Secondary Two Mathematics: An Integrated Approach
Module 2
Structures of Expressions

By

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In partnership with the
Utah State Office of Education

Module 2 – Structures of Expressions

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Ready, Set, Go Homework: Structures of Expressions 2.1

Classroom Task: 2.2 Transformers: More Than Meets the y's– A Solidify Understanding Task
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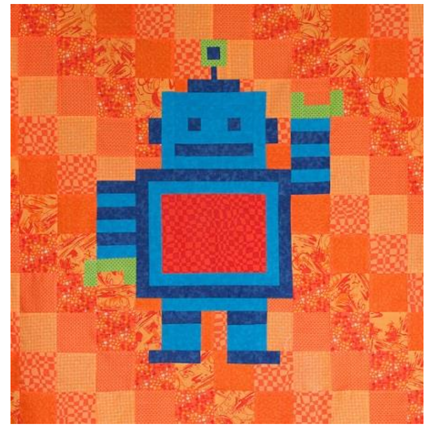
Ready, Set, Go Homework: Structures of Expressions 2.6



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2.1 Transformers: Shifty y's

A Develop Understanding Task

Optima is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x is modeled by the function, $A(x) = x^2$.

1. What is the domain of the function $A(x)$ in this context?

2. Match each statement about the area to the function that models it:

Matching Equation (A,B, C, or D)	Statement	Function Equation
	The length of any given side is increased by 5 units.	A) $A = 5x^2$
	The length of any given side is multiplied by 5 units.	B) $A = (x+5)^2$
	The area of a square is increased by 5 square units.	C) $A = (5x)^2$
	The area of a square is multiplied by 5.	D) $A = x^2 + 5$

Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers) and wondering about how changes to the equation of the function like adding 5 or multiplying by 5 affect the graph. She decided to make predictions about the effects and then check them out.

3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	Differences from the graph of $y = x^2$
$y = 5x^2$		
$y = (x+5)^2$		
$y = (5x)^2$		
$y = x^2 + 5$		

4. Optima decides to test her ideas using technology. She thinks that it is always a good idea to start simple, so she decides to go with $y = x^2 + 5$. She graphs it along with $y = x^2$ in the same window. Test it yourself and describe what you find.

5. Knowing that things make a lot more sense with more representations, Optima tries a few more examples like $y = x^2 + 2$ and $y = x^2 - 3$, looking at both a table and a graph for each. What conclusion would you draw about the effect of adding or subtracting a number to $y = x^2$? Carefully record the tables and graphs of these examples in your notebook and explain why your conclusion would be true for any value of k, given, $y = x^2 + k$.

6. After her amazing success with addition in the last problem, Optima decides to look at what happens with addition and subtraction inside the parentheses, or as she says it, "adding to the *x* before it gets squared". Using your technology, decide the effect of h in the equations: $y = (x + h)^2$ and $y = (x - h)^2$. (Choose some specific numbers for h.) Record a few examples (both tables and graphs) in your notebook and explain why this effect on the graph occurs.

7. Optima thought that #6 was very tricky and hoped that multiplication was going to be more straightforward. She decides to start simple and multiply by -1, so she begins with

$y = -x^2$. Predict what the effect is on the graph and then test it. Why does it have this effect?

8. Optima is encouraged because that one was easy. She decides to end her investigation for the day by determining the effect of a multiplier, a, in the equation: $y = ax^2$. Using both positive and negative numbers, fractions and integers, create at least 4 tables and matching graphs to determine the effect of a multiplier.

Transformers: Shifty y's – Teacher Notes

A Develop Understanding Task

Special Note to Teachers: Graphing technology is required for this task.

Purpose: The purpose of this task is to develop understanding of the effect on the graph of a quadratic function of replacing *f(x)* by *f(x) +k*, *kf(x), f(kx)* and *f(x+k).* The task begins with a brief story context to anchor student thinking about the effect of changing parameters on the graph. Students use technology to investigate the graphs, create tables and generalize about the transformations of quadratic functions.

Core Standards Focus:

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) +k*, *kf(x), f(kx)* and *f(x+k)* for specific values of *k* (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Launch (Whole Class):

Begin the task by asking students how to calculate the area of a square with a side of length 2. Use this short example to introduce the context given at the beginning of the task, and that the area of a square is a quadratic function of the length of a side of the square. Ask students to use the context to think about the answers to question 1 and 2, encouraging the use of drawings to show how the change in parameters actually changes the given square. Give them a short time to work on their own, and then discuss each of the answers, sharing reasoning about whether the change is to the length of the side or directly to area. Help students notice that if 5 is added or multiplied by the length of a side, then the units of the 5 are linear units (like inches or feet). If 5 is multiplied or added to the area, the units of the 5 are square units (like square inches or square feet). It is also useful to notice that if the 5 is "applied" to the length of the side, it is inside the argument of the function. If the 5 is "applied" to the area it is outside the square function.

Matching Equation (A,B, C, or D)	Statement	Function Equation
B	The length of any given side is increased by 5 units.	A) $A = 5x^2$
C	The length of any given side is multiplied by 5 units.	B) $A = (x+5)^2$

D	The area of a square is increased by 5 square units.	C) $A = (5x)^2$
A	The area of a square is multiplied by 5.	D) $A = x^2 + 5$

Ask students to make predictions about the changes in the graphs. Although the domain of the area context is (0,∞), they should think about the entire domain of the function for the remainder of the task. If students have not used the technology that they will be employing for the remainder of the task before, it is important to be sure that they can obtain graphs and find appropriate viewing windows before they proceed.

Explore (Small Group):
Monitor students as they work to ensure that they are able to get appropriate graphs and draw conclusions from the graphs. This is a chance to experiment mathematically, so students should have fun with testing their predictions and explaining their results. Watch for students that are relating the numeric results in the tables to the graphs to help explain why the graphs are transformed as they are. Listen particularly for explanations of why the horizontal shifts are the opposite sign of the parameter so that the reasoning can be used in the discussion.

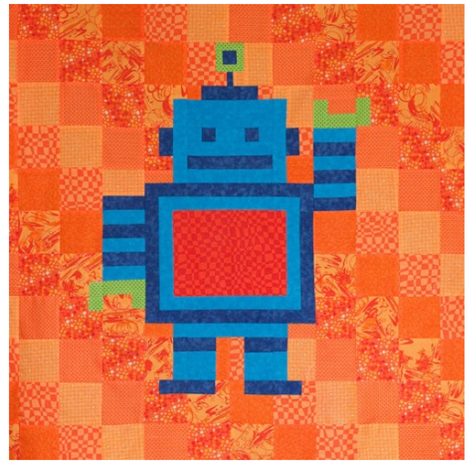
Discuss (Whole Class):
When students have completed their investigation, go through each question 4-8. Ask students to relate their tables to the graphs and explain why their conclusions make sense in each case. Some important points to highlight for each question are:

- Questions 4 and 5: Use the context to bring out the idea that the area (or the result of squaring) is the y-value or height of the graph. It should make sense that adding a number, k, to each area (or y-value) will shift the graph up k units. Ask students what happens if k is negative.
- Question 6: Although students will probably be able to see that for $k > 0$, $x + k$ shifts the function to the left and x – k shifts the function to the right, this is generally much harder to explain. Help students to draw upon the tables to compare values and articulate something like, "if you add 5 to the length of the side, then the area that you will get at $x = 1$ is the area that would have been at $x = 6$. That's why adding 5 shifts the graph 5 units to the left."

  It is also helpful to solidify the common use of the notation $f(x) = (x - k)^2$ to represent both $x + k$ and $x - k$, with the shift depending of the sign of k.
- Question 7: This should be an easy one to explain based on either the table or the context. If you change the sign of the every output (or multiply by -1), it will create a maximum where the minimum was previously, and change the intervals that were increasing to decreasing and vice-versa.
- Question 8: Students will probably notice that multiplying by a whole number makes the parabola "skinnier" and multiplying by a fraction makes the parabola wider. This may seem a little counter-intuitive until they connect to the tables and think that a multiplier of 3

multiplies each output, making the curve decrease and increase three times faster. It will be very useful for later tasks to establish that the graph of the parent function, $y = x^2$ starts at the vertex (0,0) and then counts: over 1, up 1, over 2, up 4, over 3, up 9 and repeats on the other side of the line of symmetry, x = 0. A multiplier multiplies the outputs, so the graph of $y = 2x^2$ will start at the vertex (0,0) and count: over 1, up 2, over 2, up 8, over 3 up 18 and so on. Beginning the practice of counting three points on either side of the line of symmetry will help students build fluency in quickly graphing parabolas for later work.

Aligned Ready, Set, Go: ***Structures of Expressions 2.1***

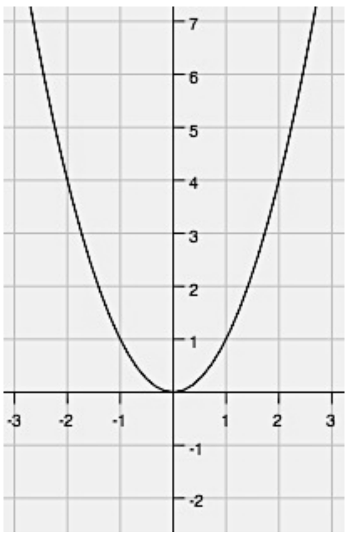


Ready, Set, Go!

Ready Topic: Finding key features in the graph of a quadratic equation

Make a point on the **vertex** and draw a dotted line for the **axis of symmetry**.
Label the coordinates of the vertex and state whether it's a **maximum** or a **minimum.**
Write the **equation for the axis of symmetry**.

1.



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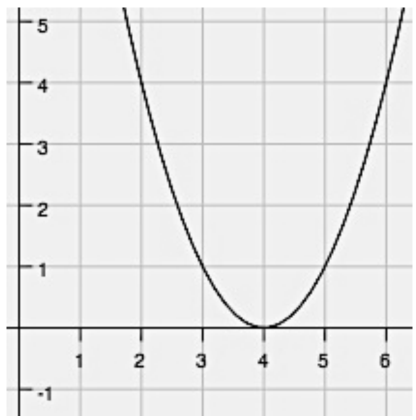

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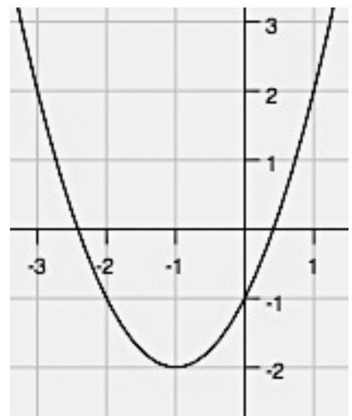

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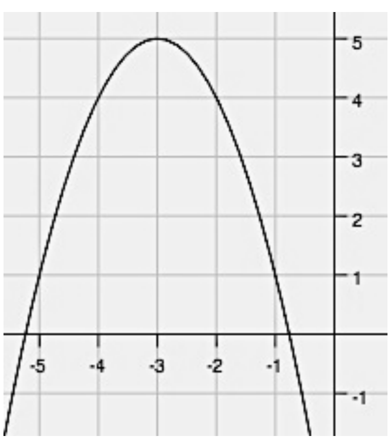

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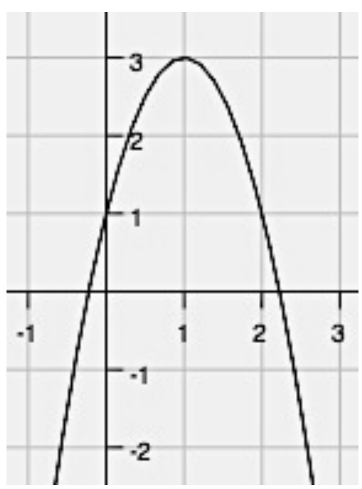

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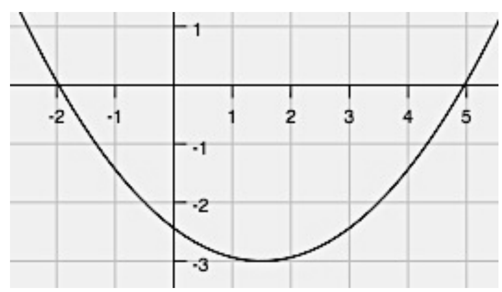
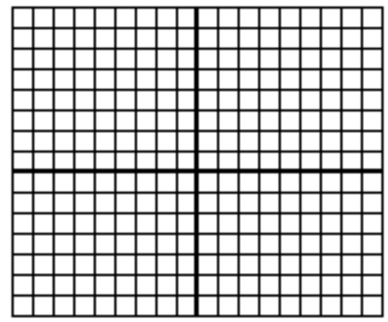


7. What connection exists between the coordinates of the vertex and the equation of the axis of symmetry?

8. Look back at #6. Try to find a way to find the ***exact*** value of the coordinates of the vertex. Test your method with each vertex in 1 - 5. Explain your conjecture.

9. How many **x-intercepts** can a parabola have?

10. Sketch a parabola that has no x-intercepts, then explain what has to happen for a parabola to have no x-intercepts.

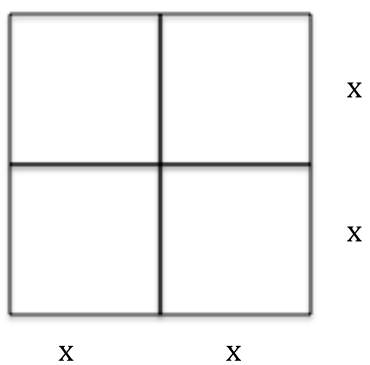


Set Topic: Transformations on quadratics

Choose the area model that is the best match for the equation.

11. $x^2 + 4$	12. $(x + 4)^2$	13. $(4x)^2$	14. $4x^2$

A.



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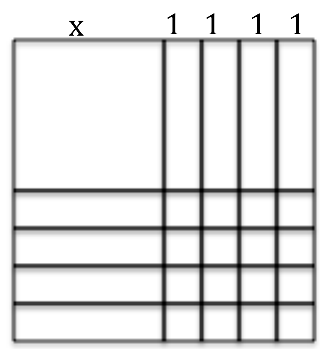

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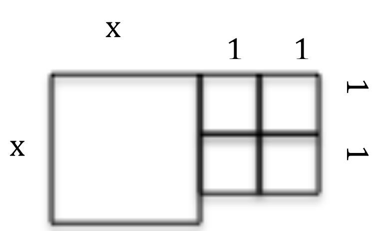
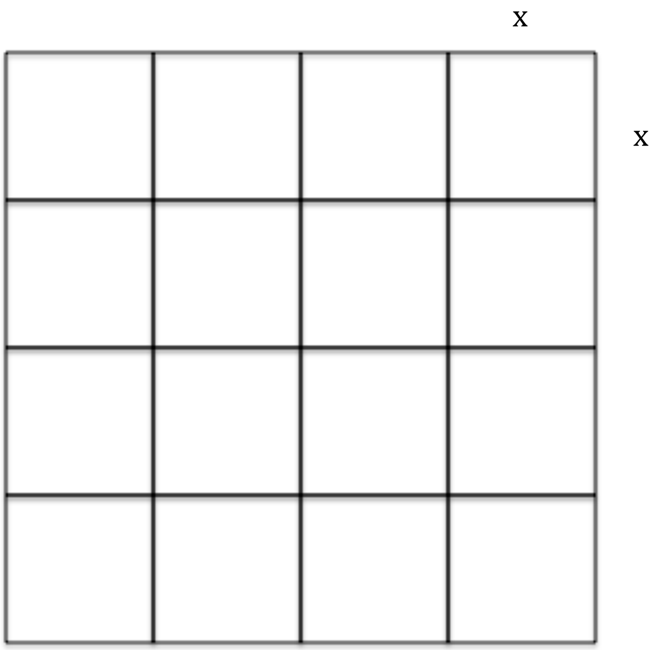
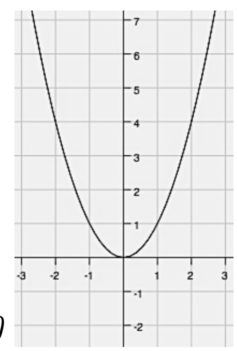


D.



A table of values for $f(x) = x^2$ is given. Compare the values in the table for $g(x)$ to those for $f(x)$. Identify what stays the same and what changes. Use this information to write the vertex form of the equation of $g(x)$. Then graph $g(x)$.



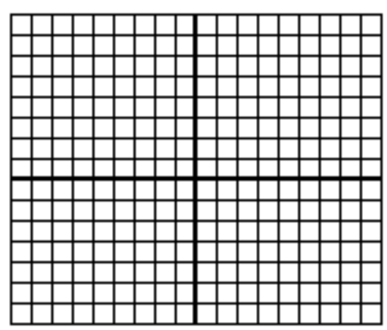
$f(x)$

Describe how the graph changed from the graph of $f(x)$.
Use words such as right, left, up, and down.

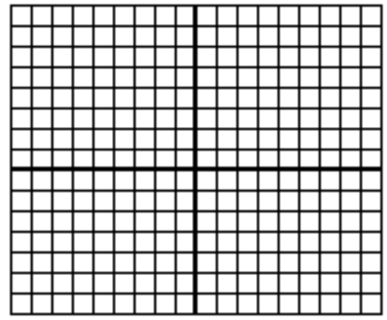
x	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9

15. $g(x) =$

x	-3	-2	-1	0	1	2	3
$g(x)$	2	-3	-6	-7	-6	-3	2



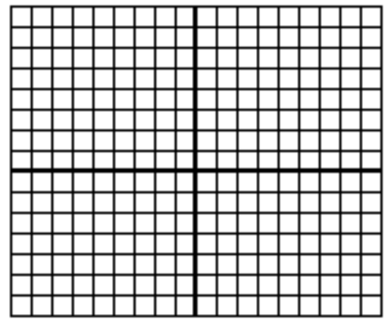
In what way did it move?
What part of the equation shows this move?



16. $g(x) =$

x	-3	-2	-1	0	1	2	3
$g(x)$	11	6	3	2	3	6	11

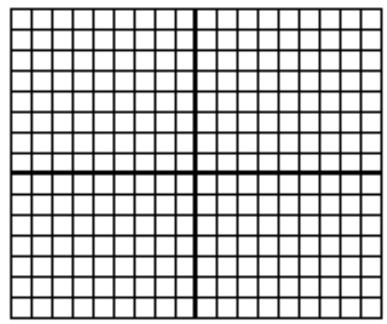
In what way did it move?
What part of the equation shows this move?



17. $g(x) =$

x	-4	-3	-2	-1	0	1	2
$g(x)$	9	4	1	0	1	4	9

In what way did it move?
What part of the equation shows this move?



18. $g(x) =$

x	0	1	2	3	4	5	6
$g(x)$	9	4	1	0	1	4	9

In what way did it move?
What part of the equation shows this move?

Go Topic: Finding square roots

Simplify.

19. $\sqrt{49a^2b^6}$

20. $\sqrt{(x+13)^2}$

21. $\sqrt{(x-16)^2}$

22. $\sqrt{(36x+25)^2}$

23. $\sqrt{(11x-7)^2}$

24. $\sqrt{9m^2(2p^3-q)^2}$

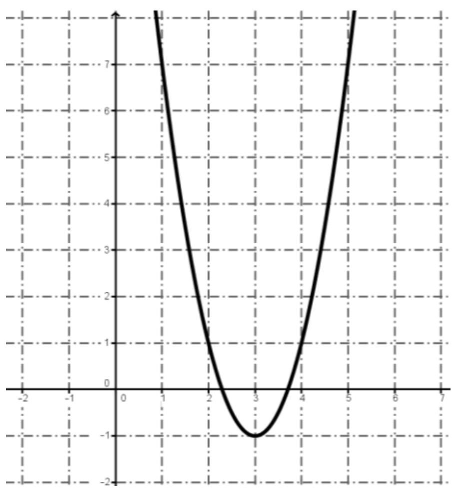
2.2 Transformers: More Than Meets the y's

A Solidify Understanding Task

Write the equation for each problem below. Use a second representation to check your equation.

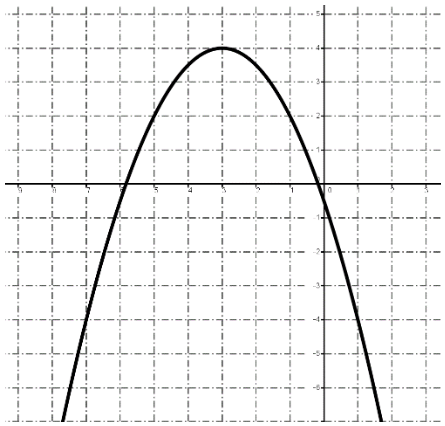
1. The area of a square with side length *x*, where the side length is decreased by 3, the area is multiplied by 2 and then 4 square units are added to the area.

2.



3.

x	*f(x)*
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23

4. 

Graph each equation without using technology. Be sure to have the exact vertex and at least two correct points on either side of the line of symmetry.

5. $f(x) = -x^2 + 3$

6. $g(x) = (x+2)^2 - 5$

7. $h(x) = 3(x-1)^2 + 2$

8. Given: $f(x) = a(x-h)^2 + k$
   a. What point is the vertex of the parabola?

   b. What is the equation of the line of symmetry?

   c. How can you tell if the parabola opens up or down?

   d. How do you identify the dilation?

9. Does it matter in which order the transformations are done? Explain why or why not.

Transformers: More Than Meets the y's– Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to extend student understanding of the transformation of quadratic functions to include combinations of vertical stretches, reflections over the *x*-axis, and vertical and horizontal shifts. Students will write equations given story contexts, graphs, and tables. They will use their knowledge of transformations to graph equations and then they will apply their understanding to a general formula for the graph of a quadratic function in vertex form.

Core Standards Focus:

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) +k*, *kf(x), f(kx)* and *f(x+k)* for specific values of *k* (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Launch (Whole Class):
Begin class by reminding students of the work they did in "Transformers: Shifty y's". Give the following equations and ask how each equation is a transformation of the parent function, $f(x) = x^2$:

a) $f(x) = x^2 - 3$
b) $f(x) = 3x^2$
c) $f(x) = \frac{1}{3}x^2$
d) $f(x) = (x - 3)^2$

Ask what equation they could write that would reflect the graph over the x-axis and what equation they could write that would shift the graph to the left 3.

Tell students that in the work today they will be combining these transformations and using them to write equations and find the graph of quadratic functions. For questions 1 – 4, they should write an equation and then use the equation to create another representation to check their work. For instance, on #2, they are given a graph. They should write an equation, use it to create a table of values and then check to see that the table and the graph match up.

Explore (Small Group):
Monitor students as they work to be sure that they are entering the problem successfully. Because students are probably not yet solid on the graph of $f(x) = x^2$, they may need to start by creating a table and graph of $f(x)$ and using it to compare to the function given in the problem. This may help them to identify the transformations and write the equation. If they are stuck on problems 5-7, they may need to start with a table. Listen for students that have productive comments to share for the discussion of questions 8 and 9.

Discuss (Whole Class):
Start the discussion with students presenting their work for problems 2 and 3. In each case, have a student identify what transformations have been made to the graph of $f(x) = x^2$ and then explain how they appear in the equation.

Move the discussion to the questions 5, 6, and 7. Have the students that present their work for each problem start by identifying the transformations from the equations and then build the graph. Have at least one of the students show a table along with the graph to demonstrate how the transformations appear in the table, as well as the graph. After each graph is presented, ask the class to identify:

a. The maximum or minimum point of the graph (the vertex):
b. Intervals on which the function is increasing or decreasing.
c. The domain and range of the function.
d. The equation of the line of symmetry.

This will lead students to generalize their experiences with questions 5-7 to answer question #8. Discuss the use of the formula and check for understanding with a couple of examples like:
$f(x) = -2(x+3)^2 + 7$.

Turn the discussion to question #9. Use student comments from the exploration to explain that if a function has multiple transformations, they are applied starting from the inside and working outward, in the following order:

1. Horizontal translation
2. Reflection, stretching, shrinking
3. Vertical Translation.

Close the discussion with a quick practice of accurately graphing quadratic functions from the equation. Model quickly identifying the location of the vertex, drawing the line of symmetry, deciding if it opens up or down and then counting the points: over 1, up $1\times a$, over 2, up $4\times a$, over 3, up $9\times a$, etc. Establish a routine of beginning the class period with using this method to quickly and accurately graph a few quadratic functions each day for the next few days to really build fluency.

Aligned Ready, Set, Go: ***Structure of Expressions 2.2***

Ready, Set, Go!

Ready

Topic: Standard form of a quadratic equation

The standard form of a quadratic equation is defined as

$$y = ax^2 + bx + c \quad (a \neq 0).$$

Identify ***a***, ***b***, and ***c*** in the following equations.

1. $y = 5x^2 + 3x + 6$

$a =$ ______
$b =$ ______
$c =$ ______

2. $y = x^2 - 7x + 3$

$a =$ ______
$b =$ ______
$c =$ ______

3. $y = 6x^2 - 5$

$a =$ ______
$b =$ ______
$c =$ ______

4. $y = -3x^2 + 4x$

$a =$ ______
$b =$ ______
$c =$ ______

5. $y = 8x^2 - 5x - 2$

$a =$ ______
$b =$ ______
$c =$ ______

Multiply and write each product in the form $y = ax^2 + bx + c$. Then identify ***a***, ***b***, and ***c***.

6. $y = x(x - 4)$

$a =$ ______
$b =$ ______
$c =$ ______

7. $y = (x - 1)(2x - 1)$

$a =$ ______
$b =$ ______
$c =$ ______

8. $y = (3x - 2)(3x + 2)$

$a =$ ______
$b =$ ______
$c =$ ______

9. $y = (x + 6)(x + 6)$

$a =$ ______
$b =$ ______
$c =$ ______

10. $y = (x - 3)^2$

$a =$ ______
$b =$ ______
$c =$ ______

11. $y = -(x + 5)^2$

$a =$ ______
$b =$ ______
$c =$ ______

Set

Topic: Writing the equation of a parabola in vertex form.

Find a value for $\boldsymbol{\omega}$ such that the graph will have the specified number of x-intercepts.

12. $\boldsymbol{y = x^2 + \omega}$
2 x-intercepts

13. $\boldsymbol{y = x^2 + \omega}$
1 x-intercept

14. $\boldsymbol{y = x^2 + \omega}$
no x-intercepts

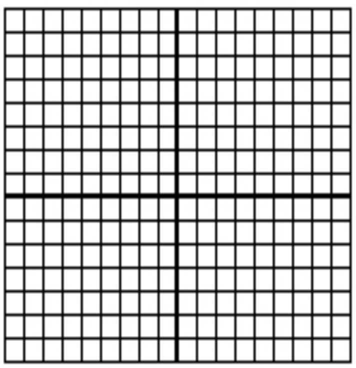
15. $\boldsymbol{y = -x^2} + \omega$
2 x-intercepts

16. $\boldsymbol{y = -x^2} + \omega$
1 x-intercept

17. $\boldsymbol{y = -x^2} + \omega$
no x-intercepts

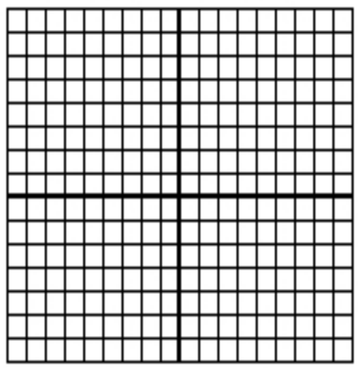
Graph the following equations. State the vertex. (Be accurate with your key points and shape!)

18. $y = (x-1)^2$



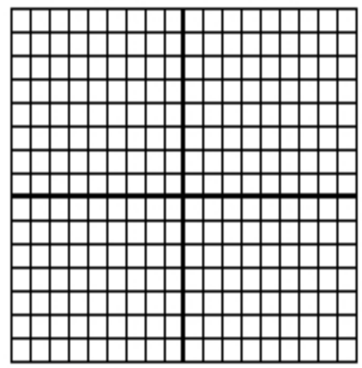
vertex? ________________

19. $y = (x-1)^2 + 1$



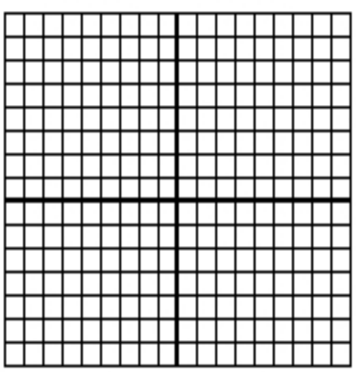
vertex? ________________

20. $y = 2(x-1)^2 + 1$



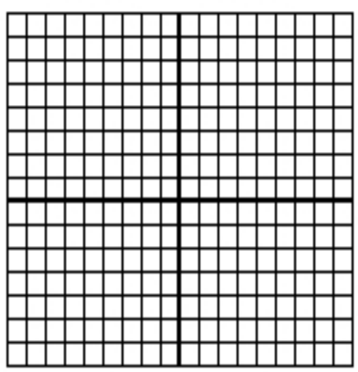
vertex? ________________

21. $y = (x+3)^2$



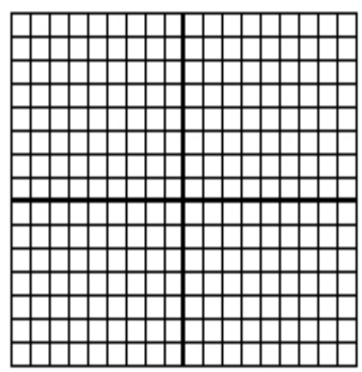
vertex? ________________

22. $y = -(x+3)^2 - 4$



vertex? ________________

23. $y = -0.5(x+1)^2 + 4$



vertex? ________________

Go

Use the table to identify the **vertex**, the equation for the **axis of symmetry**, and state the number of **x-intercept(s)** the parabola will have, if any. Will the vertex be a **minimum** or a **maximum**?

24.

x	y
-4	10
-3	3
-2	-2
-1	-5
0	-6
1	-5
2	-2

vertex ___________

A.S. ___________

x-inter __________

max or min?

25.

x	y
-2	49
-1	28
0	13
1	4
2	1
3	4
4	13

vertex ___________

A.S. ___________

x-inter __________

max or min?

26.

x	y
-7	-9
-6	3
-5	7
-4	3
-3	-9
-2	-29
-1	-57

vertex ___________

A.S. ___________

x-inter ___________

max or min?

27.

x	y
-8	-9
-7	-8
-6	-9
-5	-12
-4	-17
-3	-24
-2	-33

vertex ___________

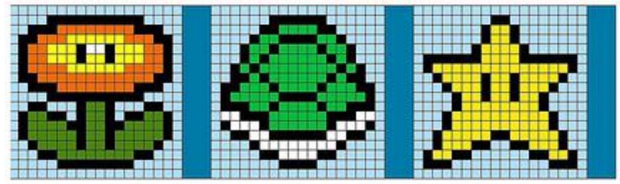
A.S. ___________

x-inter ___________

max or min?

2.3 Building the Perfect Square

A Solidify Understanding Task



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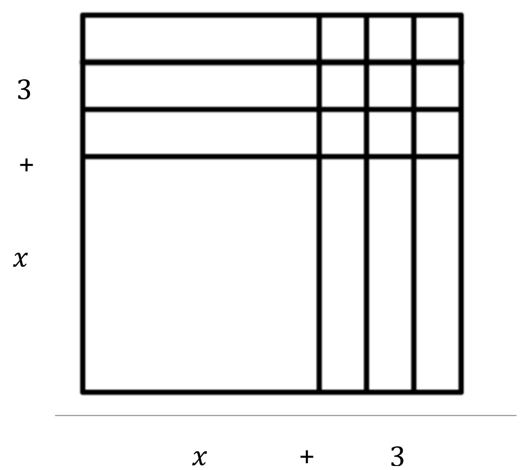
Part 1: Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that the can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to the $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

1. Block with side length: $x + 2$.

2. Block with side length: $x + 1$.

3. What patterns do you notice when you relate the diagrams to the two expressions for the area?

4. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

    a. $x^2 + 4x$

    b. $x^2 + 6x$

    c. $x^2 + 8x$

    d. $x^2 + 12x$

5. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

   Will this strategy work if b is negative? Why or why not?

   Will the strategy work if b is an odd number? What happens to c if b is odd?

Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$.

6. What would they write if the order was for 2 blocks that are 1 inch longer than the basic block? Write the area function in two equivalent forms. Verify your algebra using a diagram.

Part 2: Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x-3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

7. Fully describe the parabola that Jenny has been assigned to graph.

8. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or any of the transformations of the parabola in this form. Now what am I supposed to do?"

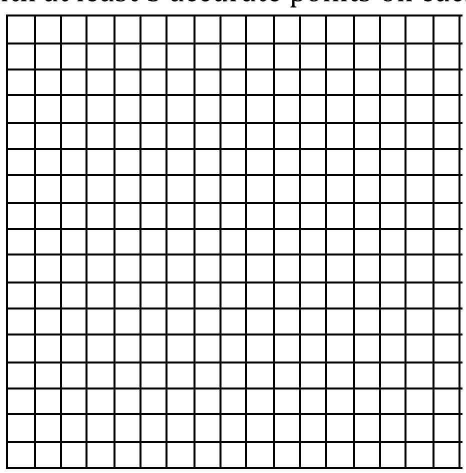
   "Wait a minute—is this the area of a perfect square?" Use your work from Part 1of this task to answer Jenny's question and justify your answer.

9. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$. Change the form of the equation, find the vertex, and graph the parabola.

   a. $y = x^2 - 6x + 9$ New form of the equation: ________________________________

   b. Vertex of the parabola: ____________

   c. Graph (with at least 3 accurate points on each side of the line of symmetry):

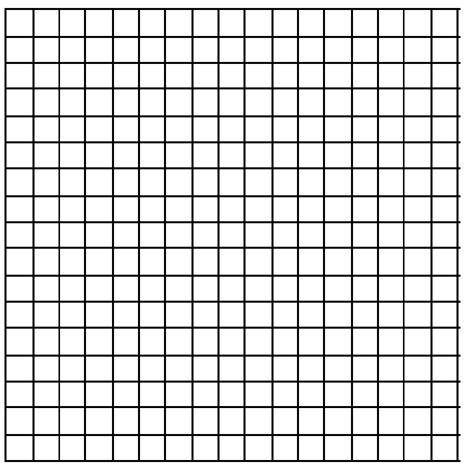


10. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x-h)^2 + k$.

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

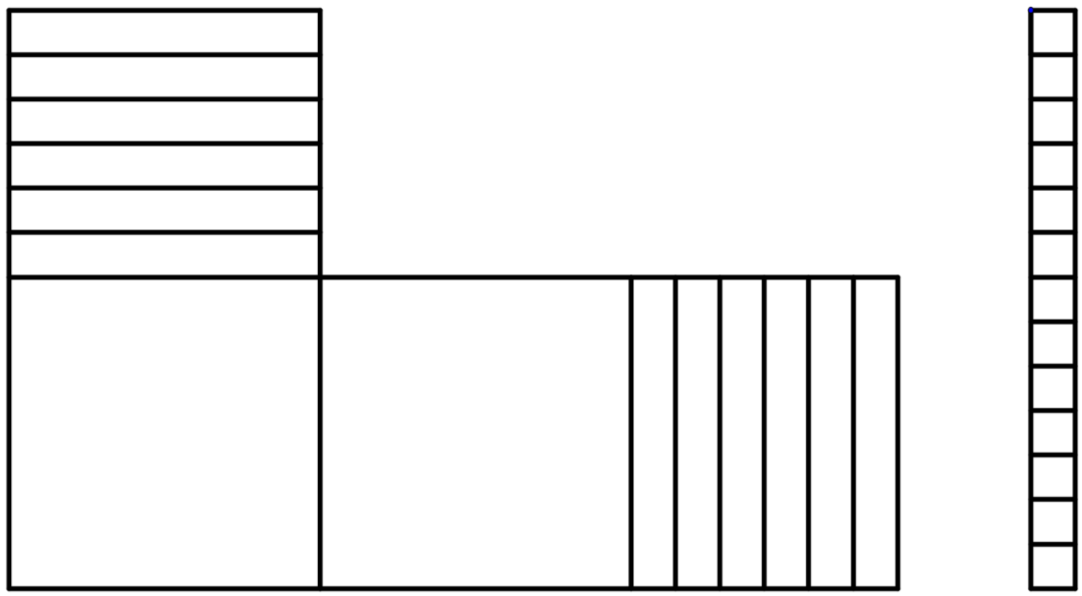
c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.



11. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$$g(x) = x^2 + 7x + 10$$

12. Jenny's last quadratic function to graph is $f(x) = 2x^2 + 12x + 13$. She draws the following diagram and says, "I'm not sure how this helps me. I don't see how to make this a square." Help Jenny to complete the square and find the vertex of the parabola using either the diagram or the equation.



Building the Perfect Square – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to extend students' understanding of graphing quadratic functions to include equations written in standard form. In the task, students will use diagrams of area models to make sense of the terms in a perfect square trinomial and discover the relationship between coefficients of a quadratic equation in standard form. They will use this understanding to complete the square to find an equivalent form of an equation. Students will also use completing the square to find the vertex form of a quadratic function and graph the associated parabola.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Launch (Whole Class):

Begin the task by reading through the initial context. Quickly ask students what the area would be if the base square was 1, 2, and 3. Ask them to work on question 1 and check that the class understands that the area function would be $A(x) = (x + 3)^2$. Let students spend a few minutes working on their own to answer question #2 and then ask students to explain their thinking. Ask students to test a few numbers for x to verify the relationship with numbers. Use both the equation and a diagram to show that the area model is a way to see the distributive property, both for variables and for numbers.

Explore (Small Group):

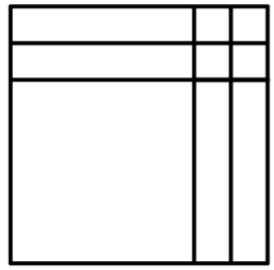
Part 1: Let students work on drawing the diagrams and completing Part 1. Ensure that students are connecting the diagrams to the algebraic expressions. Listen for their reasoning and encourage them to make generalizations that can be used in the later part of the task.

Discuss (Whole Class):

Part 1: Begin with problem #6 and ask a student to present the diagram that he/she made and the equations that he/she wrote. Ask for them to explain how they set up the diagram and especially how they figured out how many small squares to fill in. Label both the sides of the diagram and the different portions of the area. Relate the diagram to the equations.

Diagram and equation(s) for part a):

$$A(x) = (x+2)^2 = x^2 + 4x + 4$$



Depending on how students are responding, you may wish to continue to drawing the diagrams and looking for patterns in the equations. When students are ready to move on, go to question #8. Ask for their generalizations in words, and then be sure that the ideas are recorded algebraically. For instance, if a student says, “To get c, you take half of b and then square it,” be sure to model writing:

$$\left(\frac{b}{2}\right)^2$$

Be sure that students are solid on the relationship between b and c before moving on. Tell students that they will be applying these ideas to their work with graphing quadratic functions and let them get started on Part 2.

Explore:
Part 2: In this part of the task, students are working with completing the square, but now in the context of moving from standard form to vertex form for graphing a parabola. The major difference is that they will need to maintain the equality by adding and subtracting the same thing to one side of the equation (adding zero) or by adding the same thing to both sides of the equation. Either way, this can get a little tricky for students to work through initially.

Discuss:
Part 2: Focus the discussion on problem 12. Ask a student to demonstrate how they drew the diagram and knew that they needed 2 more small squares to complete the square. Of course, adding 2 squares changes the value of the expression, so adjustments need to made. This is a very important point to talk through with the class, getting several explanations from students. They will probably have different strategies for accounting for this change and it is worth discussing them. Quickly graph the parabola and go on to problem 13.

Ask a student to present their work on problem 13. The important part to talk through is how they managed to divide 7 by 2 and then square it. Some students may have used decimals, but fractions should be encouraged so that the expressions remain exact values. (Decimals could be exact values too, but students tend to round numbers off.)

At this point, it would be valuable to work several examples with the class. Some possibilities are:

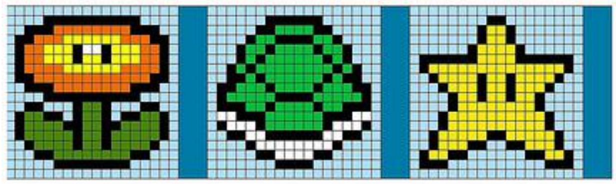
a) $y = x^2 + 6x + 8$
b) $y = x^2 + 10x + 21$
c) $y = x^2 + 5x + 8$
d) $y = x^2 + 3x + 9$

Conclude the discussion with #12. If no one in the class has solved it by using the diagram, demonstrate how to factor out the 2 and complete the square. Then, draw the diagram so they can connect the diagram with the equation. Students will probably need a few more examples of this type of problem, also. Possibilities are:

a) $y = 2x^2 + 4x + 5$
b) $y = 3x^2 + 6x + 8$

Aligned Ready, Set, Go: ***Structure of Expressions 2.3***

NAME Structures of Expressions | 2.3



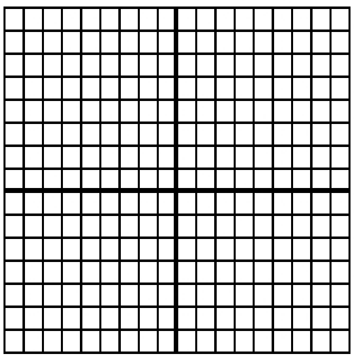
Ready, Set, Go!

Ready

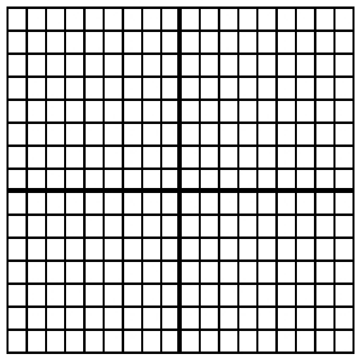
Topic: graphing lines using the intercepts

Find the x-intercept and the y-intercept.
Then graph the equation.

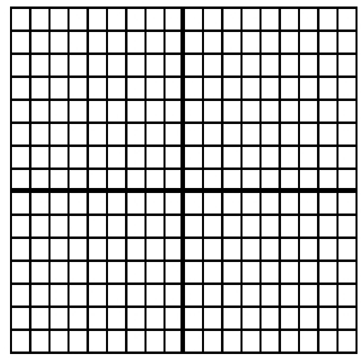
1. $3x + 2y = 12$



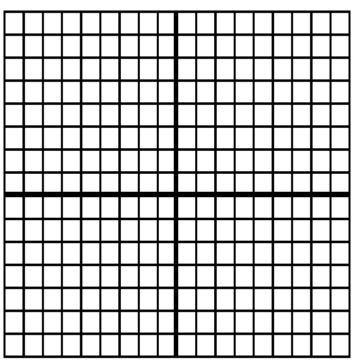
2. $8x - 12y = -24$



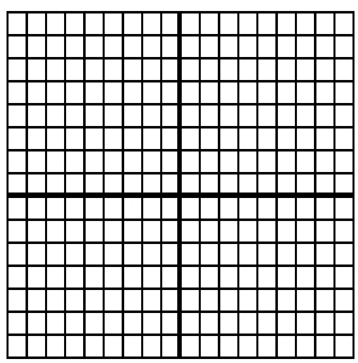
3. $3x - 7y = 21$



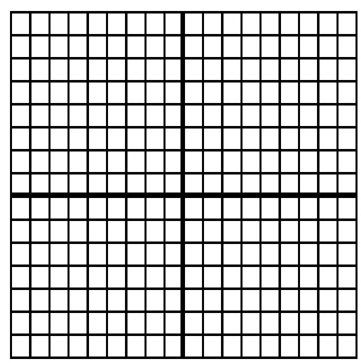
4. $5x - 10y = 20$



5. $2y = 6x - 18$



6. $y = -6x + 6$



Set

Topic: Completing the square

Multiply. Show each step. <u>Circle the pair of like terms before you simplify to a trinomial.</u>

7. $(x+5)(x+5)$ 8. $(3x-7)(3x-7)$ 9. $(9x+1)^2$ 10. $(4x-11)^2$

11. Write a rule for finding the coefficient of the x-term when multiplying and simplifying $(x + q)^2$.

Fill in the number that completes the square. Then write the trinomial in factored form.

12. x^2+8x+ ____ 2. x^2-10x+ ____ 3. x^2+16x+ ____

4. x^2-6x+ ____ 5. x^2-22x+ ____ 6. x^2+18x+ ____

On the next set of problems, leave the number that completes the square as a fraction. Then write the trinomial in factored form.

7. x^2-11x+ __________ 8. x^2+7x+ __________ 9. x^2+15x+ __________

10. $x^2+\frac{2}{3}x+$ __________ 11. $x^2-\frac{1}{5}x+$ __________ 12. $x^2-\frac{3}{4}x+$ __________

Find the value of "B," that will make a perfect square trinomial. Then write the trinomial in factored form.

16. x^2+ ______ $x+16$ 17. x^2- ______ $x+121$ 18. x^2- ______ $x+625$

19. $9x^2+$ ______ $x+225$ 20. $25x^2+$ ______ $x+49$ 21. x^2+ ______ $x+9$

22. x^2+ ______ $x+\frac{25}{4}$ 23. x^2+ ______ $x+\frac{9}{4}$ 24. x^2+ ______ $x+\frac{49}{4}$

Go

Find the intercepts of the graph of each equation. State whether it's an x-intercept or a y-intercept.

25. y = -4.5 26. x = 9.5 27. x = -8.2 28. y = 112

2.4 Factor Fixin'

A Solidify Understanding Task

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At first, *Optima's Quilts* only made square blocks for quilters and Optima spent her time making perfect squares. Customer service representatives were trained to ask for the length of the side of the block, x, that was being ordered, and they would let the customer know the area of the block to be quilted using the formula $A(x) = x^2$.

Optima found that many customers that came into the store were making designs that required a combination of squares and rectangles. So, *Optima's Quilts* has decided to produce several new lines of rectangular quilt blocks. Each new line is described in terms of how the rectangular block has been modified from the original square block. For example, one line of quilt blocks consists of starting with a square block and extending one side length by 5 inches and the other side length by 2 inches to form a new rectangular block. The design department knows that the area of this new block can be represented by the expression: $A(x) = (x + 5)(x + 2)$, but they do not feel that this expression gives the customer a real sense of how much bigger this new block is (e.g., how much more area it has) when compared to the original square blocks.

1. Can you find a different expression to represent the area of this new rectangular block? You will need to convince your customers that your formula is correct using a diagram.

Here are some additional new lines of blocks that *Optima's Quilts* has introduced. Find two different algebraic expressions to represent each rectangle, and illustrate with a diagram why your representations are correct.

2. The original square block was extended 3 inches on one side and 4 inches on the other.

3. The original square block was extended 4 inches on only one side.

4. The original square block was extended 5 inches on each side.

5. The original square block was extended 2 inches on one side and 6 inches on the other.

6. The original square block was extended 2 inches on one side and decreased by 2 inches on the other. (One of the employees thinks that this rectangle will have the same area as the original square since one side was decreased by the same amount as the other side was increased. What do you think?)

7. Both sides of the original square block were decreased by 3 inches.

Customers start ordering custom-made blocks designs by requesting how much additional area they want beyond the original area of x^2. Once an order is taken for a certain type of block, you need to have specific instructions on how to make the new design for the manufacturing team. Your instructions need to explain how to extend the sides of a square blocks to create the new line of rectangular blocks.

The customer service department has placed the following orders on your desk. For each, describe how to make the new blocks by extending the sides of a square block with an initial side length of x. Your instructions should include diagrams, written descriptions and algebraic descriptions of the area of the rectangles in using expressions representing the lengths of the sides.

8. $x^2 + 3x + 5x + 15$

9. $x^2 + 4x + 6x + 24$

Some of the orders are written in an even more simplified algebraic code. Figure out what these entries mean by finding the sides of the rectangles that have this area. Use the sides of the rectangle to write equivalent expressions for the area.

10. $x^2 + 9x + 18$

11. $x^2 + 7x + 10$

12. $x^2 + 9x + 8$

13. $x^2 + 6x + 8$

14. What relationships or patterns do you notice when you find the sides of the rectangles for a given area of this type?

One customer service representative has received an order requesting that the length of one side of the original square block be doubled and then increased by 3 inches, and that the other side be increased by 4 inches.

15. How might you represent this order using two different algebraic expressions?

16. What are the sides of the rectangle that has the area: $2x^2 + 9x + 10$?

17. A customer called and asked for a rectangle with area given by: $x^2 + 7x + 9$. The customer service representative said that the shop couldn't make that rectangle. Do you agree or disagree? How can you tell if a rectangle can be constructed from a given area?

2.4 Factor Fixin' – Teacher Notes
A Solidify Understanding Task

Note to teacher: Graph paper and colored pencils will be useful for this task.

Purpose: The purpose of this task is for students to understand equivalent expressions obtained from factoring trinomials. In the task, students use area model diagrams to identify the sides of the rectangle, and thus, the factors. Students write expressions in both factored form and standard form.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F.BF.1 Write a function that describes a relationship between two quantities.

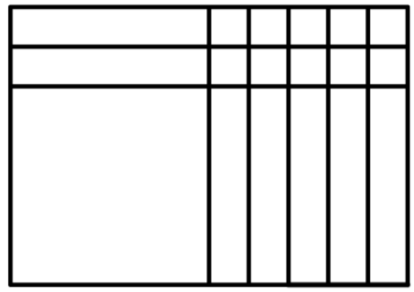
b. Combine standard functions types using arithmetic operations.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.,

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Launch (Whole Class):
This task is based on a context that students are familiar with from "Building the Perfect Square". Since they have used area model diagrams to model perfect squares, it should not be difficult to extend their thinking to rectangles. Start the task by having them work on problem 1. Ask a student to share the diagram, which should be something like this:



Ask the class how to label each of the sides and parts of the area. Ask students to connect the diagram with the equation and to justify the expression for the area. Be sure that students recognize the distributive property in both representations. Ask students if they notice any number patterns in the diagrams or the equation. At this point, it's not important to introduce the patterns unless students happen to see them. The purpose of the question is to get them to think about patterns as they work.

Explore (Small Group):
This task is meant to be a puzzler, so let students work through it a bit without leading too much. Encourage them to use the diagrams to write and justify their expressions. Listen for students that are noticing useful patterns in the numbers that will help the class with factoring. When students get to problems 6 and 7, they may have difficulty with the diagram. You may want to offer the idea of coloring the positive spaces one color and the negative spaces another. Students will probably need some prompting to think that when they have two equal sized squares of different colors, they add up to be zero. Identify students that have worked through this idea and drawn diagrams for 6 and 7 to share in the discussion.

Discuss (Whole Class):
Start the discussion with #10 and continue through the task so that students see multiple examples. In each case, have the presenting student talk about how they constructed the rectangle and found the sides. Be sure that they label their diagrams and write the factored form of the expression for area. When students get to #14, ask students to share that have found helpful relationships such as noticing that the factors of the last term must add up to the middle term. Help students to formalize the language and write it on the board so they can remember it.

When the discussion gets to #17, let students explain how they know that the expression doesn't factor. A good way to tell if a trinomial factors in general is to multiply the lead coefficient by the last term. If there are factors of that product that add to give the middle coefficient, the trinomial can be factored.

Aligned Ready, Set, Go: *Structure of Expressions 2.4*

Ready, Set, Go!

Ready Topic: Creating binomial quadratics

Multiply.

1. $x(4x-7)$ 2. . $5x(3x+8)$

3. Are the answers to problems 1 & 2 quadratics? Justify!

4. Write a rule for factoring a quadratic, written in standard form (**a*x*² + b*x* + c**) when **c** equals 0.

Multiply.

5. $(x+9)(x-9)$ 6. $(x+2)(x-2)$ 7. $(6x+5)(6x-5)$ 8. $(7x+1)(7x-1)$

9. The answers to problems 5, 6, 7, & 8 are quadratics. Which coefficient, **a, b,** or **c**, equals 0?

10. Multiply $(x-13)(x+13)$ (Show all of your steps.) Then multiply $(x-13)(x-13)$.

11. Multiply $(a-b)(a+b)$ (Show all of your steps.) Then multiply $(a+b)(a+b)$.

12. These problems represent two different types of special products. The first is called a ***difference of 2 squares***, while the second one is called a ***perfect square trinomial.*** If you can recognize these, you will make factoring easier for yourself. Explain how you will recognize these two special products. Include, how they are the same, how they are different, and how they factor.

difference of 2 squares	***perfect square trinomial***
Example:	*Example:*
same?	
different?	
factor?	*factor?*

Set Topic: factoring quadratic expressions

Factor the following quadratic expressions into two binomials.

13. $x^2-4x+45$ 14. $x^2-12x+45$ 15. $x^2-44x+45$

16. $x^2 - x - 72$

17. $x^2 + 14x - 72$

18. $x^2 - 6x + 72$

19. $x^2 - 12x + 36$

20. $x^2 - 36$

21. $x^2 - 15x + 36$

22. $15x^2 - 26x + 8$

23. $15x^2 - 2x - 8$

24. $15x^2 - 37x - 8$

25. Look back at each "row" of factoring problems. Explain how it is possible for the coefficient of the middle term to be different numbers in each problem when the "outside" coefficients are basically the same.

Go

Topic: Taking the square root of perfect squares

Only some of the expressions inside the radical sign are perfect squares. Identify which ones are perfect squares and take the square root. Leave the ones that are not perfect squares under the radical sign. Do not attempt to simplify them. (*Hint: Check your answers by squaring them. You should be able to get what you started with, if you are right.)*

26. $\sqrt{(17xyz)^2}$

27. $\sqrt{(3x-7)^2}$

28. $\sqrt{121a^2b^6}$

29. $\sqrt{x^2 + 32x + 16}$

30. $\sqrt{4x^2 + 28x - 49}$

31. $\sqrt{4x^2 + 28x + 49}$

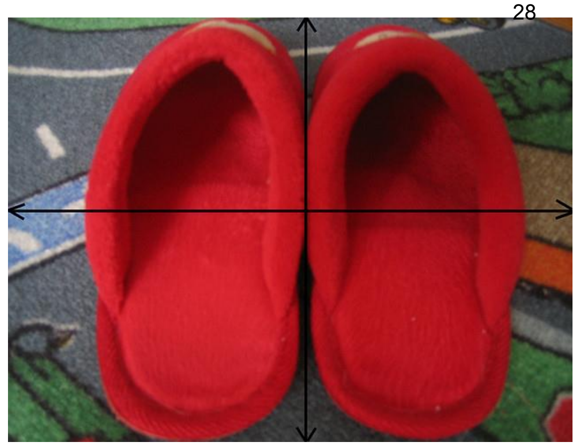
32. $\sqrt{x^2 - 16}$

33. $\sqrt{x^2 + 9}$

33. $\sqrt{x^2 + 10x + 100}$

34. $\sqrt{225x^2 + 30x + 1}$

35. $\sqrt{169x^2 - 260x + 100}$

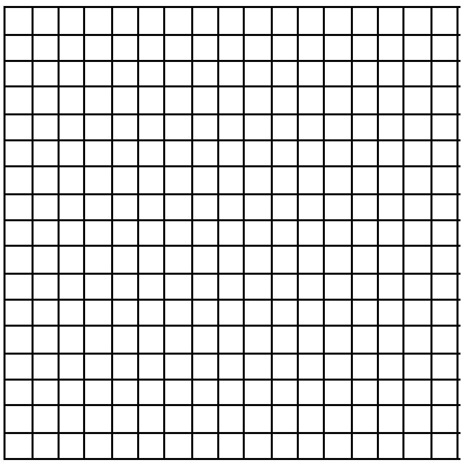


2.5 Lining Up Quadratics

A Solidify Understanding Task

Graph each function and find the vertex, the y-intercept and the x-intercepts. Be sure to properly write the intercepts as points.

1. $y = (x - 1)(x + 3)$



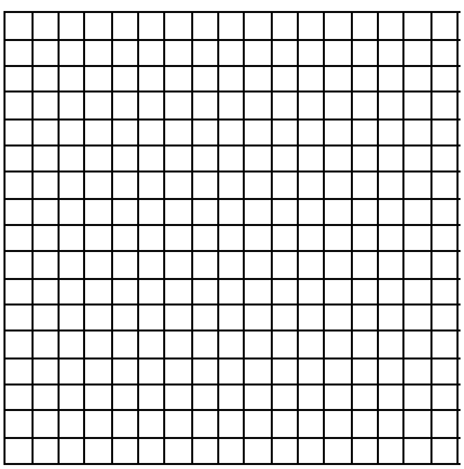
Line of Symmetry ________

Vertex ________

x-intercepts ________ ________

y-intercept _______

2. $f(x) = 2(x - 2)(x - 6)$



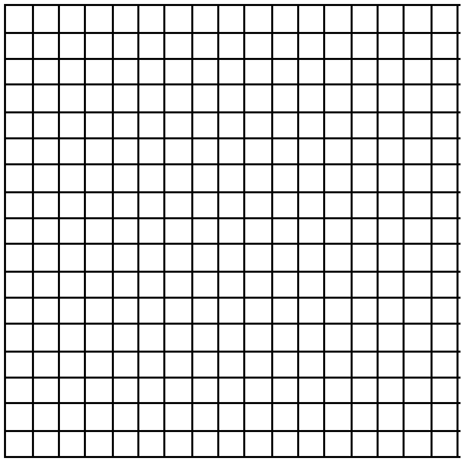
Line of Symmetry ________

Vertex ________

x-intercepts ________ ________

y-intercept _______

3. $g(x) = -x(x + 4)$



Line of Symmetry __________

Vertex _________

x-intercepts _________ _________

y-intercept ________

4. Based on these examples, how can you use a quadratic function in factored form to:
   a. Find the line of symmetry of the parabola?

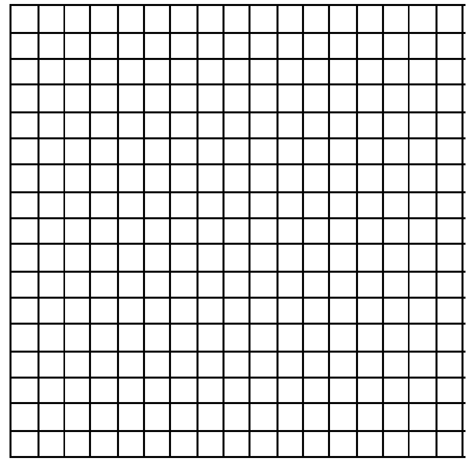
   b. Find the vertex of the parabola?

   c. Find the x-intercepts of the parabola?

   d. Find the y-intercept of the parabola?

   e. Find the vertical stretch?

5. Choose any two **linear** functions and write them in the form: $f(x) = m(x - c)$, where m is the slope of the line. Graph the two functions.



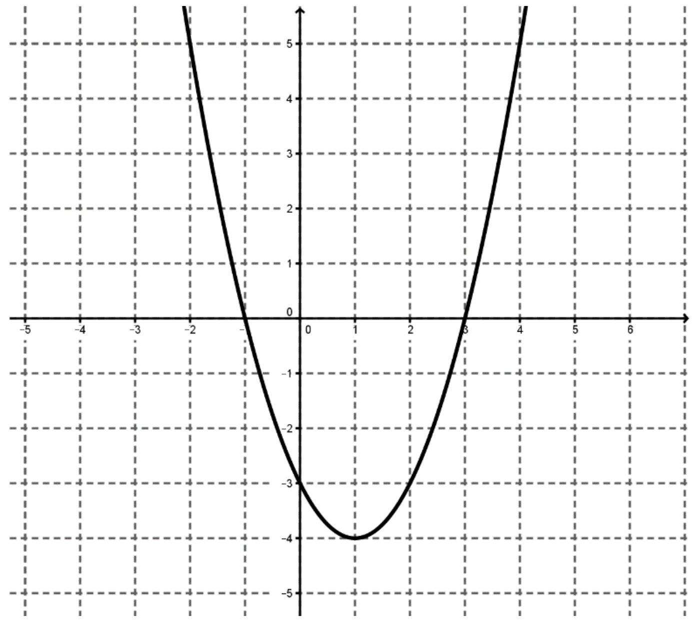
Linear function 1:

Linear function 2:

6. On the same graph as #5, graph the function *P(x)* that is the product of the two linear functions that you have chosen. What shape is created?

7. Describe the relationship between *x*-intercepts of the linear functions and the *x*-intercepts of the function *P(x).* Why does this relationship exist?

8. Describe the relationship between *y*-intercepts of the linear functions and the *y*-intercepts of the function *P(x).* Why does this relationship exist?



9. Given the parabola to the right, sketch two lines that could represent its linear factors.

10. Write an equation for each of these two lines.

11. How did you use the *x* and *y* intercepts of the parabola to select the two lines?

12. Are these the only two lines that could represent the linear factors of the parabola? If so, explain why. If not, describe the other possible lines.

13. Use your two lines to write the equation of the parabola. Is this the only possible equation of the parabola?

2.4 Lining Up Quadratics – Teacher Notes

A Solidify Understanding Task

Special Note to Teachers: Graphing technology is useful for this task.

Purpose: The purpose of this task is two-fold: First, for students to explore and generalize how the features of the equation can be used to graph the quadratic function. Second, for students to deepen students' understanding of a quadratic function as a product of two linear factors. In the task, students are asked to graph parabolas from equations in factored form. They are given several cases to provide an opportunity to notice how the x-intercepts, y-intercept, and vertical stretch are readily visible in the equation. This also sets them up to notice the relationship between the x-intercepts and the y-intercept. The task extends this thinking by asking students to start with any two linear functions, multiply them together and find function that is created, which is quadratic. They graph both the initial lines and the parabola to find the relationship between x-intercepts and y-intercept and to highlight the idea that quadratic functions are the product of two linear factors.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F.BF.1 Write a function that describes a relationship between two quantities.

b. Combine standard functions types using arithmetic operations.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Launch (Whole Class):
Begin the task by telling students that questions 1-4 are an exploration that has been set up for them to notice as many things as they can about using factored form of the equation of a quadratic function. It is recommended that graphing technology is provided for students to use in this part of the task. Tables of values could also be used to generate graphs of the functions, although this will be much slower. Ask students to work on just questions 1-4 and then call them back for a discussion.

Explore (Small Group):
While students are working, listen for students that can describe the process of finding the line of symmetry half way between the *x*-intercepts and then using the x-value to find the y-value of the vertex. This will be tricky for many students and may require a few scaffolding questions for support. Encourage students to not only draw conclusions based on the examples that are given, but to consider if their conclusions will be true for any quadratic function.

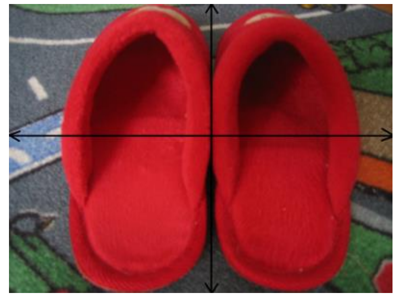
Discuss (Whole Class):
Begin the discussion with question #2. Use technology to project the graph for the class, and ask a student to explain how he/she found the x-intercepts. Ask another student to explain how he/she used the x-intercepts to find the line of symmetry, and then another student to explain how to use the line of symmetry to find the vertex. Move the discussion to question #4 and formalize the thinking of the class using algebraic notation. For each question, ask if the process will work in all cases and to justify their generalizations.

Re-launch (Whole class):
Start the second part of the task with question #5. Lead students through this question, prompting them to write two linear functions of their choice and to graph them. Explain that for question #6, students are to multiply the two functions together and graph the result. They don't need to go through the algebra of multiplying the two functions together and combining terms. You may need to show a quick example. Once everyone has this part completed, they should be ready to work together on the rest of the task.

Explore (Small Group):
Because the language in this part of the task is somewhat abstract, be prepared to help students understand what the question is asking for without giving away the answers to the questions. The most important thing to be shared in the discussion is student reasoning about why they are getting the results that they are. Since they have all chosen different linear functions, encourage students to share their results with each other and to talk about why the conclusions are the same.

Discuss (Whole Class):
After students have been given time to work through the task on their own, lead the class through the questions, sharing results and pressing for good explanations of the results.

Aligned Ready, Set, Go: ***Structures of Expressions 2.5***



Ready, Set, Go!

Ready

Topic: Multiplying binomials using a two-way table.

Multiply the following binomials using the given two-way table to assist you.
Example:

Multiply $(2x+3)(5x-7)$

	$(5x-7)$	
$(2x+3)$	**$10x^2$**	**$-14x$**
	$+15x$	**-21**

$= 10x^2 + x - 21$

1. $(3x-4)(7x-5)$
2. $(9x+2)(x+6)$
3. $(4x-3)(3x+11)$
4. $(7x+3)(7x-3)$
5. $(3x-10)(3x+10)$
6. $(11x+5)(11x-5)$
7. $(4x+5)^2$
8. $(x+9)^2$
9. $(10x-7)^2$

10. What do you notice in the "like-term" boxes in #'s 7, 8, and 9 that is different from the other problems?

Set Topic: Factored form of a quadratic function

Given the **factored form** of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

11. $y = 4(x-2)(x+6)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

12. $y = -3(x+2)(x-6)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

13. $y = (x+5)(x+7)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

14. $y = \frac{1}{2}(x-7)(x-7)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

15. $y = -\frac{1}{2}(x-8)(x+4)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

16. $y = \frac{3}{5}(x-25)(x-9)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

17. $y = \frac{3}{4}(x-3)(x+3)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

18. $y = -(x-5)(x+5)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

19. $y = \frac{2}{3}(x+10)(x+10)$

v:_____________

x-inter(s) ______________

y-inter ______________

stretch ______________

Go

Topic: Vertex form of a quadratic function

Given the **vertex form** of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

20. $y = (x+2)^2 - 4$

v:______________
x-inter(s) ______________
y-inter ______________
stretch ______________

21. $y = -3(x+6)^2 + 3$

v:______________
x-inter(s) ______________
y-inter ______________
stretch ______________

22. $y = 2(x-1)^2 - 8$

v:______________
x-inter(s) ______________
y-inter ______________
stretch ______________

23. $y = 4(x+2)^2 - 64$

v:______________
x-inter(s) ______________
y-inter ______________
stretch ______________

24. $y = -3(x-2)^2 + 48$

v:______________
x-inter(s) ______________
y-inter ______________
stretch ______________

25. $y = (x+6)^2 - 1$

v:______________
x-inter(s) ______________
y-inter ______________
stretch ______________

26. Did you notice that the parabolas in problems 11, 12, & 13 are the same as the ones in problems 23, 24, & 25 respectively? If you didn't, go back and compare the answers in problems 11, 12, & 13 and problems 23, 24, & 25.

Prove that a. $4(x-2)(x+6) = 4(x+2)^2 - 64$

b. $-3(x+2)(x-6) = -3(x-2)^2 + 48$

c. $(x+5)(x+7) = (x+6)^2 - 1$

2.6 I've Got a Fill-in

A Practice Understanding Task

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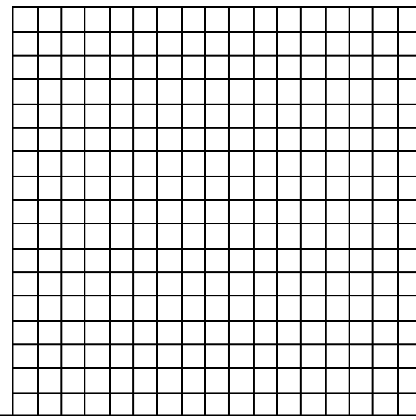
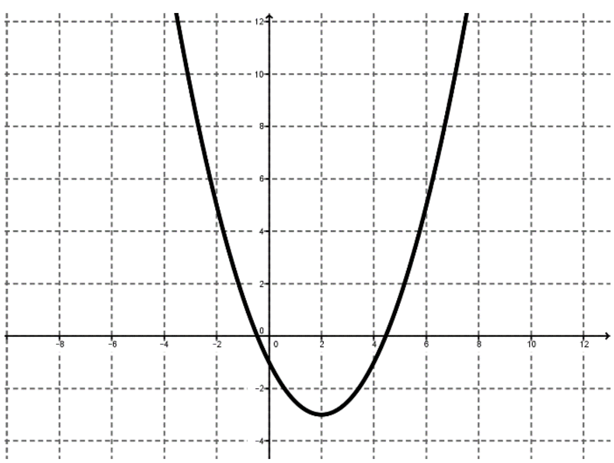
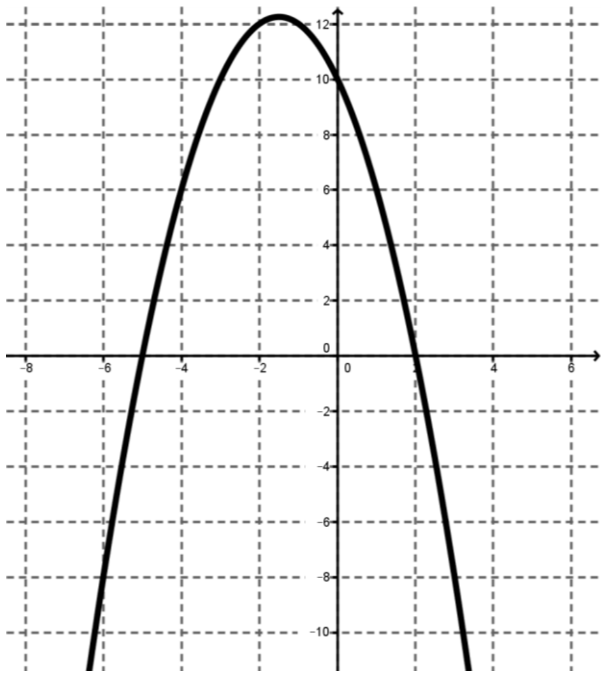
For each problem below, you are given a piece of information that tells you a lot. Use what you know about that information to fill in the rest.

1.	You get this:	Fill in this:
	$y = x^2 - x - 12$	Factored form on the equation:
		Graph of the equation:

2. You get this:	Fill in this:
$y = x^2 - 6x + 3$	Vertex form of the equation:
	Graph of the equation: 

3. You get this:	Fill in this:
	Vertex form of the equation:
	Standard form of the equation:

<table>
<tr><td>4.</td><td>You get this:</td><td>Fill in this:</td></tr>
<tr><td colspan="2" rowspan="2"></td><td>Factored form of the equation:</td></tr>
<tr><td>Standard form of the equation:</td></tr>
</table>

<table>
<tr><td>5.</td><td>You get this:</td><td>Fill in this:</td></tr>
<tr><td colspan="2" rowspan="3">$$y = -x^2 - 6x + 16$$</td><td>Either form of the equation other than standard form.</td></tr>
<tr><td>Vertex of the parabola</td></tr>
<tr><td>x-intercepts and y-intercept</td></tr>
</table>

6. You get this:	Fill in this:
$y = 2x^2 + 12x + 13$	Either form of the equation other than standard form.
	Vertex of the parabola
	x-intercepts and y-intercept

7. You get this:	Fill in this:
$y = -2x^2 + 14x + 60$	Either form of the equation other than standard form.
	Vertex of the parabola
	x-intercepts and y-intercept

2.4 I've Got a Fill-in– Teacher Notes
A Practice Understanding Task

Purpose: The purpose of this task is to build fluency in writing equivalent expressions for quadratic equations using factoring, completing the square, and the distributive property. Students will use the equations that they have constructed to analyze and graph quadratic functions.

Core Standards Focus:

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F.BF.1 Write a function that describes a relationship between two quantities.

b. Combine standard functions types using arithmetic operations.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.,

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Launch (Whole Class):
Students are familiar with the ideas in the task, so tell them that this task is to help them to be more flexible and fluent in using completing the square and factoring to graph quadratic functions. Be sure they understand the instructions and that in every case, they should be prepared to justify and explain their work. Before students get started it might be useful to note that the two graphs are both scaled by two's.

Explore (Small Group):
As students are working, listen for problems that are difficult or controversial for the discussion. In particular, watch for student approaches to problems 5-7. Listen for how they are deciding to either factor or complete the square, and then how they are working with the expressions with $a \neq 1$. Also note if students use area model diagrams or tables to support their thinking, as these are strategies that can be shared with the class.

Discuss (Whole Class):
Begin the discussion with problem #3. Ask a student to present their work, explaining how they identified the vertex and the vertical stretch of the parabola and used it to write the equation in vertex form. Ask another student to show how they used the vertex form of the equation to get the standard form. Ask the class how they could verify that the two equations are equivalent and then test them to see if they are correct.

Move the discussion to #5 and ask the class what they know about the parabola just by looking at the equation. They should be able to predict that it opens downward, that it doesn't have a vertical stretch and that the y-intercept is 16. If some students wrote the equation in vertex form and others used factored form, ask both to present. Ask the class to verify that the two forms are equivalent. Use technology to project the graph of the function and discuss how the features appear in the equations. Discuss the merits of each form and what information can be easily used in each form.

Continue the discussion with problems 6 and 7, proceeding just like problem 5.

Aligned Ready, Set, Go: *Structures of Expressions 2.6*

Ready, Set, Go!

Ready

Topic: Let's get *READY* for the test!

A golf-pro practices his swing by driving golf balls off the edge of a cliff into a lake. The height of the ball above the lake (measured in meters) as a function of time (measured in seconds and represented by the variable t) from the instant of impact with the golf club is

$58.8 + 19.6t - 4.9t^2$.

The expressions below are equivalent:

a. $-4.9t^2 + 19.6t + 58.8$ *standard form*
b. $-4.9(t-6)(t+2)$ *factored form*
c. $-4.9(t-2)^2 + 78.4$ *vertex form*

1. Which expression is the most useful for finding how many seconds it takes for the ball to hit the water? Justify your answer.

2. Which expression is the most useful for finding the maximum height of the ball? Justify your answer.

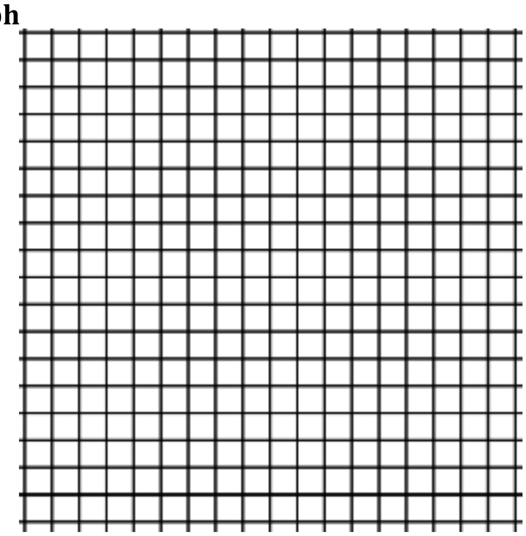
3. If you wanted to know the height of the ball at exactly 3.5 seconds, which expression would you use to find your answer? Explain why.

4. If you wanted to know the height of the cliff above the lake, which expression would you use? Explain why.

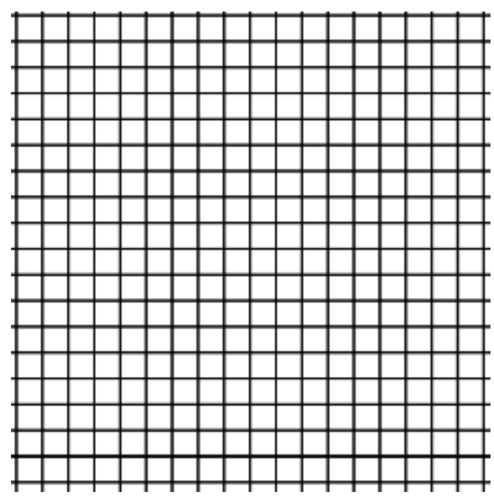
Set

One form of a quadratic function is given. Fill-in the missing forms.

5. **Standard form**	**Vertex form**	**Factored form** $y = (x+5)(x-3)$

Table (Show the vertex and at least 2 points on each side of the vertex.) Show the first differences and the second differences.	**Graph** 

6. **Standard form**	**Vertex form** $y = -3(x-1)^2 + 4$	**Factored form**

Table (Show the vertex and at least 2 points on each side of the vertex.) Show the first differences and the second differences.	**Graph** 

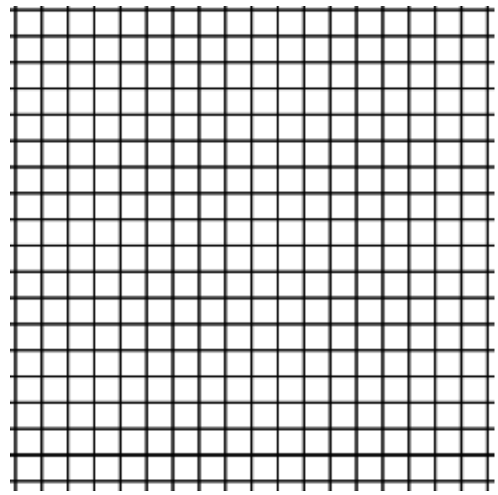
7. Standard form **Vertex form** **Factored form**

$y = -x^2 + 10x - 25$

Table (Show the vertex and at least 2 points on each side of the vertex.)

Show the first differences and the second differences.

Graph

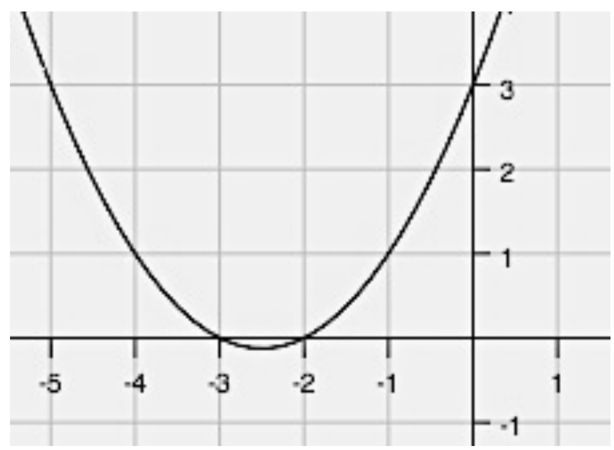


8. Standard form **Vertex form** **Factored form**

Table (Show the vertex and at least 2 points on each side of the vertex.)

Show the first differences and the second differences.

Graph



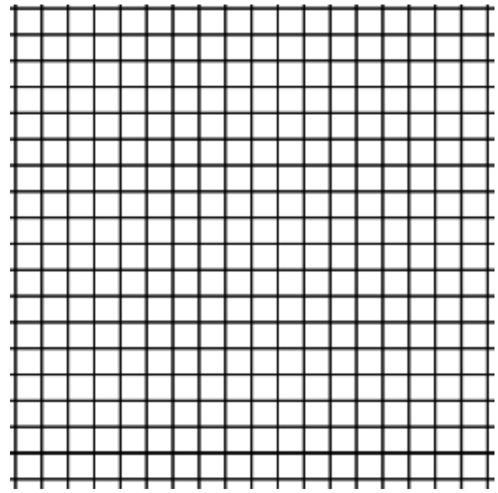
9. Standard form **Vertex form** **Factored form**

Table (Show the vertex and at least 2 points on each side of the vertex.)

x	y
0	12
1	2
2	-4
3	-6
4	-4
5	2
6	12

Show the first differences and the second differences.

Graph



Go Topic: factoring quadratics

Verify each factorization by multiplying.

10. $x^2 + 12x - 64 = (x + 16)(x - 4)$

11. $x^2 - 64 = (x + 8)(x - 8)$

12. $x^2 + 20x + 64 = (x + 16)(x + 4)$

13. $x^2 - 16x + 64 = (x - 8)(x - 8)$

Factor the following quadratic expressions. *(Hint: Some will not factor.)*

14. $x^2 - 5x + 6$

15. $x^2 - 7x + 6$

16. $x^2 - x - 6$

17. $m^2 + 16x + 63$

18. $s^2 - 3s - 1$

19. $3x^2 + 7x + 2$

20. $12n^2 - 8n + 1$

21. $3x^2 + 11x + 10$

22. $8c^2 - 11c + 3$

23. $36x^2 + 84x + 49$

24. $64x^2 - 9$

25. $25x^2 + 10x + 4$

26. Which quadratic expression above could represent the area of a square?
27. Which two in factored form could NOT be the side-lengths for a rectangle?