.3

# Ready, Set, Go!



## Ready

Topic: Determining points that satisfy equations and solving systems of equations

**Three points are given. Each point is a solution to at least one of the equations. Find the point that satisfies both equations. (This is the solution to the system!) Justify that the point is a solution to both equations and that the others are not.**

1. $\begin{cases} y = 2x - 3 \\ y = -x + 3 \end{cases}$
   a. (-2 , 5)
   b. (2 , 1)
   c. (4 , 5)

2. $\begin{cases} y = 3x + 3 \\ y = -x + 3 \end{cases}$
   a. (-1 , 0)
   b. (6 , -3)
   c. (0 , 3)

3. $\begin{cases} y = 2 \\ y = -4x - 6 \end{cases}$
   a. (7 , 2)
   b. (2 , -14)
   c. (-2 , 2)

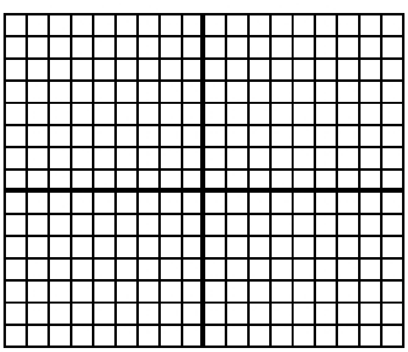
4. $\begin{cases} y = 2x + 4 \\ x + y = -5 \end{cases}$
   a. (1 , 6)
   b. (-3 , -2)
   c. (-3 , 2)

## Set

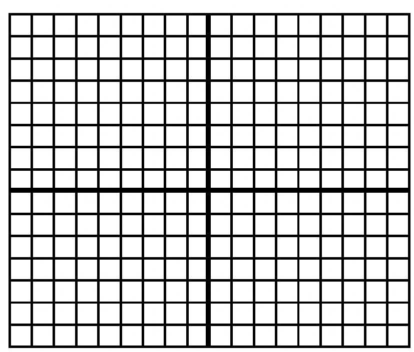
Topic: Graphing linear equations from standard form using intercepts

**Graph the following equations by finding the intercepts.**

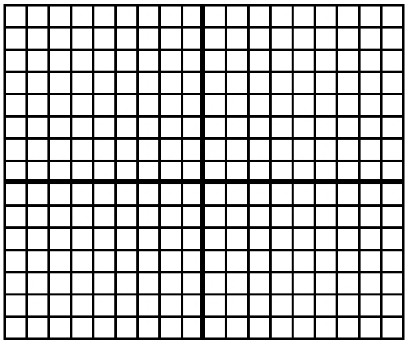
5. $5x - 2y = 10$



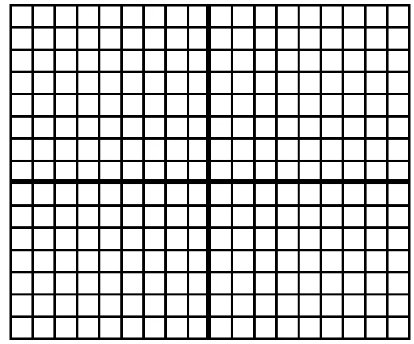
6. $3x - 6y = 24$



7. $6x + 2y = 18$



8. $-2x + 7y = -14$



## Go

Topic: Adding and multiplying fractions

**Add. Reduce your answers but leave as improper fractions when applicable.**

9. $\frac{3}{4} + \frac{1}{8}$ 10. $\frac{3}{5} + \frac{7}{10}$ 11. $\frac{2}{3} + \frac{1}{4}$ 12. $\frac{4}{7} + \frac{8}{21}$

**Multiply. Reduce your answers but leave as improper fractions when applicable.**

13. $\frac{3}{4} \times \frac{2}{9}$ 14. $\frac{4}{7} \times \frac{7}{10}$ 15. $\frac{5}{4} \times \frac{2}{9}$ 16. $\frac{3}{7} \times \frac{8}{21}$

Need help? Check out these video lessons.

http://www.youtube.com/watch?v=cuNpXve18Pc

http://www.youtube.com/watch?v=6zixwWZ88tk

http://www.youtube.com/watch?v=oHNR0FK_lDE

# 2.4 Pampering and Feeding Time
*A Practice Understanding Task*



Carlos and Clarita have been worried about space and start-up costs for their pet sitters business, but they realize they also have a limit on the amount of time they have for taking care of the animals they board. To keep things fair, they have agreed on the following time constraints.

- *Feeding Time*: Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.

- *Pampering Time*: The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

Write inequalities for each of these additional time constraints. Shade the solution set for each constraint on separate coordinate grids.

# 2.4 Pampering and Feeding Time – Teacher Notes
*A Practice Understanding Task*

**Purpose**: In the context of examining combinations of cats and dogs that satisfy these additional "Pet Sitter" constraints students will practice the following mathematics: (Note that each of these ideas have surfaced and have been examined and solidified in previous tasks. This is an opportunity for students to practice these ideas in a new setting.)

- Constraints can be represented by inequalities. The solution set of an inequality consists of a set of points that lie in a half-plane bounded by the equation associated with the inequality.

- Students should already have well-practiced procedures for writing equations of lines through given points from a graphical perspective. In addition, they should practice writing the equation of the boundary line using the terms defined by the language of the constraints. Students will determine which half-plane to shade to represent the solution to the inequality by using their specific "test points" or by identifying which points are implied by the inequality sign.

- A shaded half-plane in the coordinate grid is a representation of the solution set for an inequality in two-variables. The points on the boundary line are also included in the solution set—indicated by drawing a solid line for the boundary—unless the boundary is a strict inequality and then the boundary is represented by a dotted line.

**Core Standards Focus**:

**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) . . .

**Related Standards:**

**Launch (Whole Class):**

Students should be able to get started on this task without additional support, since it is similar in nature to the work they did on "Too Big or Not Too Big, That is the Question." Ask students to relate the work of previous tasks in this module to the work they are going to do today.

**Explore (Small Group):**

There is some extraneous language in the wording of these constraints, so watch for students who are getting hung up on the actual information needed to define each constraint. Make sure that students are attending to the language in these constraints that suggest they are inequalities, "up to 8 hours each day". Since time is given in both minutes and hours, students will have to convert between units of time. If hours are used, the coefficients of the terms in the inequalities will be fractions.

Watch for students who have not yet proceduralized the process of writing linear inequalities in two variables or for determining the half-plane formed by the boundary line in which the solution set should be shaded. Students should be moving away from testing individual points and towards the more efficient strategy of using the inequality relationship to determine the half-plane in which the solution set lies.

**Discuss (Whole Class):**

Identify issues that would warrant a whole class discussion, such as writing the inequalities in either standard form or slope-intercept form, and have more expert students share their work on each of the constraints. It might be that the arithmetic of working with fractional coefficients is more of the issue in this task than the solution set to an inequality.

**Aligned Ready, Set, Go: Systems 2.4**

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# Ready, Set, Go!

## Ready

Topic: Substitution and Solving Equations

**Determine whether $h = 3$ is a solution to each problem.**

1. $3(h-4) = -3$

2. $3h = 2(h+2) - 1$

3. $2h - 3 = h + 6$

4. $3h > -3$

5. $\frac{3}{5} = h \times \frac{1}{5}$

**Determine the value of $x$ that makes each equation true.**

6. $4x - 2 = 8$

7. $3(x+5) = 20$

8. $2x + 3 = 2x - 5$

## Set

Topic: Creating equations, solving real world problems, solve systems of equations

A phone company offers a choice of three text-messaging plans. Plan A gives you unlimited text messages for $10 a month; Plan B gives you 60 text messages for $5 a month and then charges you $0.05 for each additional message; and Plan C has no monthly fee but charges you $0.10 per message.

9. Write an equation for the monthly cost of each of the three plans.

10. If you send 30 messages per month, which plan is cheapest?

11. What is the cost of each of the three plans if you send 50 messages per month?

12. Determine the values for which each plan is the cheapest?

Name: Systems 2.4

## Go

Topic: Solve literal equations

**Re-write each of the following equations for the indicated variable.**

13. $3x + 5y = 30$ for y

14. $24x + 6y = 360$ for x

15. $\frac{1280 - 80d}{32} = c$ for d

16. $C = \frac{5}{9}(F - 32)$ for F

17. $y = mx + b$ for $b$

18. $Ax + By = C$ for $y$

Need help? Check out these related videos.

What does it mean to be a solution?
http://patrickjmt.com/an-intro-to-solving-linear-equations-what-does-it-mean-to-be-a-solution/
http://patrickjmt.com/solving-linear-equations/

Solving for a variable.
http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-for-a-variable

# 2.5 All For One, One For All

*A Solidify Understanding Task*



Carlos and Clarita have found a way to represent combinations of cats and dogs that satisfy each of their individual "Pet Sitter" constraints, but they realize that they need to find combinations that satisfy all of the constraints simultaneously. Why?

1. Begin by listing the **system of inequalities** you have written to represent the *start-up costs* and *space* "Pet Sitter" constraints.

2. Find at least 5 combinations of cats and dogs that would satisfy both of the constraints represented by this system of inequalities. How do you know these combinations work?

3. Find at least 5 combinations of cats and dogs that would satisfy one of the constraints, but not the other. For each combination, explain how you know it works for one of the inequalities, but not for other?

4. Shade a region on a coordinate grid that would represent the **solution set to the system of inequalities.** Explain how you found the region to shade.

5. Rewrite your systems of inequalities to include the additional constraints for *feeding time* and *pampering time*.

6. Find at least 5 combinations of cats and dogs that would satisfy all of the constraints represented by this new system of inequalities. How do you know these combinations work?

7. Find at least 5 combinations of cats and dogs that would satisfy some of the constraints, but not all of them. For each combination, explain how you know it works for some inequalities, but not for others?

8. Shade a region of a coordinate grid that would represent the solution set to the system of inequalities consisting of all 4 "Pet Sitter" constraints. Explain how you found the region to shade.

9. Shade a region in quadrant 1 of a coordinate grid that would represent all possible combinations of cats and dogs that satisfy the 4 "Pet Sitter" constraints. This set of points is referred to as the **feasible region** since Carlos and Clarita can feasibly board any of the combinations of cats and dogs represented by the points in this region without exceeding any of their constraints on time, money or space.

10. How is the feasible region shaded in #9 different from the solution set to the system of inequalities shaded in #8?

# 2.5 All for One, One for All—Teacher Notes

*A Solidify Understanding Task*

**Special Note to Teachers:** There are two possible ways to implement this task. The first version is to have students work on finding the solution set to a system of inequalities that involves two of the "Pet Sitter" constraints, then move to working on finding the solution set to the system of inequalities that involve all 4 constraints. As students work to find the feasible region for all 4 constraints, they will need to find the points of intersection of the boundary lines. It is sufficient to let students estimate the points of intersection from the graph, or they can use previously developed algebraic strategies to find the points of intersection, if such strategies are available. The next task, "Get to the Point" focuses on algebraic processes for finding the points of intersection. If you would prefer, you can implement this task in two parts. In part 1(questions 1-4), have students work on finding the solution set to the system of inequalities that involves just two of the "Pet Sitter" constraints. Before proceeding to part 2, students could work on the next task "Get to the Point" where they will examine specific strategies for finding the points of intersections of the boundary lines. The students can then return to part 2 of this task (questions 5-10) to consider the system of inequalities that includes all four "Pet Sitter" constraints. If you use the first version of this task, you need to decide if your students will benefit from the graphical, numerical and algebraic work of question 1 of "Get to the Point!, or if they already have the necessary understanding for algebraically solving a system of equations. If they have not already found the points of intersection for some of the pairs of constraints using an algebraic method, questions 2 and 3 of "Get to the Point!" will have them do so.

**Purpose**: In the context of examining combinations of cats and dogs that satisfy all of the "Pet Sitter" constraints students will solidify the following mathematics:

- The solution set to a system of inequalities represents the set of points that satisfy all of the inequalities simultaneously. That is, a point in the solution set of a system of inequalities makes all of the inequalities true. Contexts, such as "Pet Sitters" may imply additional constraints, such as $y \geq 0$ or $x \geq 0$, that are not explicitly stated.

- A shaded region on the coordinate grid is used to represent the solution set for a system of inequality in two-variables. In this case, the points on the boundary lines that trace out the region are also included in the solution set, indicated by drawing a solid line for the boundaries. A shaded polygonal region is used to represent the solution to a system of constraints.

**Core Standards Focus**:

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

**Related Standards: N.Q.1, N.Q.2, A.CED.2**

**Launch (Whole Class): *Q1-Q4 (or part 1)***

Introduce the concept of a system of inequalities as a set of inequalities that represent conditions that have to be satisfied simultaneously. Invite students to create a system of inequalities to represent the *start-up costs* and *space* constraints. Students will have written the individual inequalities representing these two constraints as part of their class work for "Too Big or Not Too Big". Give them a couple of minutes to collect their individual inequalities into a system of inequalities.

Students may have written their inequalities in slope-intercept form or in standard form. Select students to present different forms of the inequalities that could be included in the system, perhaps including a system that contains more than one form for writing the inequalities. If the same constraint is represented by inequalities written in different forms, ask if both inequalities should be included in the system.

**Explore (Small Group):**

Watch for strategies for determining if a combination fits both constraints. Do students guess and check points in both inequalities, or do they look for overlapping regions? How do they make use of the points of intersection between the boundary lines representing the constraints? Watch for efforts to determine where the point of intersection is located, but note that specific algebraic methods for doing so will be treated in future tasks, so a visual estimate of the coordinates of the point of intersection will work for the time being. Some students may guess the coordinates of the point of intersection and then test to see if the point actually satisfies both constraints.

Press students to find *all* of the points that work, not just those obvious ones that lie far away from the boundary lines and closer to the origin. You might remind students that the eventual goal is to collect the most daily income, so they will want to identify combinations of cats and dogs that not only work for both constraints, but will also bring in the most income—points which intuitively lie farther away from the origin and nearer to the boundary lines.

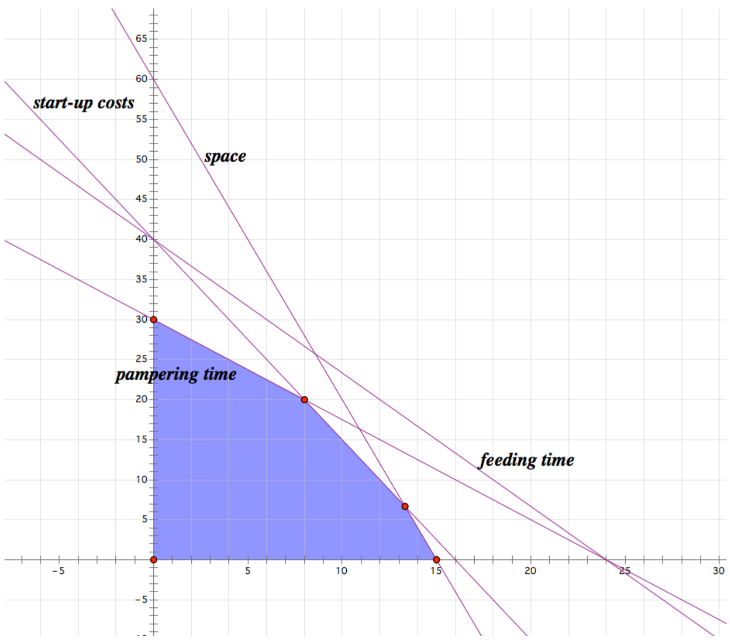
**Discuss (Whole Class):**

This discussion should focus on shading the overlapping regions of the two half-planes which represent the solution sets of the individual inequalities, and that this shaded region is the solution set for the system of inequalities. Point out that the solution to the system of constraints lie only in the first quadrant, and are in fact just a finite set of discrete points in the first quadrant, since we can't have a fraction of a cat or a dog. For convenience, we will shade a polygonal region in the first quadrant to represent the solution set to the system of constraints, since this region does include the points of interest.

**Launch (Whole Class): *Q5-Q10 (or part 2)***
Invite students to create a new system of inequalities to represent all four "Pet Sitter" constraints, including *feeding time* and *pampering time*. Give them a couple of minutes to write this new system of inequalities.

**Explore (Small Group):**
Watch for strategies for determining if a combination fits all 4 constraints. Do students guess and check points in all four inequalities, or do they look for overlapping regions? As students work on this part of the task they should notice that all four of the constraints are not necessary—the *feeding time* constraint lies outside of the region determined by the other three constraints.

**Discuss (Whole Class):**
Share successful strategies for finding the polygonal feasible region. The feasible region lies in quadrant 1, and is therefore a subset of the actual solution set for the system of inequalities that extends into quadrants II, III and IV. Make sure that both the solution set to the system of inequalities *and* the solution set for the constraints (i.e., feasible region) are identified. Bring out that the feasible region is a solution set to the system of inequalities that includes $x \geq 0$ and $y \geq 0$ as implied constraints.




**Aligned Ready, Set, Go: Systems 2.5**

Name: Systems | 2.5

# Ready, Set, Go!

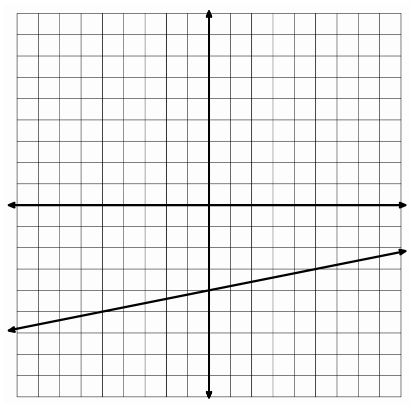


## Ready

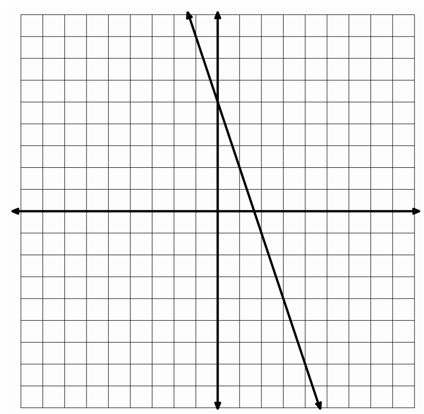
Topic: Graphing two variable inequalities

**For each inequality and graph, pick a point and use it to determine which half-plane should be shaded, then shade the correct half-plane.**

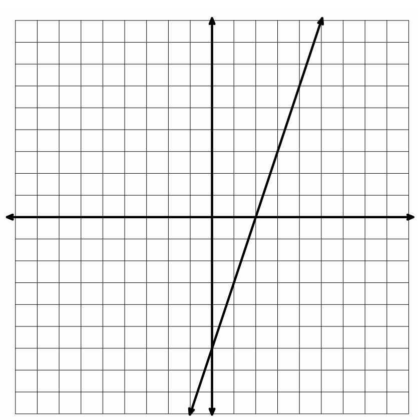
1. $y \le \frac{1}{5}x - 4$



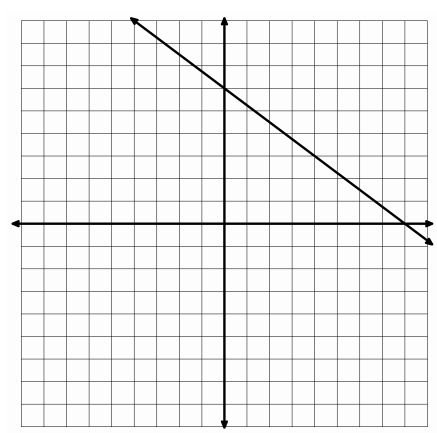
2. $y \ge -3x + 5$



3. $5x - 2y \le 10$



4. $3x + 4y \ge 24$



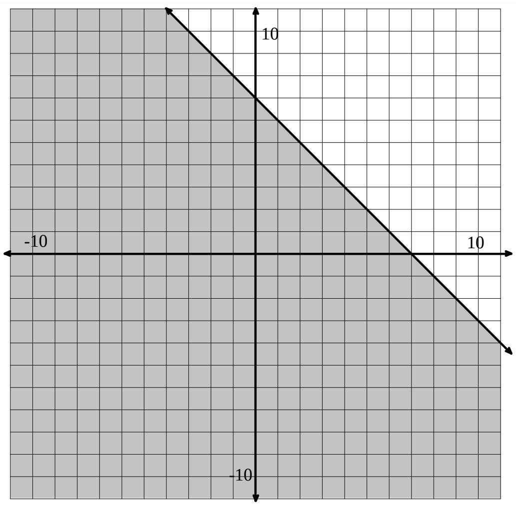
Name: Systems | 2.5

## Set

Topic: Writing two variable inequalities

**Given the graph with the regions that are shaded write the inequality or system of inequalities.**

5.

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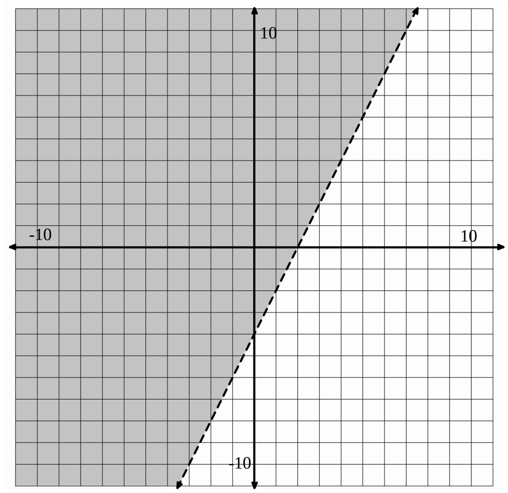


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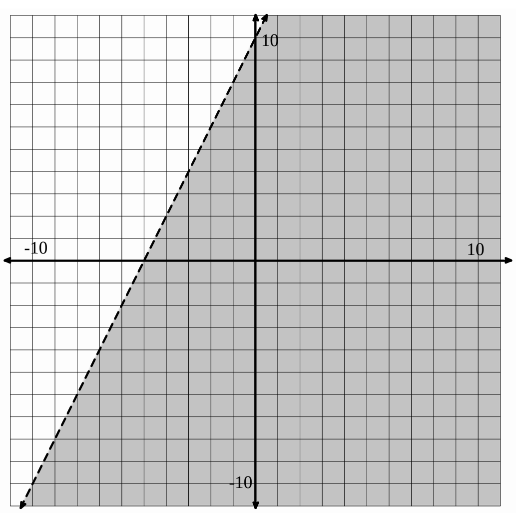


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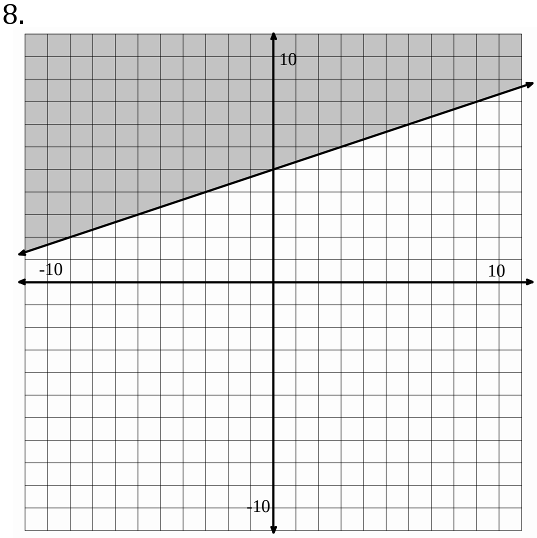


## Go

Topic: Proportional relationships

**For each proportional relationship below, one representation is provided. Show the remaining representations and explain any connections you notice between representations.**

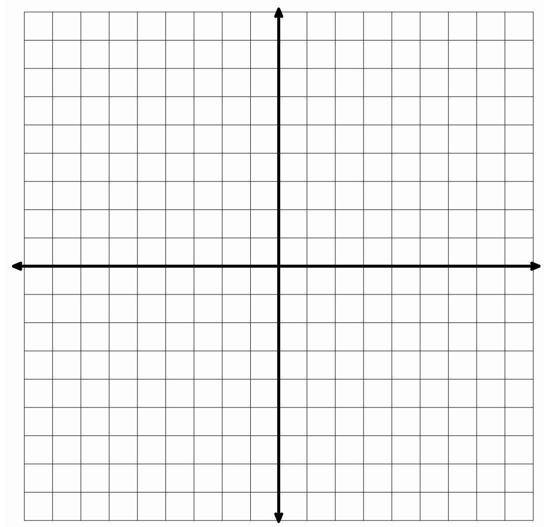
9. **Equation**:

**Table**

| Days | Cost |
|---|---|
| 1 | 8 |
| 2 | 16 |
| 3 | 24 |
| 4 | 32 |

**Create a context**

**Graph**



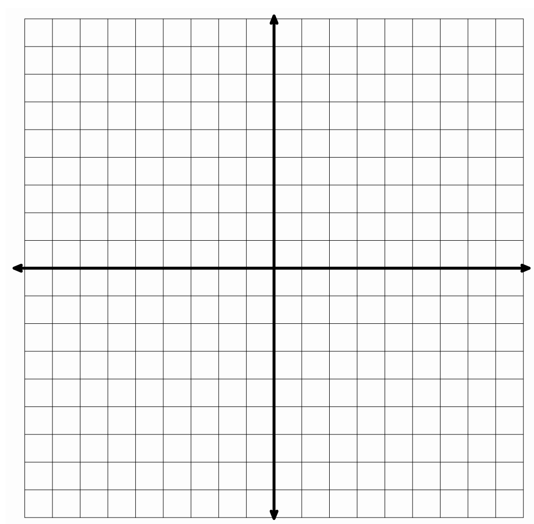
10. **Equation**:

**Table**

**Create a context**

Claire earns $9 per week allowance.

**Graph**



Name: 

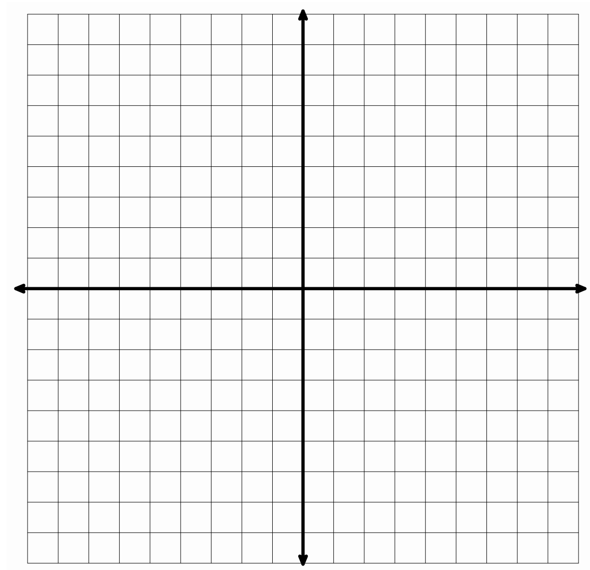
11. **Equation**: $y = 3x$

**Table**

| | |
|---|---|

**Create a context**

**Graph**



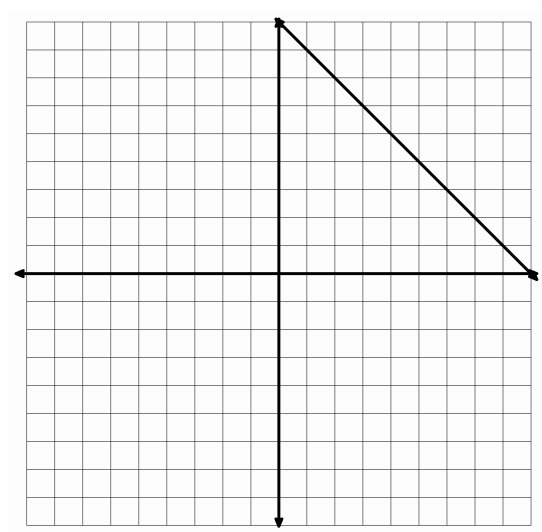
12. **Equation**:

**Table**

| Days | Cost |
|---|---|

**Create a context**

**Graph**



Need Help? Check out these related videos.

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-linear-inequalities-in-two-variables-3

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# 2.6 Get to the Point!

*A Solidify Understanding Task*

Carlos and Clarita need to clean the storage shed where they plan to board the pets. They have decided to hire a company to clean the windows. After collecting the following information, they have come to you for help deciding which window cleaning company they should hire.

- *Sunshine Express Window Cleaners* charges $50 for each service call, plus $10 per window.
- "*Pane"less Window Cleaners* charges $25 for each service call, plus $15 per window.

1. Which company would you recommend, and why? Prepare an argument to convince Carlos and Clarita that your recommendation is reasonable. (It is always more convincing if you can support your claim in multiple ways. How might you support your recommendation using a table? A graph? Algebra?)

Your presentation to Carlos reminds him of something he has been thinking about—how to find the coordinates of the points where the boundary lines in the "Pet Sitter" constraints intersect. He would like to do this algebraically since he thinks guessing the coordinates from a graph might be less accurate.

2. Write equations for the following two constraints.

    - *Space*
    - *Start-up Costs*

    Find where the two lines intersect algebraically. Record enough steps so that someone else can follow your strategy.

3. Now find the point of intersection for the two time constraints.

    - *Feeding Time*
    - *Pampering Time*

# 2.6 Get to the Point! – Teacher Notes

*A Solidify Understanding Task*

**Purpose**: This task is designed to solidify graphical, numerical and algebraic strategies for solving a system of two linear equations. While the point of intersection on a graph represents the solution to the system, it can be difficult to identify the exact coordinates of this point of intersection. A table can provide an efficient "guess and check" strategy for closing in on the coordinates of a point of intersection when the coordinates are not integers. A table might also suggest an algebraic strategy: since we are looking for an input value for which both output values are the same, we can set the two equations equal to each other to find the $x$-value for which the two $y$-values are equal. Once this "set the equations equal to each other" strategy is established, the notion of substituting one of the expressions for $y$ into the other equation can be suggested, since setting both equations equal to each other is equivalent to substituting one expression for $y$ into the other equation.

**Core Standards Focus**:

**A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Related Standards: N.Q.2, A.CED.2, A.CED.2, A.REI.10**

**Launch (Whole Class):** ***Q1***

Read the initial window-cleaning context with the class, and point out the statement that the students should provide multiple representations to justify their recommendation as to which window cleaning company Carlos and Clarita should use.

**Explore (Small Group):**

Your students may already be familiar with strategies for finding the point of intersection of the two lines from the window cleaning company constraints. The context points out an important issue—which company they should select depends on the number of windows in the storage shed. Since we don't know this number, the recommendation should be made in terms of this unknown amount. In this case, both companies charge $100 to wash 5 windows. *"Pane"less Window Cleaners* is cheaper if the twins have fewer than 5 windows to be cleaned, and *Sunshine Express* is cheaper if they have more.

Press students to explore graphical, numerical and algebraic solutions and to be able to describe the connections between each (see purpose statement above).

**Discuss (Whole Class):**

Begin by having a graph of the scenario presented in order to identify that the point of intersection determines the number of windows for which both companies would cost the same, and that the graph can be used to determine which company is cheaper on either side of the point of intersection.

Next examine a table showing an input column for the number of windows, and two columns for the amount charged by each company to clean that number of windows. Connect the row where the outputs are the same to the point of intersection on the graph. Add a row to this table to represent the general case, as follows. Note, also, the *y*-intercept of the lines represents the initial charge for each company, if no windows are cleaned.

| Number of Windows | *Sunshine Express* Cost | *"Pane"less* Cost |
|---|---|---|
| 0 | 50 | 25 |
| 1 | 60 | 40 |
| 2 | 70 | 55 |
| 3 | 80 | 70 |
| 4 | 90 | 85 |
| 5 | 100 | 100 |
| 6 | 110 | 115 |
| 7 | 120 | 130 |
| | | |
| $N$ | $50 + 10N$ | $25 + 15N$ |

Use this table to discuss how setting the two expressions $50 + 10N$ and $25 + 15N$ equal to each other would be equivalent to finding the row where both companies charge the same amount. This is also a substitution method if the expressions are treated as parts of the equations $C = 50 + 10N$ and $C = 25 + 15N$. Then the expression $50 + 10N$ can be substituted into the equation $C = 25 + 15N$ for $C$. As possible, use student work to discuss each algebraic strategy.

**Launch (Whole Class): *Q2 & Q3***

Have students turn their attention to solving for points where two of the "Pet Sitter" constraints intersect. While there are many points of intersection between various constraints, we will consider two such points in this task.

**Explore (Small Group):**

The coordinates of the point of intersection for the *space* and *start-up cost* constraints are not whole numbers. This should motivate an algebraic solution strategy. Students who are working with the constraints written in standard form may find a substitution strategy more efficient that solving both equations for a variable and setting them equal to each other. Watch for both algebraic strategies.

**Discuss (Whole Class):**

Make sure that both algebraic strategies for solving systems of two linear equations (i.e., substitution and setting expressions equal) get presented and discussed. Discuss the issue of the solution to the *space* and *start-up costs* system of equations not having whole number coordinates. While we can't have 13 1/3 dogs and 6 2/3 cats as a reasonable solution to the "Pet Sitters" scenario, this is the solution to the system of equations. As we have noticed, the solution to the contextualized situation may be a point in the interior of the region. However, it is important to note that this is an example of the modeling standard—we have decontextualized the situation to find a mathematical model that will help us reason about the "Pet Sitters" context. Eventually, any conclusions we make using the mathematical model will have to be interpreted in terms of the original context.

**Aligned Ready, Set, Go: Systems 2.6**

Name: Systems | 2.6



# Ready, Set, Go!

## Ready

Topic: Determine patterns

**Find the next two values in the pattern. Describe how you determined these values.**

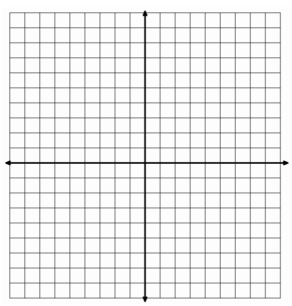
1. 3, 6, 9, 12, ____, ____ Description:
2. 3, 6, 12, 24, ____, ___ Description:
3. 24, 20, 16, 12, ___, ___ Description:
4. 24, 12, 6, 3, ___, ___ Description:

## Set

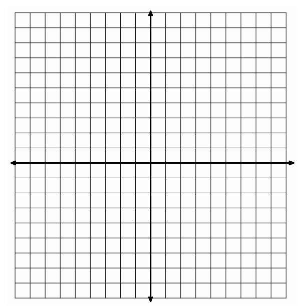
Topic: Solve systems of equations using substitution

**For questions 5-8 solve the system of equations using substitution. Check your work by graphing.**

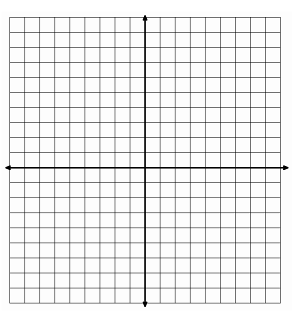
5. $\begin{cases} x + 2y = 9 \\ 3x + 5y = 20 \end{cases}$



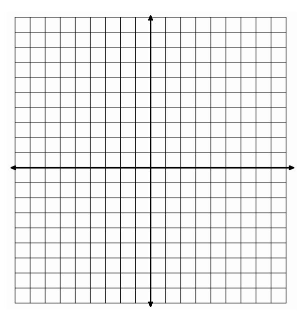
6. $\begin{cases} -4y + 8x = 16 \\ 3y + 21x = 15 \end{cases}$



7. $\begin{cases} x + 2y = -1 \\ 3x + 5y = -1 \end{cases}$



8. $\begin{cases} y = 2x - 3 \\ x + y = -5 \end{cases}$



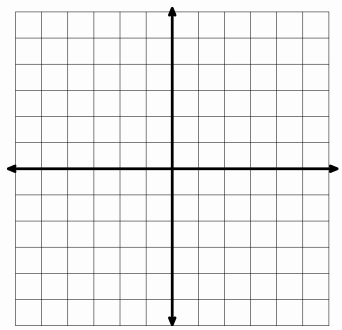
9. Tickets to a show cost $10 in advance and $15 at the door. If 120 tickets are sold for a total of $1390, how many of the tickets were bought in advance?

## Go

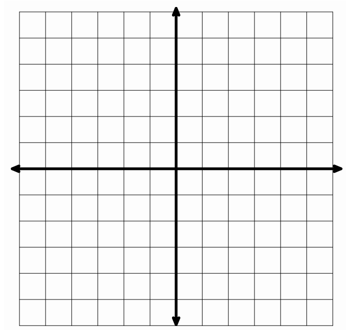
Topic: Graph two variable inequalities

**Graph the following inequalities.**

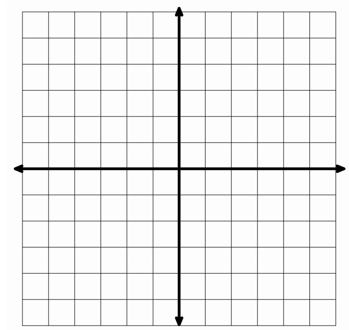
10. $y \leq 3x - 4$



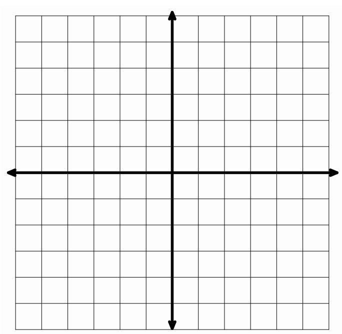
11. $y \leq -2x + 3$



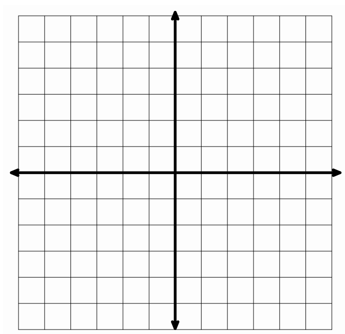
12. $y \geq 4x - 3$



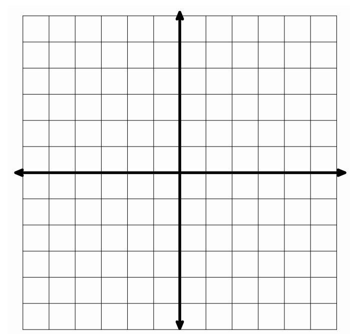
13. $3x + 4y < 12$



14. $6x + 8y \leq 24$



15. $5x + 43 \leq 15$



Need help? Check out these related videos.

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-systems-by-substitution-3

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/solving-and-graphing-linear-inequalities-in-two-variables-1

http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphing-inequalities-2

# 2.7 Shopping for Cats and Dogs

*A Develop Understanding Task*




Clarita is upset with Carlos because he has been buying cat and dog food without recording the price of each type of food in their accounting records. Instead, Carlos has just recorded the total price of each purchase, even though the total cost includes more that one type of food. Carlos is now trying to figure out the price of each type of food by reviewing some recent purchases. See if you can help him figure out the cost of particular items for each purchase, and be prepared to explain your reasoning to Carlos.

1. One week Carlos bought 3 bags of *Tabitha Tidbits* and 4 bags of *Figaro Flakes* for $43.00. The next week he bought 3 bags of *Tabitha Tidbits* and 6 bags of *Figaro Flakes* for $54.00. Based on this information, figure out the price of one bag of each type of cat food. Explain your reasoning.

2. One week Carlos bought 2 bags of *Brutus Bites* and 3 bags of *Lucky Licks* for $42.50. The next week he bought 5 bags of *Brutus Bites* and 6 bags of *Lucky Licks* for $94.25. Based on this information, figure out the price of one bag of each type of dog food. Explain your reasoning.

3. Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. Based on this information, figure out the price of each item. Explain your reasoning.

4. One week Carlos bought 2 packages of dog bones and 4 packages of cat treats for $18.50. Because the finicky cats didn't like the cat treats, the next week Carlos returned 3 unopened packages of cat treats and bought 2 more packages of dog bones. After being refunded for the cat treats, Carlos only had to pay $1.00 for his purchase. Based on this information, figure out the price of each item. Explain your reasoning.

5. Carlos has noticed that because each of his purchases have been somewhat similar, it has been easy to figure out the cost of each item. However, his last set of receipts has him puzzled. One week he tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, figure out the price of each bag of the cheaper cat and dog food. Explain your reasoning.

Summarize the strategies you have used to reason about the price of individual items in the problems given above. What are some key ideas that seem helpful?

# 2.7 Shopping for Cats and Dogs – Teacher Notes

*A Develop Understanding Task*

**Purpose**: Using shopping receipts as a context, students will develop a strategy for solving a system of two linear equations using elimination of variables. Students will recognize that we can obtain an equivalent system of equations by replacing one or both equations in the system using one of the following steps:

- Replace an equation in the system with a constant multiple of that equation
- Replace an equation in the system with the sum or difference of the two equations
- Replace an equation with the sum of that equation and a multiple of the other

Each of these strategies can be justified in terms of the purchase price of a combination of two items.

The goal of these two procedures is to obtain a system of equations in which the coefficient of one of the variables is the same in both equations. Then the difference in the results depends solely on the difference in the amounts of one quantity, rather than two. In terms of the shopping context, if we buy the same amount of one item, but different amounts of the other, then the difference in the purchase price is due entirely to the item of which we purchased different amounts.

**Core Standards Focus**:

**A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Related Standards: N.Q.1, A.SSE.1a, A.CED.2, A.CED.3**

**Launch (Whole Class):**

Read through the context about using shopping receipts to find the purchase price of individual items when only the purchase price of a combination of the items is known. Encourage students to carefully record their arguments for how they know the price of a single item when examining two similar, but different, purchases of combinations of the same items.

**Explore (Small Group):**

Listen for students making sense of the scenarios through arguments such as the following:

- For problem #1, since Carlos bought the same number of bags of *Tabitha Tidbits*, the difference in the total cost must be due to the two extra bags of *Figaro Flakes.*
- For Problem #2, if we double the first purchase we get almost the second purchase, except we need to include one more bag of *Brutus Bites,* which would account for the difference in total cost.

- For Problem #3, if we double the second purchase we get almost the first purchase, except for 2 additional cat brushes, which would account for the difference in total cost.

- For problem #4, returning items is like making a negative purchase, so we bought 7 fewer packages of cat treats in the second purchase than we bought in the first purchase, so those 7 packages must account for the difference in the total cost.

- For problem #5, we need to think of possible related purchases so that the amount purchased of one of the items is the same for two different combinations of cat food and dog food. We could, for example, consider the total cost if Carlos had bought twice as much cat and dog food on Monday and three times as much of each on Thursday. Then he would have purchased 6 small bags of cat food on each day, and the difference in the total cost would be due to the extra bag of dog food purchased on Monday.

Make sure that students figure out the purchase price for each item in each scenario. This will require them to substitute the price of the first item they figure out back into the context to find the price of the second item.

**Discuss (Whole Class):**

Instead of discussing each individual problem, ask students to generate a list of key ideas that helped them to solve each scenario. The list of key ideas should include:

- If the number of one type of item is the same in both purchases, then the difference in total cost is due to the difference in the number of the second item purchased.

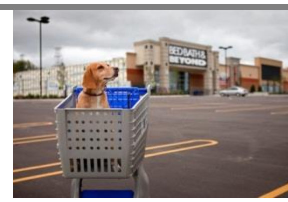
- If we know the total cost of a combination of two items, we know the total cost of buying twice as much, or three times as much, or any other multiple of the original purchase.

- Two different combinations of the same items can be added together (i.e., purchased together) and the total cost will be the sum of the costs of each separate purchase.

In the next task students will make these ideas more algebraic by writing the different purchases as systems of equations and using this intuitive reasoning to solve the system.

**Aligned Ready, Set, Go: Systems 2.7**

Name: Systems 2.7



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# Ready, Set, Go!

## Ready

Topic: Exponents

**Write the following in exponential notation.**

1. $4 \times 4 \times 4 \times 4 \times 4$
2. $3x \cdot 3x \cdot 3x \cdot 3x$

**Find each value.**

3. $2^3$
4. $3^3$
5. $2^5$
6. $(-2)^3$
7. $4^3$

## Set

Topic: Solving systems

8. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for \$2.84. Peter also buys three candy bars, but can only afford one additional fruit roll-up. His purchase costs \$1.79. What is the cost of a candy bar and a fruit roll-up individually?

9. A farmer noticed that his chickens were loose and were running around with the cows in the cow pen. He quickly counted 100 heads and 270 legs. How many chickens did he have and how many cows?

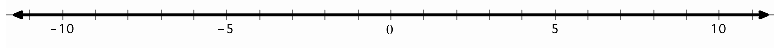
Name: Systems 2.7

## Go

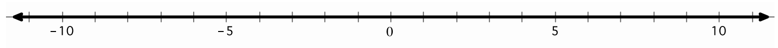
Topic: Solve one variable inequalities.

**Solve the following inequalities. Write the solution set in *interval notation* and graph the solution set on a number line.**

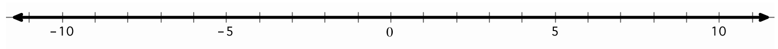
10. $4x + 10 < 2x + 14$



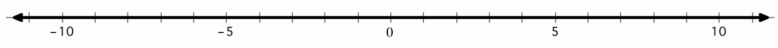

11. $2x + 6 > 55 - 5x$




12. $2(\frac{x}{4} + 3) > 6(x - 1)$




13. $9x + 4 \leq -2(x + \frac{1}{2})$




**Solve each inequality. Give the solution in *inequality notation* and *set notation*.**

14. $-\frac{x}{3} > -\frac{10}{9}$

15. $5x > 8x + 27$

16. $\frac{x}{4} > \frac{5}{4}$

17. $3x - 7 \geq 3(x - 7)$

18. $2x < 7x - 36$

19. $5 - x < 9 + x$

Need help? Check out these related videos?

Exponential notation:
http://www.khanacademy.org/math/algebra/exponents-radicals/v/understanding-exponents

Solving inequalities:
http://www.khanacademy.org/math/algebra/solving-linear-inequalities/v/solving-inequalities
http://www.khanacademy.org/math/algebra/solving-linear-inequalities/v/multi-step-inequalities-2

Set notation and interval notation:
http://patrickjmt.com/using-interval-notation-to-express-inequalities-ex1/

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# 2.8 Can You Get to the Point, Too?

*A Solidify Understanding Task*

**Part 1**

In "Shopping for Cats and Dogs," Carlos found a way to find the cost of individual items when given the purchase price of two different combinations of those items. He would like to make his strategy more efficient by writing it out using symbols and algebra. Help him formalize his strategy by doing the following:

- For each scenario in "Shopping for Cats and Dogs" write a **system of equations** to represent the two purchases.
- Show how your strategies for finding the cost of individual items could be represented by manipulating the equations in the system. Write out intermediate steps symbolically, so that someone else could follow your work.
- Once you find the price of one of the items in the combination, show how you would find the price of the other item.

**Part 2**

Writing out each system of equations reminded Carlos of his work with solving systems of equations graphically. Show how each scenario in "Shopping for Cats and Dogs" can be represented graphically, and how the cost of each item shows up in the graphs.

**Part 3**

Carlos also realized that the algebraic strategy he created in part 1 could be used to find the points of intersection for the "Pet Sitters" constraints. Use the **elimination of variables** method developed in part 1 to find the point of intersection for each of the following pairs of "Pet Sitter" constraints.

- *Start-up costs* and *space* constraints
- *Pampering time* and *feeding time* constraints
- Any other pair of "Pet Sitter" constraints of your choice

# 2.8 Can You Get to the Point, Too? – Teacher Notes

*A Solidify Understanding Task*

**Purpose**: This task solidifies the strategies for solving systems of equations that surfaced during the previous task. Students will begin by writing a system of equations to represent the shopping scenarios. Students will recognize that we can obtain an equivalent system of equations by replacing one or both equations in the system using one of the following steps:

- Replace an equation in the system with a constant multiple of that equation
- Replace an equation in the system with the sum or difference of the two equations
- Replace an equation with the sum of that equation and a multiple of the other

The goal of these steps is to obtain a system of equations in which the coefficient of one of the variables is the same in both equations. Then, when we subtract one of the equations from the other, we will obtain an equation that contains only one variable. This equation can be solved for its variable and the result can be substituted back into one of the original equations to obtain an equation that can be solved for the other variable.

**Core Standards Focus**:

**A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Related Standards: N.Q.1, A.SSE.1a, A.CED.2, A.CED.3**

**Launch (Whole Class):**

Provide a model of how students might work with systems of equations using the intuitive reasoning they developed in the previous task by working through scenario 1 from "Shopping for Cats and Dogs" together. Write out the system using equations in standard form:

$$\begin{cases} 3T + 4F = 43.00 \\ 3T + 6F = 54.00 \end{cases}$$

Since the coefficients of $T$ are the same in both equations, we will subtract equation #1 from equation #2 to get $2F = 11.00$. We can solve this equation for $F$ by dividing both sides of the equation by 2 to get $F = 5.50$, which must be the price of a bag *Figaro Flakes.* We can substitute this amount into either equation to solve for the price of *Tabitha Tidbits.* For example, substituting 5.50 into the first equation for $F$ yields $3T + 22.00 = 43.00$. Therefore, $3T = 21.00$, or $T = 7.00$.

**Explore (Small Group):**

Watch and listen for the ways students write and solve the systems of equations represented in each of the other scenarios. Encourage them to connect their intuitive reasoning with the shopping scenarios to the symbolic reasoning with variables. Part 2 of the task gives students an opportunity to connect this work to solving a system of linear equations graphically, and part 3 extends the elimination method to more complicated systems where getting the same coefficient for one of the variables in both equations is more difficult.

**Discuss (Whole Class):**

Invite students to articulate a general strategy for solving systems of equations by eliminating a variable. Have students demonstrate this strategy with one of the more challenging systems from the "Pet Sitters" context, such as the following system that involves the *space* constraint and the *pampering time* constraint.

$$\begin{cases} 24x + 6y = 360 \\ \frac{1}{3}x + \frac{4}{15}y = 8 \end{cases}$$

One possible strategy for solving this system would be to multiply the bottom equation by 15 to obtain whole number coefficients.

$$\begin{cases} 24x + 6y = 360 \\ 5x + 4y = 120 \end{cases}$$

Then multiply the top equation by 4 and the bottom equation by 6 to get the *y*-coefficient the same in both equations.

$$\begin{cases} 96x + 24y = 1440 \\ 30x + 24y = 720 \end{cases}$$

Subtracting the bottom equation from the top yields the single variable equation $66x = 720$. Solving this equation for *x* gives $x = \frac{720}{66} = 10\frac{10}{11}$. The complete solution is $\left(10\frac{10}{11}, 16\frac{4}{11}\right)$. Fortunately, this is not one of the important points of intersection in the "Pet Sitters" context, since it lies outside the feasible region.

**Aligned Ready, Set, Go: Systems 2.8**

Name: Systems | 2.8

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# Ready, Set, Go!

## Ready

Topic: Evaluate exponents

**Simplify and evaluate the following.**

1. $3^{-2}$ 2. $(0.5)^{-2}$ 3. $2^{4}$ 4. $4^{-2}$

**Write the following expression three different ways (one way can include the simplified value).**

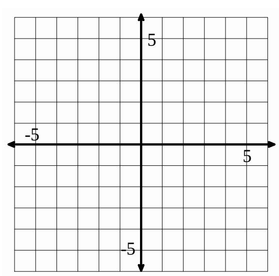
5. $(2^3)(4)$ 6. $(3^3)(2^3)$

## Set

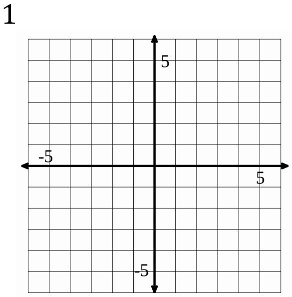
Topic: Solve systems of equations

**Solve the following systems of equations using *elimination* of variables, then justify graphically.**

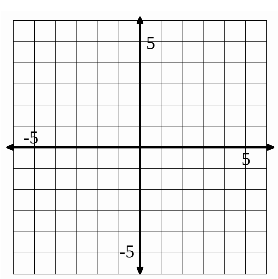
7. $\begin{cases} 2x + 0.5y = 3 \\ x + 2y = 8.5 \end{cases}$




8. $\begin{cases} 3x + 5y = -1 \\ x + 2y = -1 \end{cases}$




9. $\begin{cases} 3x + 5y = -3 \\ x + 2y = -\frac{4}{3} \end{cases}$




10. A 150-yard pipe is cut to provide drainage for two fields. If the length of one piece $(a)$ is three yards less than twice the length of the second piece $(b)$, what are the lengths of the two pieces?

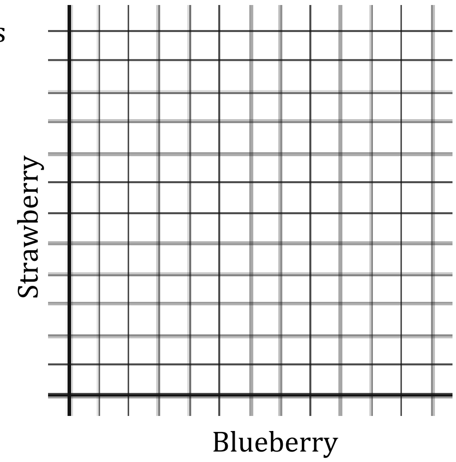
Name: Systems | 2.8

## Go

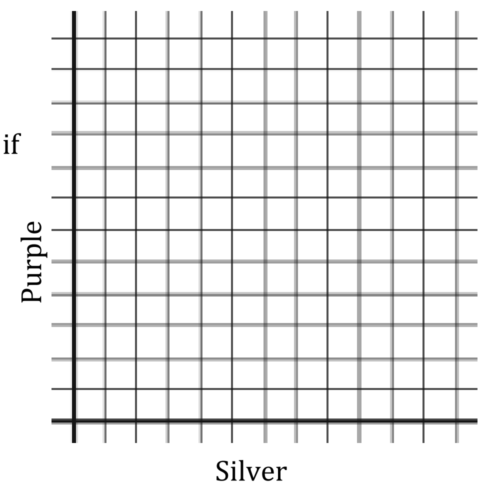
Topic: Graph two variable linear inequalities

**Graph the following linear inequalities on the graphs below. Include constraints.**

11. Ben has enough money to buy up to eight yogurts. If his favorite flavors are blueberry and strawberry, what are all the possible combinations he can buy? Graph the inequality that shows all possible combinations of his favorite flavors.




12. Peggy is buying a balloon bouquet. Her favorite colors are silver and purple. The silver balloons are $1 and the purple balloons are $0.80. Graph an inequality that shows how many of each color balloon she can put in her bouquet if she doesn't spend more than $20.

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Negative exponents
http://patrickjmt.com/negative-exponents/
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/zero--negative--and-fractional-exponents

Solving systems by elimination
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-systems-by-elimination-2

Solving systems by graphing
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/solving-linear-systems-by-graphing


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# 2.9 Food for Fido and Fluffy

*A Solidify Understanding Task*

Carlos and Clarita have found two different cat foods that seem to appeal to even the most finicky of cats: *Tabitha Tidbits* and *Figaro Flakes.* Each ounce of *Tabitha Tidbits* contains 2 grams of protein, 4 grams of carbohydrates and 4 grams of fat. Each ounce of *Figaro Flakes* contains 3 grams of protein, 4 grams of carbohydrates and 2 grams of fat. Since *Tabitha Tidbits* is fairly expensive, while *Figaro Flakes* is very cheap, the twins have decided to create a new cat food by mixing the two. After studying some nutritional guidelines for cats, Carlos and Clarita have decided to create a mixture based on the following constraints.

- *Amount of Protein*: Each meal should contain at least 12 grams of protein.
- *Amount of Carbohydrates:* Each meal should contain more than 16 grams of carbohydrates.
- *Amount of Fats:* Each meal should contain no more than 18 grams of fat.
- *Size of a Feeding:* Each meal should consist of less than 10 ounces of food.

For the work that follows, let *T* represent the number of ounces of *Tabitha Tidbits* in a meal and let *F* represent the number of ounces of *Figaro Flakes.*

1. Write an inequality for each of the constraints.

2. On separate coordinate grids, graph the solution set for each of the inequalities you wrote in #1. How do you know on which side of the boundary line you should shade the half-plane that represents the solution set?

3. Decide if the boundary line for each inequality represented in #2 should be a solid line or a dotted line. Which words or phrases in the constraints suggested a solid line? A dotted line?

4. Find at least 5 combinations of *Tabitha Tidbits* and *Figaro Flakes* Carlos and Clarita can mix together to create a nutritious cat meal. Show that these points lie within a feasible region for these constraints.

5. *Brutus Bites* is a brand of dog food that contains 4 grams of protein and 6 grams of fat per ounce. *Lucky Licks* is another brand of dog food that contains 12 grams of protein and 4 grams of fat per ounce. Carlos wants to make a meal for dogs that contains at least 8 grams of protein and no more than 6 grams of fat. Write and solve a system of inequalities that Carlos can use to determine a combination of *Brutus Bites* and *Lucky Licks* that will satisfy these constraints.

# 2.9 Food for Fido and Fluffy – Teacher Notes

*A Solidify Understanding Task*

**Purpose**: The intent of this task is to solidify a procedure for solving a system of linear inequalities using the following steps:

- Define variables for the modeling context (for example, *T* and *F* in the *Tabitha Tidbits* and *Figaro Flakes* cat food scenario)
- Write inequalities in terms of these variables for each of the constraints
- Graph each of the boundary lines represented by the constraints, using a solid line for constraints that include the points on the boundary, and a dotted line for strict inequalities
- Shade the appropriate half-plane represented by each constraint, either by using a "test point" to determine on which side of the boundary line to shade, or by using the logical implication of the inequality sign
- Find the region of intersection for all of the constraints, (including constraints implied by the context, if the system of inequalities is modeling a real-world scenario)
- This region of intersection—points common to all of the half-planes—is the solution set to the system of inequalities

**Core Standards Focus**:

**A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**Related Standards: N.Q.2, A.SSE.1, A.CED.2**

**Launch (Whole Class):**

Read the *Tabitha Tidbits* and *Figaro Flakes* scenario together. Model, with student input, how to write the constraint for the amount of protein in the mixture. Help students identify the language in the constraint that implies it is an inequality ("at least") and determine together what inequality notation they would use to represent this language ("≥"). Assign students to work on questions 1-5 with a partner.

**Explore (Small Group):**

Watch for how students are interpreting the inequality language in each constraint ("at least", ≥; "more than", >; "no more than", ≤; "less than", <). Listen for how students are determining the half-plane they should shade for each inequality: using a test point, or using the inequality relationship to determine in which half-plane the solution set lies. Note that this is the first task in this module that includes both "greater than" and "less than" inequalities. Also listen for how students are determining whether the points on the boundary line should be included (indicated by a solid line) or not (indicated by a dotted line). Good questions to ask would be of the form, "Can the mixture contain exactly 15 grams of fat?"

As students finish the *Tabitha Tidbits* and *Figaro Flakes* scenario, they should move on to the *Brutus Bites* and *Lucky Licks* scenario. This last context will be discussed as a whole group, so all students need not finish it before moving to the whole class discussion.

**Discuss (Whole Class):**
Start by having students describe, in their own words, a step-by-step procedure for solving a system of linear equations. Press for clear explanations and details, such as those given in the purpose statement above. Once a list of steps has been articulated, work through the *Brutus Bites* and *Lucky Licks* scenario together, using the student-generated set of steps. Have students clarify their list of steps if they find themselves doing something different than the steps listed. For example, did they define variables (*B* for ounces of *Brutus Bites* and *L* for ounces of *Lucky Licks*) before writing the constraints?

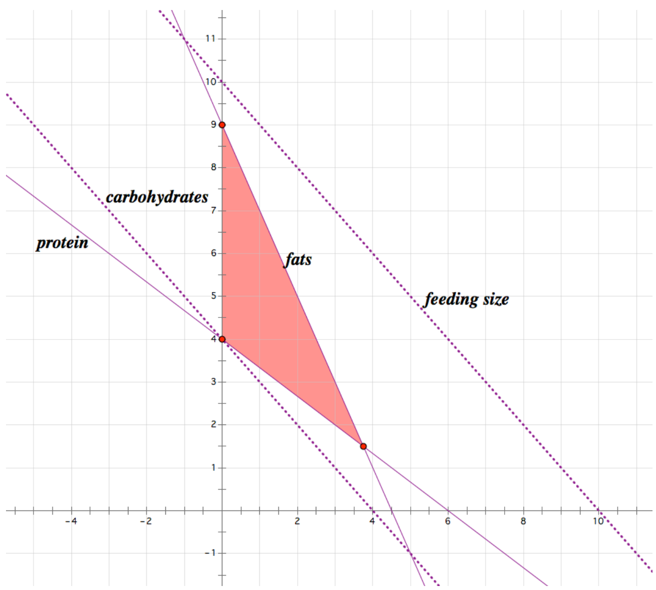
Here are the linear systems and feasible regions for the two different scenarios.

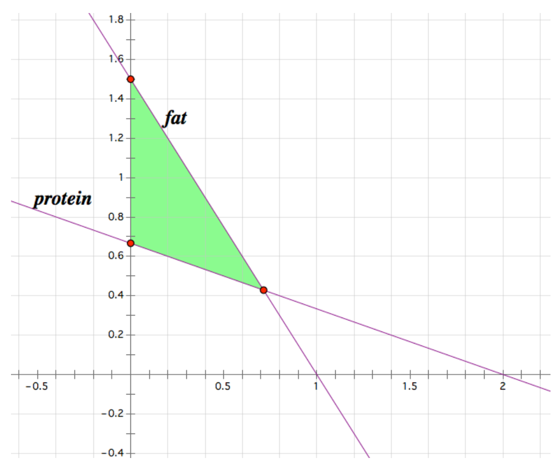
*Tabitha Tidbits* and *Figaro Flakes*

$$\begin{cases} 2T+3F \geq 12 \\ 4T+4F > 16 \\ 4T+2F \leq 18 \\ T+F < 10 \end{cases}$$

*Brutus Bites* and *Lucky Licks*

$$\begin{cases} 4B+12L \geq 8 \\ 6B+4L \leq 6 \end{cases}$$




**Aligned Ready, Set, Go: Systems 2.9**

Name: Systems | 2.9

# Ready, Set, Go!



## Ready

Topic: Solving two variable inequalities

1. A theater wants to take in at least $2000 for a certain matinee. Children's tickets cost $5 each and adult tickets cost $10 each.

a. Write an inequality describing the number of tickets that will allow the theater to meet their goal of $2000.

b. If the theater has a maximum of 350 seats, write an inequality describing the number of both types of tickets the theater can sell.

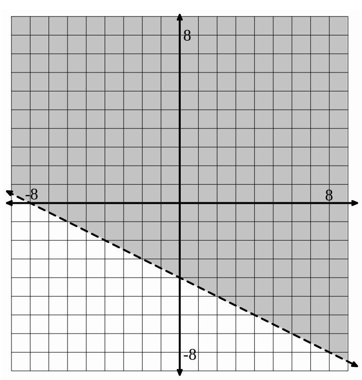
c. Find the number of children and adult tickets that can be sold so that all seats are sold and the $2000 goal is reached.

## Set

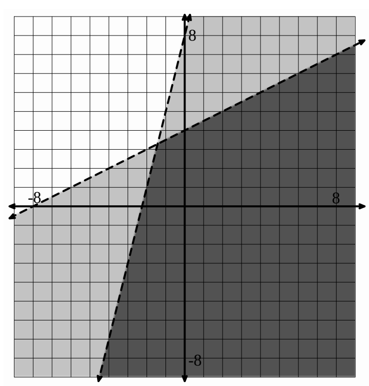
Topic: Writing equations of two variable inequalities

**Given the graph with the regions that are shaded write the inequality or system of inequalities.**

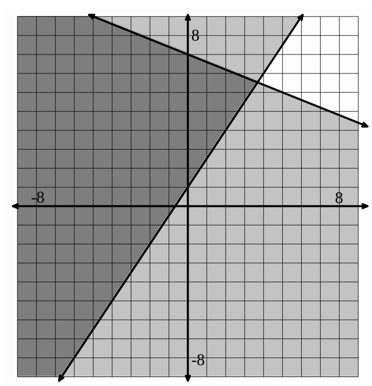
2.




3.




4.

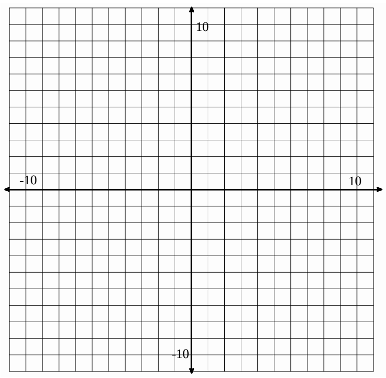


## Go

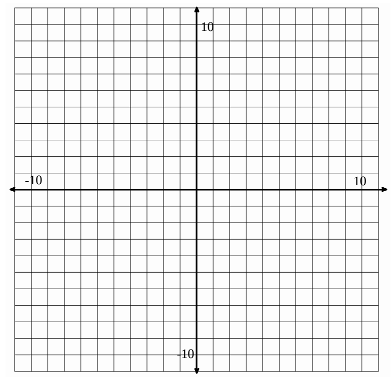
Topic: Graph two variable inequalities

**Graph each set of inequalities below. Include the shaded region of both, plus indicate the region that is true for all inequalities.**

5. $\begin{cases} x - y < -6 \\ 2y \geq 3x + 18 \end{cases}$




6. $\begin{cases} 5x - y \geq 5 \\ 2y - x \geq -10 \end{cases}$




**Solve the following systems of equations.**

7. Nadia and Peter visit the candy store. Nadia buys three candy bars and four fruit roll-ups for \$2.84. Peter also buys three candy bars, but can only afford one additional fruit roll-up. His purchase costs \$1.79. What is the cost of a candy bar and a fruit roll-up individually?

8. $\begin{cases} 5x - 10y = 15 \\ 3x - 2y = 3 \end{cases}$

9. $\begin{cases} 5x - y = 10 \\ 3x - 2y = -1 \end{cases}$

Need help? Check out these related videos.

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/graphing-systems-of-inequalities-2

# 2.10 Taken Out of Context

*A Practice Understanding Task*



Write a shopping scenario similar to those in "Shopping for Cats and Dogs" to fit each of the following systems of equations. Then use the elimination of variables method you invented in "Can You Get to the Point, Too" to solve the system. Some of the systems may have interesting or unusual solutions. See if you can explain them in terms of the shopping scenarios you wrote.

1. $\begin{cases} 3x + 4y = 23 \\ 5x + 3y = 31 \end{cases}$

2. $\begin{cases} 2x + 3y = 14 \\ 4x + 6y = 28 \end{cases}$

3. $\begin{cases} 3x + 2y = 20 \\ 9x + 6y = 35 \end{cases}$

4. $\begin{cases} 4x + 2y = 8 \\ 5x + 3y = 9 \end{cases}$

Three of Carlos' and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, three packages of pencils and two markers for $7.50. Jan buys two notebooks, six packages of pencils and five markers for $15.50. Fran buys a notebook, two packages of pencils and two markers for $6.25. How much do each of these three items cost?

Explain in words or with symbols how you can use your intuitive reasoning about these purchases to find the price of each item.

# 2.10 Taken Out of Context – Teacher Notes

*A Practice Understanding Task*

**Purpose**: This task gives students practice in solving systems of two linear equations in two variables. In addition, a couple of ideas that have not been examined in this module will arise. One of the systems has no solutions (an inconsistent system) since the lines are parallel. One of the systems has an infinite set of solutions—all the points on the line (a dependent system)—since both equations actually describe the same line. Students should note how to identify equivalent forms of the same line when written in standard form, as well as how to identify parallel lines when written in standard form.

In the final part of the task students extend the strategy for solving a system of linear equations to a situation that contains three equations and three unknowns. This work provides additional practice with the conceptual ideas of solving a linear system of equations.

**Core Standards Focus**:

**A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Related Standards:**

**Launch (Whole Class):**

Point out that while the systems provided in questions 1-4 do not come from a context, we can impose a context, such as the shopping scenario suggested in the instructions, to create a story that can help us reason through the mathematics of solving the system, particularly as unusual situations arise that may seem confusing when working only with the equations.

Point out also that question 5 will ask students to extend the intuitive reasoning of the "shopping scenario" to purchases involving combinations of three items.

**Explore (Small Group):**

Problems 1 and 4 are straightforward; they can be solved using the strategies developed and reviewed in this module. Some students may choose to use only a graphical or substitution method. Allow these methods to be used as a check on their work, but encourage all students to practice the elimination method.

Ask students to explain the dilemmas that occur when solving problems 2 and 3 algebraically using the shopping scenario. (Possible explanation for #2: Since we bought exactly twice as much, the prices of the individual items cannot be determined. Any combination of prices that will make the first equation true will also make the second equation true, so the solution to the system is the whole collection of ordered-pairs that satisfy either equation. Possible explanation for #3: We purchased three times as much in the second purchase than in the first, but the total cost was not three times as much. This suggests that the way the total cost was determined in the second purchase was inconsistent with the way the total price was determined in the first purchase—

perhaps a large discount in the price of one of both of the items. We cannot determine the price of either item, since the prices did not remain consistent throughout the scenario.)

While a graphical method can be used to check the reasonableness of the solutions to the systems in problems 1 and 4, it provides additional insight into what is happening in problems 2 or 3. Encourage "fast finishers" to graph #2 and #3 to present during the whole class discussion.

**Discuss (Whole Class):**

Unless students have encountered difficulty with solving questions 1 and 4, you can focus the discussion on the new issues that arise in questions 2, 3 and 5. For each of problems 2 and 3 have students explain what is happening using a graphical method and a verbal description of the associated shopping scenario. In #2 students should note that while the graphs of the lines are the same, the shopping scenarios represented by the equations are not, although the one purchase is just a constant multiple of the other. Attempting to use the elimination method would result in both variables and the total cost being eliminated, resulting in the identity 0 = 0. In #3 students should note that the lines are parallel by examining the slope-intercept form of the two lines, and recognize that this results because the coefficients of *x* and *y* in the two equations are proportional, but the total costs are not.

Have students share their intuitive reasoning on #5, and then connect this to the algebraic work of solving the system of three equations in three variables by elimination.

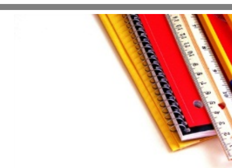
$$\begin{array}{c} \text{Stan} \\ \text{Jan} \\ \text{Fran} \end{array} \begin{cases} n+3p+2m=7.50 \\ 2n+6p+5m=15.50 \\ n+2p+2m=6.25 \end{cases}$$

Since Stan bought almost the same items as Fran, the difference in the total cost is due entirely to the additional package of pencils, which must cost $1.25. If you double Stan's purchase it will be almost the same as Jan's, except Jan would have purchased one more marker for $0.50. Now that we know the price of two of the items we can find the price of the third. Stan purchased 3 packages of pencils for $3.75 and 2 markers for $1.00, so a notebook must cost $2.75. These prices also check in the other two purchases.

Algebraically, if we subtract Fran's equation from Stan's we get $p$ = 1.25. If we double Stan's equation and subtract it from Jan's we get $m$ = 0.50. We can substitute these values into any one of the three equations to solve for *n.* There are other algebraic ways we could approach this system, such as eliminating $n$ in two of the equations to form a new $2 \times 2$ system. It is not necessary to formalize this strategy in this course.

**Aligned Ready, Set, Go: Systems 2.10**

Name: Systems | 2.10



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# Ready, Set, Go!

## Ready

Topic: Systems of Inequalities

**For each of the systems of inequalities, determine if the given coordinates are solutions to the system.**

1. $\begin{cases} y \le 3x - 5 \\ y \ge x + 2 \end{cases}$

   a. ( 6 , 10 )

   b. ( 1 , 4 )

   c. ( 8 , 15 )

2. $\begin{cases} y > -2x + 9 \\ y \ge 5x - 6 \end{cases}$

   a. ( -2 , -5 )

   b. ( -1 , 12 )

   c. ( 5 , 0 )

3. $\begin{cases} y < -\dfrac{1}{2}x + 9 \\ y > 6x - 10 \end{cases}$

   a. ( -2 , -5 )

   b. ( 7 , 3 )

   c. ( -8 , 10 )

## Set

Topic: Determine the number of solutions in a system of equations

**Express each equation in slope-intercept form. *Without graphing*, state whether the system of equations has zero, one or infinite solutions. How do you know?**

4. $\begin{cases} 3x - 4y = 13 \\ y = -3x - 7 \end{cases}$

5. $\begin{cases} 3x - 3y = 3 \\ x - y = 1 \end{cases}$

6. $\begin{cases} 0.5x - y = 30 \\ 0.5x - y = -30 \end{cases}$

7. $\begin{cases} 4x - 2y = -2 \\ 3x + 2y = -12 \end{cases}$

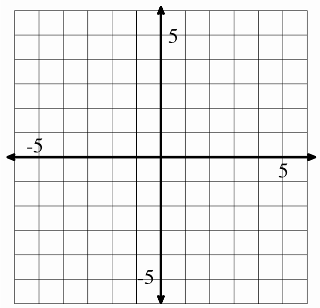
Name: Systems | **2.10**

## Go

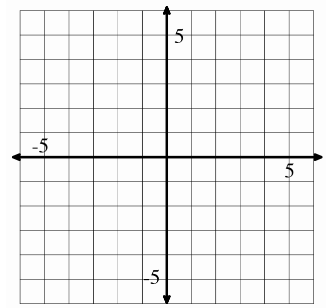
Topic: Graph two variable inequalities

**Graph the following inequalities. Be sure to label your axes and scale. Justify the region you shade by showing three points in the region as being solutions to the problem. Show a point you have tested to prove your shaded region is accurate.**

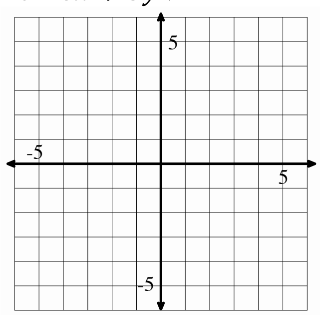
8. $3x - 4y \geq 12$



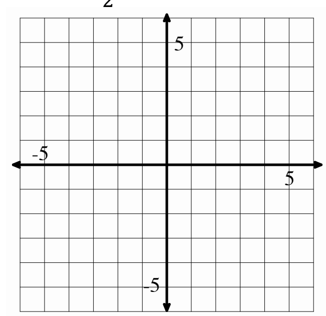

9. $x + 6y < 6$



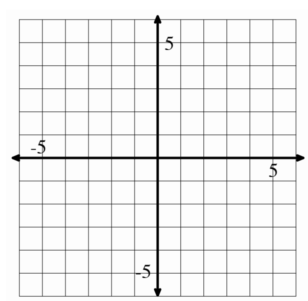

10. $6x + 5y > 1$




11. $x - \frac{1}{2}y \geq 3$




12. On the same set of axes, graph $y < x + 2$ and $y > x + 5$. What values do these two have in common?

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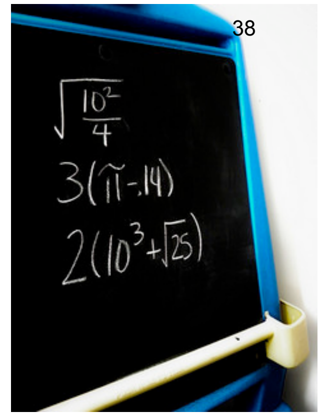
Testing a solution to an equation
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/testing-a-solution-for-a-system-of-equations

Number of solutions
http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/special-types-of-linear-systems

Solving inequalities
http://www.khanacademy.org/math/algebra/solving-linear-inequalities/v/solving-inequalities






# 2.11 More Things Taken Out of Context

*A Practice Understanding Task*

Solve the following systems of inequalities:

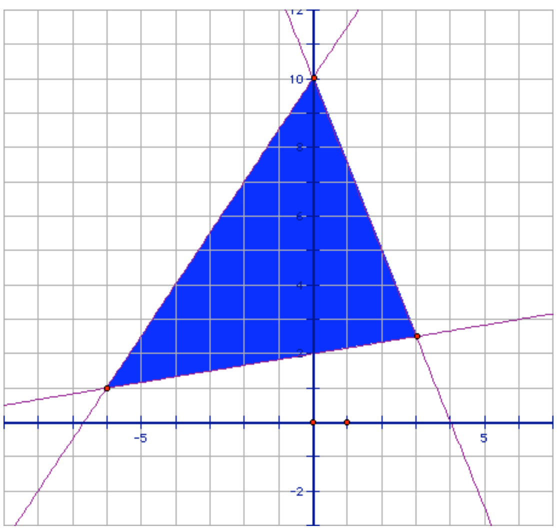
1. $\begin{cases} -5x + 3y \le 45 \\ 2x + 3y > 24 \end{cases}$

2. $\begin{cases} -10x + 6y \le 90 \\ 6x + 9y > 36 \end{cases}$

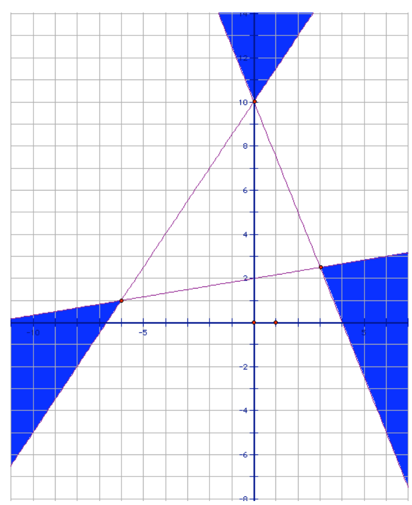
3. Is the point (-3, 10) a solution to the system in problem #1? Why or why not?

4. How are the inequalities representing the boundaries of the solution sets in problems #1 and #2 similar to each other? What accounts for these similarities?

5. Write the system of inequalities whose solution set is shown below:




6. Amanda is examining Frank's work on #5, when she exclaims, "You have written all of your inequalities backwards. The solution set to your system would look like this."



What do you think about Amanda's statement?

# 2.11 More Things Taken Out of Context — Teacher Notes

*A Practice Understanding Task*

**Purpose**: The purpose of this task is to practice solving systems of linear inequalities by identifying the overlapping (or intersecting) region of the half-planes that form the solution sets of each of the two-variable inequalities in the system. Students practice this in problems 1 and 2 by finding the overlapping region and in 3 by creating a system of inequalities whose solution is given as a shaded region in the coordinate plane. Students also must recognize the difference between a strict inequality and one that includes the points on the boundary line as part of the solution set. That is, in problem 4, they must distinguish the difference between < and ≤, and between > and ≥ as relationships. Students also get additional practice in recognizing parallel or equivalent lines for linear equations written in standard form.

**Core Standards Focus**:

**A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

**Related Standards:**

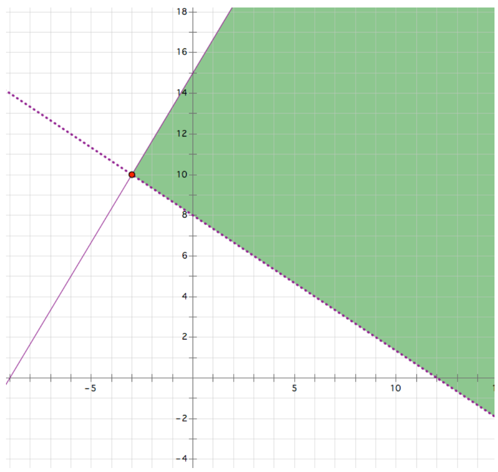
**Launch (Whole Class):**

Students should need very little introduction to this task. Point out the questions 1, 2 and 5 are practice problems, and that questions 3, 4 and 6 will extend their thinking.

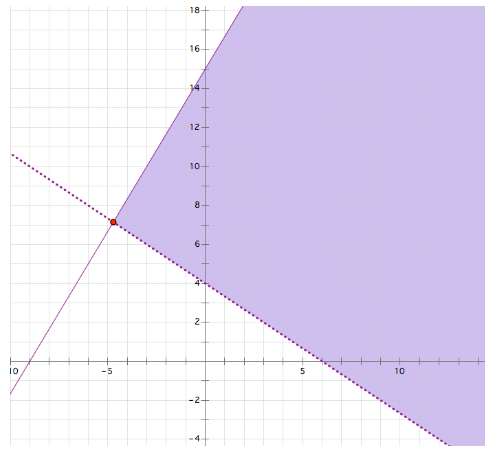
**Explore (Small Group):**

The main issue to watch for is whether students can find and shade the correct regions in the coordinate plane for 1 and 2, and if they can write a system of three linear inequalities in two variables for 5 so that the overlapping region of the three half-planes is the triangle given. This will require students to also be able to write the equations of the boundary lines shown. Watch for how students do this: using the slope and y-intercept of each line to write the equation in slope-intercept form, or using the two points of intersection that lie on each line. Also note if students are willing to leave their equations in slope-intercept form when they write their system of inequalities (which is fine) or if they feel compelled to transform their equations into standard form (which is not necessary, but may feel more "correct" to students since this is the form in which most systems have appeared throughout the module).

Solution set for problem 1:



Solution set for problem 2:



System for problem 5:

$$\begin{cases} y \le \frac{3}{2}x + 10 \\ y \le -\frac{5}{2}x + 10 \\ y \ge \frac{1}{6}x + 2 \end{cases} \qquad \text{or} \qquad \begin{cases} -3x + 2y \le 20 \\ 5x + 2y \le 20 \\ -x + 6y \ge 12 \end{cases}$$

**Discuss (Whole Class):**

If there are no issues with shading the appropriate solution set for questions 1 and 2, begin the discussion with question 5. Have a student who wrote the equations in slope-intercept form present their system of inequalities, followed by a student who wrote the equations in standard form. If both methods are not present in your class, bring up the question of which is "correct". It is important for students to recognize that the form in which we write the equation of the boundary line is not important.

Turn the discussion to the ideas represented in questions 3, 4 and 6.

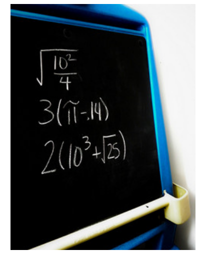
For question 3, students should be able to conclude that the point (-3, 10) is not a solution to the system since it doesn't satisfy the second (strict) inequality in the system in #1.

For question 4, students should recognize the first inequalities in each system contain the same boundary line since one inequality is just a constant multiple of the other. The other boundary lines in the two systems are parallel to each other since the coefficients of the *x*-terms and the *y*-terms are proportional, but the constant terms are not.

For question 6, students should recognize that if Frank did indeed turn all of his inequality statements around (we assume Amanda means he switched around his "greater than" and "less than" signs), the solution to Frank's system would be an empty set rather than the solution proposed by Amanda, since all three half-planes would not share any points in common.

**Aligned Ready, Set, Go: Systems 2.11**

Name: More Things Taken out of Context | 2.11





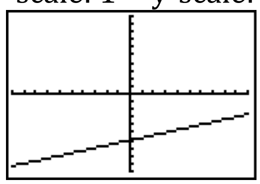
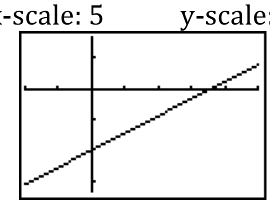
## Ready, Set, Go!

### Ready

Topic: Determine a good viewing window for graphs

*When sketching a graph of a function, it is important that we see important points. For linear functions, we want a window that shows important information related to the story. Often, this means including both the x- and y-intercepts.*

Example: $g(x) = \frac{1}{3}x - 6$

| Window: [ -10, 10] by [ -10,10]<br>x- scale: 1 y-scale: 1 | Window: [-10, 25] by [ -10, 5]<br>x-scale: 5 y-scale: 5 |
|---|---|
|  |  |
| NOT a good window | Good window |

**For the following equations, state a window that would be satisfactory for the given equation. Then sketch a graph in the boxes provided.**

1. $f(x) = 3x - 100$

x: [ , ] by y: [ , ]

x-scale: y-scale:

2. $5x + 7y = 15$

x: [ , ] by y: [ , ]

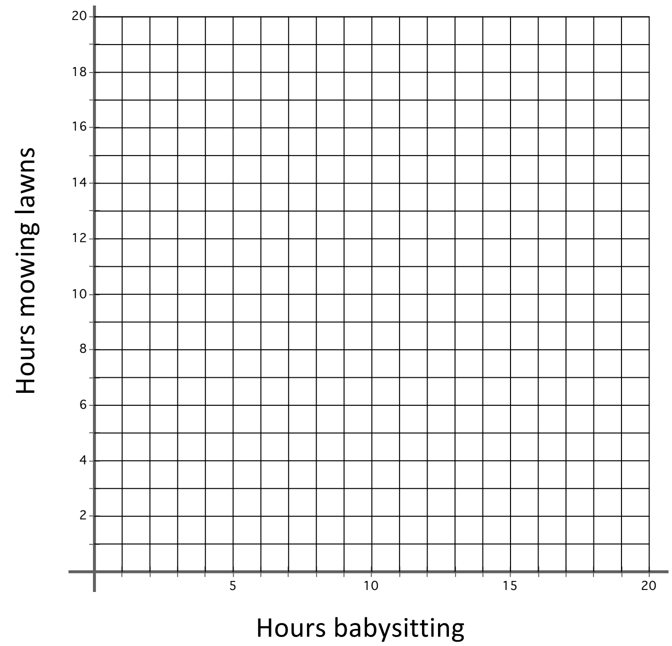
x-scale: y-scale:

# Name: More Things Taken out of Context | 2.11

## Set

Topic: Creating and solving two variable inequalities

3. Patty makes $8 per hour mowing lawns and $12 per hour babysitting. She wants to make at least $100 per week but can work no more than 12 hours a week. Write and graph a system of linear inequalities. Finally, list 2 possible combinations of hours that Patty could work at each job.




## Go

Topic: Solve systems of equations

**Solve each system of equations using any method you prefer**

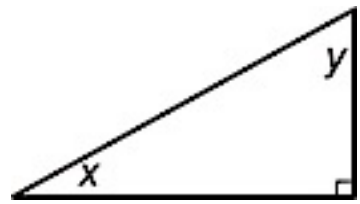
4. $\begin{cases} 3x + 5y = -3 \\ x + 2y = -\frac{4}{3} \end{cases}$

5. $\begin{cases} x - y = -\frac{12}{5} \\ 2x + 5y = -2 \end{cases}$

# Name: More Things Taken out of Context 2.11

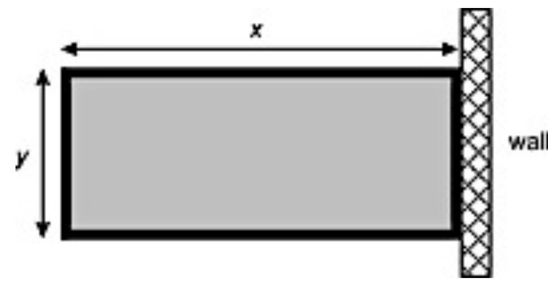
**Create a system of equations and use it to solve the following questions.**

6. Of the two non-right angles in a right triangle, one measures twice as many degrees as the other. What are the angles?

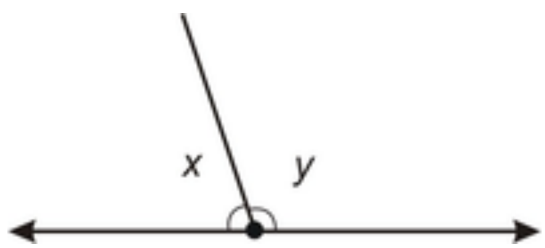



7. The sum of two numbers is 70 and the difference is 11. What are the numbers?

8. A rectangular field is enclosed by a fence on three sides and a wall on the fourth side. The total length of the fence is 320 yards. If the field has a total perimeter of 400 yards, what are the dimensions of the field?




9. A ray cuts a line forming two angles. The difference between the two angles is $18°$. What does each angle measure?




Need Help? Check out these related videos:

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/system-of-inequalities-application



# 2.12 Pet Sitters Revisited

*A Develop Understanding Task*

Carlos and Clarita have successfully found a way to represent *all* of the combinations of cats and dogs that they can board based on *all* of the following constraints.

- *Space*: Cat pens will require 6 ft$^2$ of space, while dog runs require 24 ft$^2$. Carlos and Clarita have up to 360 ft$^2$ available in the storage shed for pens and runs, while still leaving enough room to move around the cages.

- *Feeding Time*: Carlos and Clarita estimate that cats will require 6 minutes twice a day—morning and evening—to feed and clean their litter boxes, for a total of 12 minutes per day for each cat. Dogs will require 10 minutes twice a day to feed and walk, for a total of 20 minutes per day for each dog. Carlos can spend up to 8 hours each day for the morning and evening feedings, but needs the middle of the day off for baseball practice and games.

- *Pampering Time*: The twins plan to spend 16 minutes each day brushing and petting each cat, and 20 minutes each day bathing or playing with each dog. Clarita needs time off in the morning for swim team and evening for her art class, but she can spend up to 8 hours during the middle of the day to pamper and play with the pets.

- *Start-up Costs*: Carlos and Clarita plan to invest much of the $1280 they earned from their last business venture to purchase cat pens and dog runs. It will cost $32 for each cat pen and $80 for each dog run.

Now they are trying to determine how many of each type of pet they should plan to accommodate. Of course, Carlos and Clarita want to make as much money as possible from their business, so they need to pay attention to both their daily income as well as their daily costs. They plan to charge $8 per day for boarding each cat and $20 per day for each dog. They estimate that each cat will require $2.00 per day in food and supplies, and that each dog will require $4.00 per day in costs.

After surveying the community regarding the pet boarding needs, Carlos and Clarita are confident that they can keep all of their boarding spaces filled for the summer.

So the question is, how many of each type of pet should they prepare for in order to make as much money as possible?

What combination of cats and dogs do you think will make the most money? What recommendations would you give to Carlos and Clarita, and what argument would you use to convince them that your recommendation is reasonable?

To get started on this task, you might want to look for collections of points where the daily profit is the same. For example, can you find a collection of points where for each point the daily profit is $120? What about $180?

# 2.12 Pet Sitters Revisited – Teacher Notes

*A Develop Understanding Task*

**Purpose**: While this module has used the "Pet Sitter" context to elicit important mathematics relative to solving systems of linear equations and linear inequalities, solving linear programming problems in which a selected quantity, such as profit, is maximized or minimized across a set of constraints is not an essential mathematical goal for this course. The "Pet Sitters" context is being revisited here to give students an opportunity to use all that they have learned throughout this module from the perspective of the modeling standard for high school mathematics. To successfully answer the question posed by Carlos and Clarita, "What combination of cats and dogs will make the most money?" students have and will continue to negotiate the modeling cycle which consists of the following features:

- Identifying variables in the situation and selecting those that represent essential features
- Formulating a model by creating and selecting geometric, graphical, tabular, algebraic or statistical representations that describe relationships between the variables
- Analyzing and performing operations on these relationships to draw conclusions
- Interpreting the results of the mathematics in terms of the original situation
- Validating the conclusions by comparing them with the situation, and then either improving the model, or
- Reporting on the conclusions and the reasoning behind them

This version of "Pet Sitters" introduces the quantity to be maximized—*profit*. Profit is determined by revenue minus costs, and in this case is a function of the number of cats and dogs being boarded. While all points in the feasible region represent combinations of cats and dogs that satisfy all of the constraints, the profit varies from point to point. Intuitively, points farther from the origin would represent higher profits, suggesting that points along the boundary lines would be among the most profitable combinations. When students turn their attention towards finding a set of points for which the profit is the same for all of the points in the set they may notice that all of these points lie on a line. This is because the "profit equation" for a particular profit $P$ is of the form $16x + 6y = P$, the standard form of a linear equation. While each value of $P$ produces a different linear equation, all of the "profit equation lines" are parallel to each other, since the coefficients of $x$ and $y$ remain the same. As the profit increases, the series of parallel lines move farther away from the origin until they exit the feasible region at one of the vertex points of the polygon that represents the feasible region. Therefore, the maximum profit will occur at one of the vertex points of the feasible region, provided the vertex point represents a possible combination of cats and dogs (i.e., a whole number of each, rather than a fraction of either). It is not necessary for students to fully develop this theory—whatever arguments they use to justify their recommendation for the number of cats and dogs Carlos and Clarita can board can and should be critiqued by their peers for appropriateness.

**Core Standards Focus**:

**Mathematical Practice Standard 4: Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, graphs, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served the purpose.

**Related Standards: N.Q.1, N.Q.2, A.SEE.1a, A.CED.2, A.CED.3, A.REI.10, A.REI.12, F.IF.4, F.IF.5, F.IE.1b, F.IE.5, A.REI.5, A.REI.6**

**Launch (Whole Class):**

Review the "Pet Sitters" scenario down to the question "What combination of cats and dogs do you think will make the most money?" Highlight the new issues in this version of "Pet Sitters"—although they charge a daily boarding fee, the profit the twins earn each day is determined not only by the daily revenue, but also by the additional costs incurred each day for food, litter, and other supplies. Give students a few minutes to work independently to determine possible strategies for maximizing the profit. After a few minutes, have students compare their strategies with others and then work with a partner to consider how to improve their strategies. At this point, introduce the "To get started . . ." idea from the last paragraph of the handout, and point out the language in the second to last paragraph about making a convincing argument.

**Explore (Small Group):**

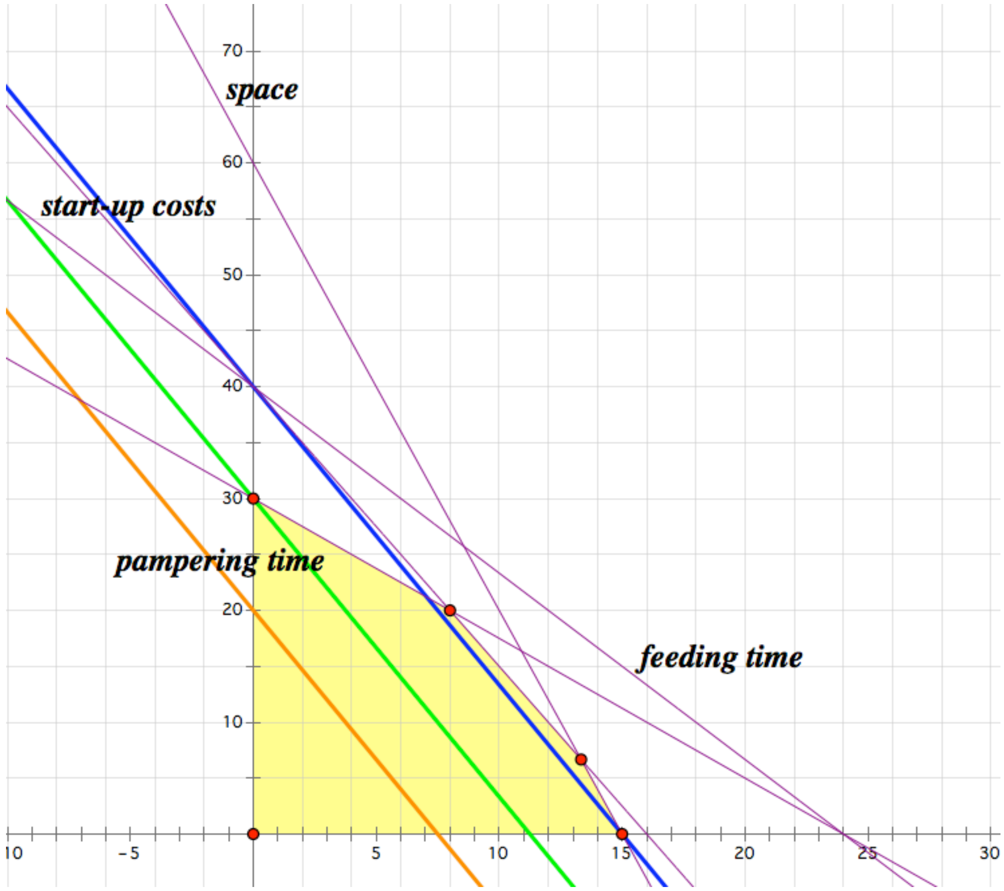
It is anticipated that many students will begin with a guess and check strategy: pick a particular combination of dogs and cats as represented by the points in the feasible region and then calculate the profit. Students will probably come to realize that points on the boundary lines produce higher profits than points in the interior. As students find sets of points with the same profit (as suggested by the "hint" at the end of the task) they will notice that these points form a line and that sets of points for different profits form different lines that are all parallel to each other. They may speculate as to ways to use this set of parallel lines to maximize the profit.

Watch for students who calculate daily revenue instead of daily profit, particularly when writing "profit equations" for a profit of $120 or $240. Suggest that they plot several "profit lines" and consider what is happening to the lines as the profit increases.
Look for students who can articulate why points on the boundary lines yield higher profits. Also look for students who have successfully plotted some profit lines, and perhaps have an argument based on the parallel profit lines as to why the maximum profit will occur at a vertex point.

**Discuss (Whole Class):**

The intent of this lesson is to surface intuitive ways to think about the maximum profit, and not necessarily to formalize the idea that the maximum profit occurs at one of the vertex points. Share strategies that have surfaced which point students towards the maximum profit occurring along one of the boundary lines or at a vertex. Also discuss how the "profit equations" are lines that lie within the feasible region and that every point on the "profit line" represents a combination of cats and dogs for which the profit is the same as any other combination represented by other points on the same line.

***The "Pet Sitters" Feasible Region with a $120 profit line (orange), a $180 profit line (green), and a $240 profit line (blue).***




(Note that the profit lines are parallel, and as the profit increases the profit line moves farther away from the origin. At what point would a sequence of parallel profit lines "exit" the feasible region?)

**Aligned Ready, Set, Go: Systems 2.12**

# Ready, Set, Go!

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## Ready

Topic: Solve exponential equations

**Find the value of *x* for each situation.**

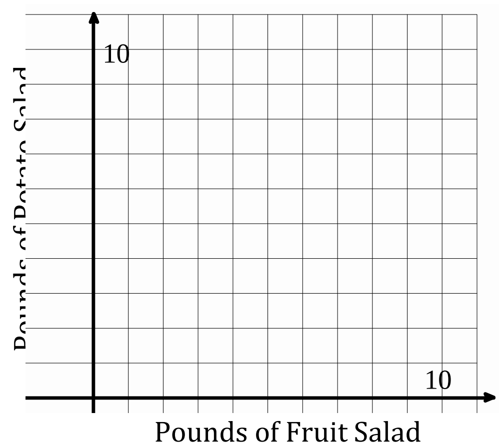
1. $2^x = 8$ 2. $3^x = 27$ 3. $2^x = 4$ 4. $(-2)^x = -8$

## Set

Topic: Create and solve two variable inequalities

5. Jane is buying fruit salad and potato salad for a picnic. Fruit salad costs $2.00 per pound and potato salad costs $4.00 per pound. Jane needs to buy at least 6 pounds of salads and she doesn't want to spend more than $20. Write and graph a system of linear inequalities. Also, list 2 possible combinations of salad Jane could buy.

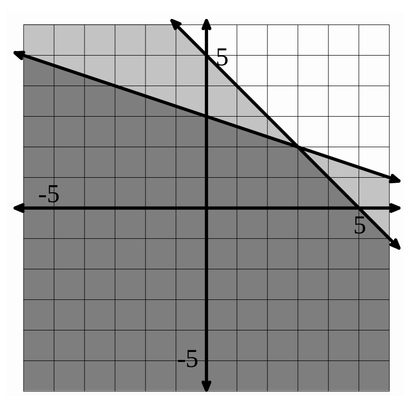
Let x = pounds of fruit salad and
y = pounds of potato salad.




## Go

Topic: Find the solution region of the following systems of inequalities.

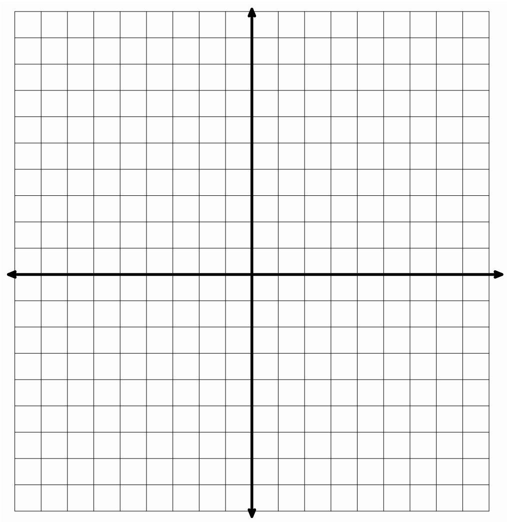
6. Write the system of inequalities that is represented in the graph to the right.



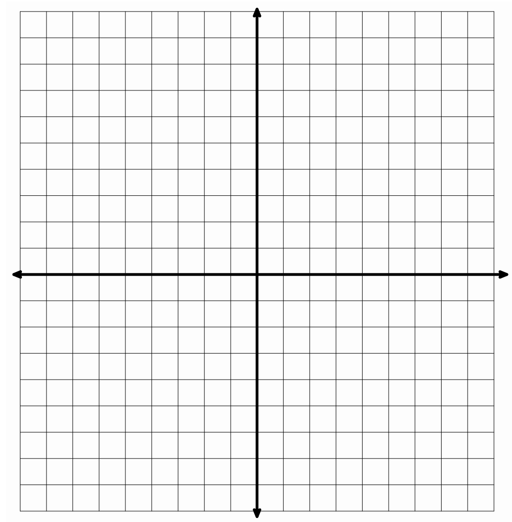
Name: Systems | 2.12

**Graph each set of inequalities and determine the solution region.**

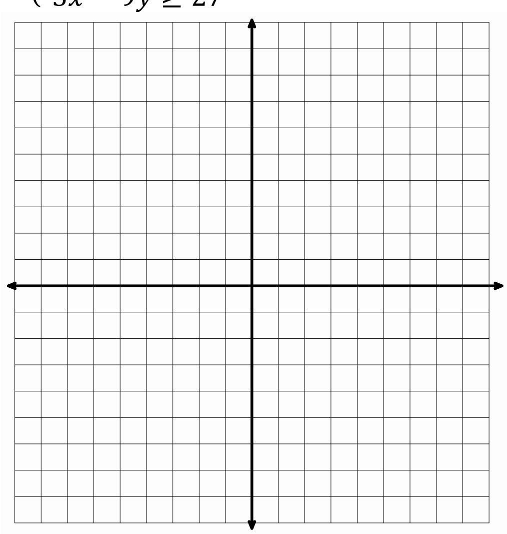
7. $\begin{cases} x - y < -6 \\ -2y \geq 3x - 18 \end{cases}$



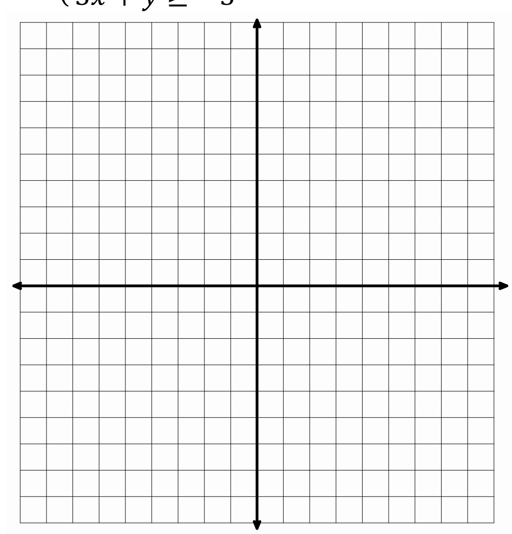
8. $\begin{cases} 5x - y \geq 5 \\ 2y - x \geq 10 \end{cases}$



9. $\begin{cases} 5x + 2y \geq -10 \\ 3x - 2y \leq 18 \\ 3x - 9y \geq 27 \end{cases}$



10. $\begin{cases} 2x - 3y \leq 24 \\ x + 4y \leq 8 \\ 3x + y \geq -3 \end{cases}$



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Exponents http://patrickjmt.com/exponents-intro-to-evaluating-a-few-truefalse-questions/

Rules for exponents http://patrickjmt.com/basic-exponent-properties/

Solving a system of inequalities http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/systems-of-linear-inequalities

# 2.13H To Market with Matrices

## *A Solidify Understanding Task*

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Carlos learned about matrices when Elvira, the manager of the school cafeteria, was asked to substitute teach during one of the last days of school before summer vacation. Now that he has worked out a strategy for solving systems of equations by elimination of variables, he is wondering if matrices can help him keep track of his work.

Carlos is reconsidering the following scenario from "Shopping for Cats and Dogs", while trying to record his thinking using matrices.

> *One week Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. What is the price of each item?*

Carlos realizes that he can represent this scenario using the following matrix:

$$\begin{array}{r} \\ \textit{purchase } 1 \\ \textit{purchase } 2 \end{array} \begin{array}{c} \begin{array}{ccc} \textit{leashes} & \textit{brushes} & \textit{total} \end{array} \\ \begin{bmatrix} 6 & 6 & \$45.00 \\ 3 & 2 & \$19.00 \end{bmatrix} \end{array}$$

He also realizes that he can represent the cost of each item with a matrix that looks like this:

$$\begin{array}{r} \\ \textit{purchase } 1 \\ \textit{purchase } 2 \end{array} \begin{array}{c} \begin{array}{ccc} \textit{leashes} & \textit{brushes} & \textit{total} \end{array} \\ \begin{bmatrix} 1 & 0 & \$4.00 \\ 0 & 1 & \$3.50 \end{bmatrix} \end{array}$$

So, now he is trying to find a sequence of matrices that can fill in the gaps between the first matrix and the last. He knows from his previous work with solving systems of equations that he can do any of the following manipulations with equations—and he realizes that each of these manipulations would give him a new row of numbers in a corresponding matrix.

- Replace an equation in the system with a constant multiple of that equation
- Replace an equation in the system with the sum or difference of the two equations
- Replace an equation with the sum of that equation and a multiple of the other

1. Help Carlos find a sequence of matrices that starts with the matrix that represents the original purchases, and ends with the matrix that represents purchasing one leash or purchasing one brush. For each matrix in your sequence, write out the justification that allows you to write that matrix based on the three manipulations we can perform on the equations in a system. For example, the following matrix transformation can be justified by writing "I replaced the first row of the matrix by multiplying the first row by $\frac{1}{6}$."

$$\begin{bmatrix} 6 & 6 & 45.00 \\ 3 & 2 & 19.00 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 7.50 \\ 3 & 2 & 19.00 \end{bmatrix}$$

2. Find and justify a sequence of matrices that could be used to solve the following scenario.

*One week Carlos tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, can you figure out the price of each bag of the cheaper cat and dog food?*

# 2.13H To Market with Matrices – Teacher Notes

*A Solidify Understanding Task*

**Purpose**: The purpose of this task is to introduce matrices as a way of keeping track of the steps for solving a system of equations by elimination. In general, matrices can be used to "organize and manipulate" data (see CCSSM N.VM.6). When used in solving systems of equations, the "data" represented by the matrices consists of the coefficients of the variables in each equation, augmented with the sums of the linear combinations represented by each equation. However, the manipulations performed on the matrices are not the arithmetic operations of addition, subtraction, and multiplication introduced in the *Getting Ready Module*. Rather, individual rows of the matrix are replaced with numbers that represent an equivalent equation. It is important for students to recognize the difference between *matrix arithmetic*—which combines two matrices to produce a sum, difference or product matrix—and *row reduction of matrices*—which produces a matrix that represents an equivalent system of equations.

**Core Standards Focus**:

**UT Honors:** Solve systems of linear equations using matrices.

**Related Standards: A.REI.6**

**Launch (Whole Class):**
Before distributing the task handout to students, remind them of their work on the tasks "Shopping for Cats and Dogs" and "Can You Get to the Point, Too?" where they developed a strategy for solving a system of linear equations by elimination of variables. Their strategy involved writing equivalent systems of equations by modifying one or more of the equations in the system.

Review their work on these previous two tasks by asking students to regenerate the list of things they can do to the equations in a system to obtain an equivalent system. For example, "You can replace an equation in a system with a constant multiple of that equation." What else can you do?

At this point distribute the task handout and remind students of their work with matrices in the task "Cafeteria Consumption and Costs" from the *Getting Ready Module.* In that task they used matrices to organize information such that the location of a number in the rectangular array described two different attributes of that number. Point out that in the shopping scenarios revisited in this task, a row will represent a particular purchase, and a column will represent a particular item purchased, so a number's location in the matrix will tell the amount of a particular item acquired during a particular purchase. Also point out that the matrices used to solve systems of equations by row reduction are *augmented* with a column that represents the sum of the linear combination of terms in the equations. In this case, the final column in the matrix will represent the final purchase price. Read and discuss the task until students are ready to work on question 1.

[Note: Have students work on question 1, then discuss it, before having them work on question 2.]

**Explore (Small Group): Question 1**

While there is an efficient way to approach row reduction of matrices (see the notes in the Discuss section), it is not necessary to press for an efficient strategy at this point in time. Rather, it is important for students to recognize that they are doing the same arithmetic work they have done previously when solving systems of equations, but now they are recording the results of that work in the form of a matrix. The example in problem 1 is intended to be a hint that it is possible, and perhaps useful, to get a 1 in the first column of the first row, but students can approach the problem in other ways. For example, if students are connecting the matrix representation to the context, they may approach the problem in a way that is similar to the way they intuitively solved the shopping scenario—double purchase 2, subtract purchase 2 from purchase 1 to eliminate "leashes" (e.g., get a 0 in column 1 of row 2 of the matrix), and then divide the second purchase (row 2 of the matrix) by 2 to get the cost of 1 brush. Students who take this approach might record their thinking using matrices as follows:

$$\begin{bmatrix} 6 & 6 & 45.00 \\ 3 & 2 & 19.00 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 6 & 45.00 \\ 6 & 4 & 38.00 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 6 & 45.00 \\ 0 & 2 & 7.00 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 6 & 45.00 \\ 0 & 1 & 3.50 \end{bmatrix}$$

In their previous work with solving systems, once students found the value of one of the variables in the system, they would substitute that value into the other equation to find the value of the other variable. They may wonder how to represent this work with matrices. In the above example, we now know the value of 1 brush is $3.50. To find the value of 1 leash, we could multiply the bottom row of the matrix by -6 and add the result to the top row to eliminate brushes from the first purchase. We can then divided the top row by 6—the number of brushes purchased for $24.00—to get the cost of a single brush.

$$\begin{bmatrix} 6 & 6 & 45.00 \\ 0 & 1 & 3.50 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 0 & 24.00 \\ 0 & 1 & 3.50 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 4.00 \\ 0 & 1 & 3.50 \end{bmatrix}$$

Because each matrix needs to be a $2 \times 3$ matrix, students will need to record two purchases (e.g., two rows) in each matrix, and they should be able to justify the amounts purchased and cost of each purchase represented by each row. Keep this connection at the forefront of students' work by asking them to describe what each row of their matrices represents in terms of purchases.

Students might approach this work in a variety of ways, and they may not record their work appropriately using matrices. Work towards consensus on what their work should look like—including a written description of what they did to transform one matrix into another.

Watch for groups that use the step hinted at in question 1 as their first step in their sequence of matrices. Note how they use the 1s in the first row—representing 1 leash and 1 brush. Do they use these 1s to eliminate either the number of leashes or number of brushes in the second purchase (i.e., get 0s in either the first or second column of the second row of the matrix)? When students have "played around" with matrix notation for a sufficient amount of time to have access to a whole class discussion—and perhaps have encountered a number of issues that can be resolved during a whole class discussion—feel free to begin discussing question 1 as a whole class.

**Discuss (Whole Class): Question 1**
Share successful matrix transformations and appropriate matrix notation that students may have generated, even if they do not lead to a complete strategy. Make sure students describe what they are doing in terms of the shopping context. If possible, have a student share how they can use a 1 in a particular column to get 0 in that same column in the other row.

Help students transition from the language of solving systems by elimination to the language of solving systems with matrices. That is, the following rules for solving systems of equations:

- Replace an equation in the system with a constant multiple of that equation
- Replace an equation in the system with the sum or difference of the two equations
- Replace an equation with the sum of that equation and a multiple of the other

can be revised to:

- Replace a row in a matrix with a constant multiple of that row
- Replace a row in a matrix with the sum or difference of that row and another row of the matrix
- Replace a row in a matrix with the sum of that row and a constant multiple of another row of the matrix

(Note that the middle statement is a special case of the bottom statement, and therefore is redundant, but still may be helpful to state explicitly.)

Another elementary row operation that may not have come up explicitly in this task is the idea that you can switch any two rows in a matrix. This is justified by the fact that the order in which you write the equations in a linear system does not matter. You can introduce this idea now, or allow it to arise more naturally in the next task.

Help students recognize that the goal of working with the matrices is to get 1s in the diagonal of the portion of the matrix representing the coefficients of the variable terms, and 0s in the remaining positions of that portion of the matrix, so that the last column of the matrix will give the value of a different variable for each row of the matrix.

**Explore (Small Group): Question 2**
Allow students additional time to work on the shopping scenario is question 2. Watch for more efficient strategies to emerge, based on the previous discussion.

**Discuss (Whole Class): Question 2**
Share and discuss strategies that illustrate the following steps for an efficient and consistent way to row reduce a matrix. Have students summarize the discussion by articulating ideas similar to the following. (Note that these ideas will be formalized in the next task.)

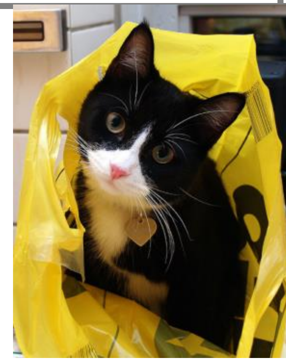
To row reduce a matrix:

- Perform elementary row operations to yield a "1" in the first row, first column.
- Create zeros in all of the other rows of the first column by adding the first row times a constant to each other row.

- Perform elementary row operations to yield a "1" in the second row, second column.
- Create zeros in all of the other rows of the second column by adding the second row times a constant to each other row.
- Perform elementary row operations to yield a "1" in the third row, third column.
- Create zeros in all of the other rows of the third column by adding the third row times a constant to each other row.
- Continue this process until the first $m \times m$ entries form a square matrix with 1s in the diagonal and 0s everywhere else.

**Aligned Ready, Set, Go: Systems 2.13H**

Name: To Market with Matrices | 2.13H



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## Ready, Set, Go!

### Ready

Topic: Solving Systems by Substitution and Elimination

**Solve each system of equations using any algebraic method.**

1. $\begin{cases} 3x - y = 1 \\ 3x + 2y = 16 \end{cases}$

2. $\begin{cases} x + 2y = 5 \\ 3x + 5y = 14 \end{cases}$

3. $\begin{cases} 4x + 2y = -8 \\ x - 2y = -7 \end{cases}$

4. $\begin{cases} 2x + 3y = 2 \\ 3x - 4y = -14 \end{cases}$

5. $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

6. $\begin{cases} 2x + y = 0 \\ 5x + 3y = 1 \end{cases}$

Name: To Market with Matrices | 2.13H

## Set

Topic: Row reductions in Matrices

7. Create a matrix to match each step in the solving of the system of equations given. Also, write a description of what happened to the equation and the matrix between steps.

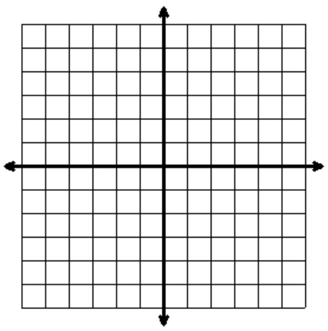
| | System of Equations | Description | Matrix |
|---|---|---|---|
| *Given System* | $\begin{cases} 3x + 2y = 40 \\ x - 7y = -2 \end{cases}$ | | $\left[\begin{array}{cc\|c} 3 & 2 & 40 \\ 1 & -7 & -2 \end{array}\right]$ |
| | ↓ | $-3R_2 \rightarrow R_2$ | ↓ |
| *Step 1* | $\begin{cases} 3x + 2y = 40 \\ -3x + 21y = 6 \end{cases}$ | ↓ | $\left[\begin{array}{cc\|c} & 2 & 40 \\ -3 & & 6 \end{array}\right]$ |
| | ↓ | ____________ | ↓ |
| *Step 2* | $\begin{cases} 3x + 2y = 40 \\ 0x + 23y = 46 \end{cases}$ | ↓ | $\left[\begin{array}{cc\|c} & & 40 \\ 0 & & \end{array}\right]$ |
| | ↓ | ____________ | ↓ |
| *Step 3* | $\begin{cases} 3x + 2y = 40 \\ 0x + y = 2 \end{cases}$ | ↓ | $\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$ |
| | ↓ | ____________ | ↓ |
| *Step 4* | $\begin{cases} 3x + 0y = 36 \\ 0x + y = 2 \end{cases}$ | ↓ | $\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$ |
| | ↓ | ____________ | ↓ |
| *Step 5* | $\begin{cases} x + 0y = 12 \\ 0x + y = 2 \end{cases}$ | | $\left[\begin{array}{cc\|c} & & \\ & & \end{array}\right]$ |

## Go

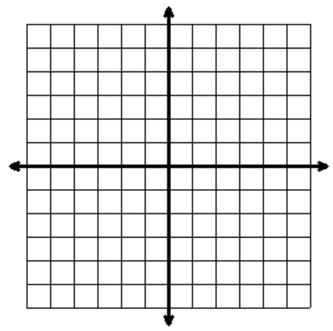
Topic: Solving Systems of Equations by Graphing

**Solve each system of equations by graphing.**

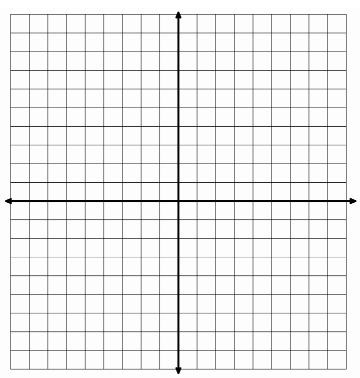
8. $\begin{cases} y = 3x - 3 \\ y = -3x + 3 \end{cases}$



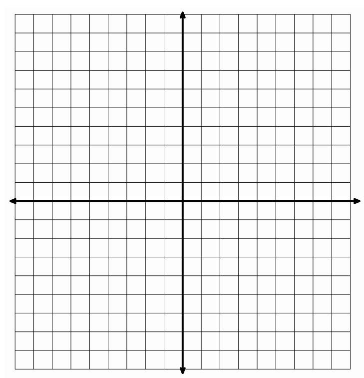
9. $\begin{cases} y = 4x - 1 \\ y = -x + 4 \end{cases}$



10. $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$



11. $\begin{cases} 4x - y = 7 \\ 3x + 2y = 8 \end{cases}$



Need help? Check out these related videos:

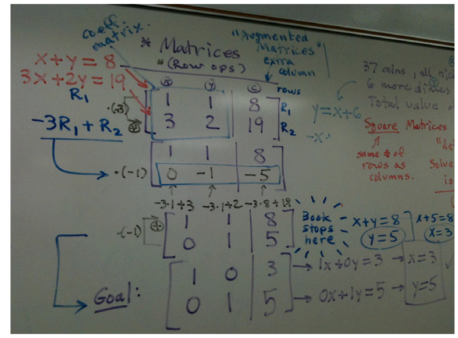
http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/solving-linear-systems-by-substitution

http://patrickjmt.com/row-reducing-a-linear-system-of-equations/

http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/v/graphings-systems-of-equations

# 2.14H Solving Systems with Matrices

## *A Practice Understanding Task*

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In the task "To Market with Matrices" you developed a strategy for solving systems of linear equations using matrices. An efficient and consistent way to carry out this strategy can be summarized as follows:

To row reduce a matrix:

- Perform elementary row operations to yield a "1" in the first row, first column.
- Create zeros in all of the other rows of the first column by adding the first row times a constant to each other row.
- Perform elementary row operations to yield a "1" in the second row, second column.
- Create zeros in all of the other rows of the second column by adding the second row times a constant to each other row.
- Perform elementary row operations to yield a "1" in the third row, third column.
- Create zeros in all of the other rows of the third column by adding the third row times a constant to each other row.
- Continue this process until the first m × m entries form a square matrix with 1s in the diagonal and 0s everywhere else.

Practice this strategy by creating a sequence of matrices for each of the following that begins with the given matrix and ends with the left portion of the matrix (the first m × m entries) in row-reduced form. Write a description of what you did to get from one matrix to another in each step of your sequence of matrices.

1. $\begin{bmatrix} 2 & 4 & 0 \\ 3 & 5 & -2 \end{bmatrix}$

2. $\begin{bmatrix} 4 & -2 & 2 \\ 1 & 3 & 11 \end{bmatrix}$

3. $\begin{bmatrix} 4 & -2 & 1 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & 2 & 7 \end{bmatrix}$

4. Each of the above matrices represents a system of equations. For each problem, write the system of equations represented by the original matrix. Determine the solution for each system using the row-reduced matrix you obtained, and then check the solutions in the original system.

5. Solve the following problem by using a matrix to represent the system of equations described in the scenario, and then changing the matrix to row-reduced form to obtain the solution.

   *Three of Carlos' and Clarita's friends are purchasing school supplies at the bookstore. Stan buys a notebook, three packages of pencils and two markers for $7.50. Jan buys two notebooks, six packages of pencils and five markers for $15.50. Fran buys a notebook, two packages of pencils and two markers for $6.25. How much do each of these three items cost?*

6. Create a linear system that is either dependent (both equations in the system represent the same line) or inconsistent (the equations in the system represent non-intersecting lines). What happens when you try to row reduce the $2 \times 3$ matrix that represents this linear system of equations?

# 2.14H Solving Systems with Matrices—Teacher Notes

*A Practice Understanding Task*

**Purpose**: The purpose of this task is to practice an efficient, organized and consistent strategy for transforming a matrix into row-reduced form. Students continue to connect the matrix representation to the system of linear equations it represents, and to the scenarios that give rise to both representations. In question 6 students consider how dependent systems and inconsistent systems create a row of 0s (no 1s in the diagonal) when rewritten in row-reduced form.

**Core Standards Focus**:

**UT Honors:** Solve systems of linear equations using matrices.

**Related Standards: A.REI.6**

**Launch (Whole Class):**
Review the "elementary row operations" that can be performed on matrices when used to represent linear systems of equations: (these emerge from the work with matrices in the previous task "To Market With Matrices")

- Replace a row in a matrix with a constant multiple of that row
- Replace a row in a matrix with the sum or difference of that row and another row of the matrix
- Replace a row in a matrix with the sum of that row and a constant multiple of another row of the matrix

(Note that the middle statement is a special case of the bottom statement, and therefore is redundant, but still may be helpful to state explicitly.)

Another elementary row operation that may not have come up explicitly in the previous task is the idea that you can switch any two rows in a matrix. This is justified by the fact that the order in which you write the equations in a linear system does not matter. This idea becomes useful when students are working on the "efficient and consistent" steps to row reduce a matrix formally introduced in this task. That is, sometimes the easiest way to get 1s on the diagonal of the matrix is to exchange rows. You can introduce this idea directly, or let it arise naturally as students discuss different ways to get 1 in the top row, first column for problems 1 and 2.

Before setting students to work on the problems, discuss the steps of the "efficient and consistent" row reduction strategy given in the task by asking students why this strategy would <u>always</u> work. Ask such questions as, "Why are the 1s on the diagonal helpful?" and "Why do you want to get 0s in one column before you move onto the next?"

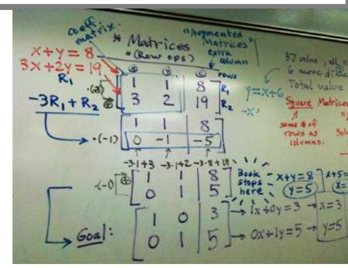
**Explore (Small Group):**
Students should see this list of steps as a helpful strategy, rather than the only way to row reduce a matrix. As students work on problems 1-3, have them consider why this strategy works in more detail, and examine alternative ways for accomplishing the steps listed, such as switching rows or dividing a row by a constant. Help students articulate how the 1s in the diagonal positions are used to get the 0s in the other rows of the same column (e.g., using the rule "Replace a row in a matrix with the sum of that row and a constant multiple of another row of the matrix" where the row being replaced is the row where you want to get a 0, the other row is the row that has 1 in the same column, and the constant multiple is the additive inverse of the number in the position where you want to get the 0).

Questions 3 and 5 give students an opportunity to extend the process for solving linear systems using matrices to situations involving 3 equations in 3 unknowns. This allows students to reconsider the ideas behind the row reduction strategy as they expand it to matrices of larger dimensions.

**Discuss (Whole Class):**
Give feedback to individuals or groups, as needed during their work on the task. As you monitor their work, identify any key issues or misconceptions that may benefit from a whole class discussion. Key issues for a whole class discussion might include firming up why the steps listed work all of the time, why you need to get 0s in all of the non-diagonal positions of a column before moving to working on another column, and what happens when the matrix represents a dependent or inconsistent system.

**Aligned Ready, Set, Go: Systems 2.14H**

# Name: Solving Systems with Matrices | 2.14H

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## Ready, Set, Go!

### Ready

Topic: Solving systems of equations using matrices.

1. In an earlier assignment you worked the following problem:

   *"A theater wants to take in $2000 for a certain matinee. Children's tickets cost $5 each and adult tickets cost $10 each. If the theater has a maximum of 350 seats, write a system of equations that can be solved to determine the number of both children and adult tickets the theater can sell."*

   Set up a matrix that goes with the situation described above.

### Set

**Assume that the matrices below represent linear systems of equations. Practice the strategy you used for reducing a given matrix so that the left portion of the matrix (the 2 rows and first 2 columns of entries) has ones on the diagonal. Write a description of what you did to get from one matrix to another in each step of your sequence of matrices.**

2. $\left[\begin{array}{cc|c} 3 & 2 & -6 \\ 1 & 2 & 2 \end{array}\right] R_1 - R_2 \rightarrow R_2 \left[\begin{array}{cc|c} 3 & 2 & -6 \\ 2 & 0 & -8 \end{array}\right] R_2 \div 2 \rightarrow R_2 \left[\begin{array}{cc|c} 3 & 2 & -6 \\ 1 & 0 & -4 \end{array}\right] \rightarrow$

3. $\left[\begin{array}{cc|c} -3 & 1 & -12 \\ 2 & 3 & -14 \end{array}\right] 3R_1 - R_2 \rightarrow R_2 \left[\begin{array}{cc|c} -3 & 1 & -12 \\ -11 & 0 & 22 \end{array}\right] \rightarrow$

4. $\left[\begin{array}{cc|c} 7 & 2 & 24 \\ 8 & 2 & 30 \end{array}\right] \rightarrow$

5. $\left[\begin{array}{cc|c} 5 & 1 & 9 \\ 10 & -7 & -18 \end{array}\right] \rightarrow$

# Name: Solving Systems with Matrices 2.14H

## Go

Topic: Solving systems of equations

**Solve the following systems of equations with a method of your choice.**

6. $\begin{cases} x - y = 11 \\ 2x + y = 19 \end{cases}$

7. $\begin{cases} 8x + y = -16 \\ -3x + y = -5 \end{cases}$

8. $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

9. $\begin{cases} -7x + y = -19 \\ -2x + 3y = -19 \end{cases}$

Need help? Check out these related videos:

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/solving-linear-systems-by-graphing

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/solving-linear-systems-by-substitution

http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/solving-systems-of-equations-by-elimination