Secondary Two Mathematics: An Integrated Approach
Module 1
Quadratic Functions

By

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In partnership with the Utah State Office of Education

Module 1 – Quadratic Functions

Classroom Task: 1.1 Something to Talk About – A Develop Understanding Task
An introduction to quadratic functions, designed to elicit representations and surface a new type of pattern and change (**F.BF.1, A.SSE.1, A.CED.2**)
Ready, Set, Go Homework: Quadratic Functions 1.1

Classroom Task: 1.2 I Rule – A Solidify Understanding Task
Solidification of quadratic functions begins as quadratic patters are examined in multiple representations and contrasted with linear relationships (**F.BF.1, A.SSE.1, A.CED.2**)
Ready, Set, Go Homework: Quadratic Functions 1.2

Classroom Task: 1.3 Scott's Macho March – A Solidify Understanding Task
Focus specifically on the nature of change between values in a quadratic being linear. (**F-BF, F-LE**)
Ready, Set, Go Homework: Quadratic Functions 1.3

Classroom Task: 1.4 Rabbit Run – A Solidify Understanding Task
Focus on maximum/minimum point as well as domain and range for quadratics (**F.BF.1, A.SSE.1, A.CED.2**)
Ready, Set, Go Homework: Quadratic Functions 1.4

Classroom Task: 1.5 Look out Below – A Solidify Understanding Task
Examining quadratic functions on various sized intervals to determine average rates of change (**F.BF.1, A.SSE.1, A.CED.2**)
Ready, Set, Go Homework: Quadratic Functions 1.5

Classroom Task: 1.6 Tortoise and Hare – A Solidify Understanding Task
Comparing quadratic and exponential functions to clarify and distinguish between type of growth in each as well as how that growth appears in each of their representations (**F.BF.1, A.SSE.1, A.CED.2, F.LE.3**)
Ready, Set, Go Homework: Quadratic Functions 1.6

Classroom Task: 1.7 How Does it Grow – A Practice Understanding Task
Incorporating quadratics with the understandings of linear and exponential functions (**F.LE.1, F.LE.2, F.LE.3**)
Ready, Set, Go Homework: Quadratic Functions 1.7

1.1 Something to Talk About
A Develop Understanding Task

Cell phones often indicate the strength of the phone's signal with a series of bars. The logo below shows how this might look for various levels of service.

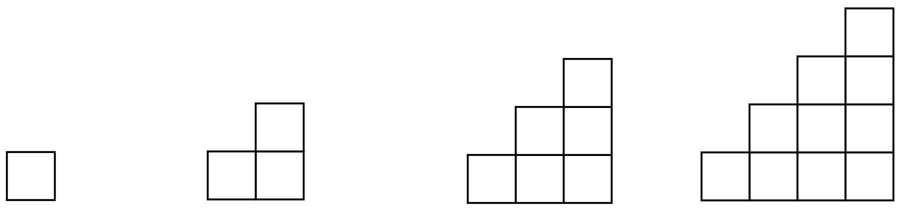


Figure 1 Figure 2 Figure 3 Figure 4

1. Assuming the pattern continues, draw the next figure in the sequence.

2. How many blocks will be in the size 10 logo?

3. Examine the sequence of figures and find a rule or formula for the number of tiles in any figure number.

Something to Talk About – Teacher Notes
A Develop Understanding Task

Purpose: The purpose of this task is to surface ideas and representations for quadratic functions. The task is designed to elicit tables, graphs, and equations, both recursive and explicit to describe a growing pattern. The classroom discussion will focus on the growth shown in the various representations, developing the idea that quadratic functions show linear rates of change.

Core Standards Focus:

F.BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

Related Standards:

Launch (Whole Class):
Begin the task by drawing students' attention to the pattern and asking students to draw the next figure in the sequence (question #1). Have a student show figure 5 and talk about what they noticed about the pattern to draw figure 5. Ask students to work on the rest of the task.

Explore (Small Group):
As students are working in their groups, pay attention to ways that they are using the pattern. Watch for students that are using tables to keep track of the growth of the pattern and find number of blocks in figure 10. Listen for students that are using reasoning about the pattern to predict figure 10. Some students may be writing various equations, based upon the way they see the pattern growing. If they are noticing the part that is added onto the pattern each time, they may write a recursive equation. Since this is an important representation for the discussion, encourage this thinking, even if the notation isn't entirely correct. If students are paying attention to the relationship between the figure number and the number of blocks in the figure, they may try to write an explicit equation for the pattern. You may expect students to think about the "empty

space" or the squares that could be used to complete a rectangle. They may notice that if the figure is copied and rotated, it can be fit upon itself to form a rectangle like so:

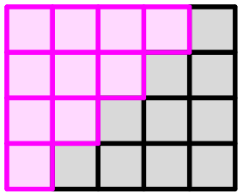


Figure 4

This reasoning may help them to find the explicit formula for pattern. As students work, encourage as many representations as possible.

Discuss (Whole Class):
When students have finished #2 and had a chance to work on #3 (some students may not have found explicit equations), begin the discussion with the number of squares in figure 10 (problem #2). Begin by having a group that used a table to present their work.

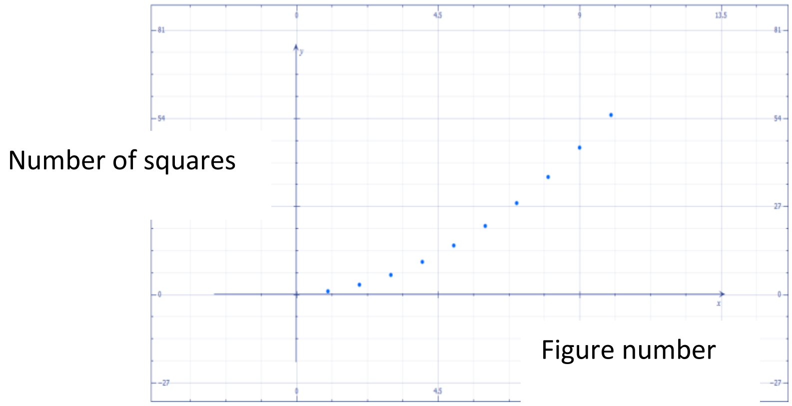
Figure No.	Number of squares
1	1
2	3
3	6
4	10
5	15
6	21
7	27
8	36
9	45
10	55
n	

First Difference

2
3
4
5

Ask students what patterns they see in the table. Ask how those same patterns show up in the figures. One important pattern to notice is the first difference, or the change in outputs on each line of the table.

Ask a student that has created a graph to show their representation. An example is shown below. Have a brief discussion of why the points in this graph should not be connected based upon this context.



Ask students what they know about the relation based upon the table and graph. Some conclusions that should be drawn:

- The relation is a function.
- The function is discrete.
- The domain and range of the function are both the set of natural numbers.
- The function is not linear because it does not have a constant rate of change.
- The function is not exponential because it does not have a constant ratio between terms.
- The function is always increasing.
- The minimum value of the function is (1,1).

Students may refer to the relationship as a sequence, comparing it to arithmetic and geometric sequences. This idea is appropriate but can be extended to include the broader categories of linear and exponential functions.

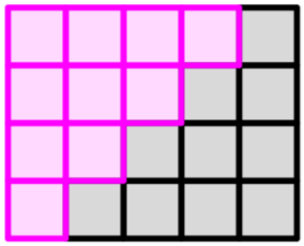
Turn the discussion to finding an equation to represent the pattern. Ask students to share their recursive equations. You may wish to start with a student that has used a recursive idea but has not used function notation. The idea is that the function starts at 1, and the n^{th} term can be found by taking the previous term and adding n. Ask students to show how this can be seen in the table, moving from one output row to the next. In function notation, this is represented by:

$$f(1) = 1, f(n) = f(n-1) + n$$

Ask a student how they can use the equation to describe the figures as the pattern goes from figure 4 to figure 5.

It is much harder to use the table to find the explicit equation than the recursive equation. Most students that are successful in writing an explicit formula for the pattern use the visual pattern, thinking about the rectangle made by copying and rotating the figure. Even if they have not written an equation, have a student that considered this strategy to show how they used the drawing for figure 4, as below:

"Using figure 4, I notice that the rectangle formed has width of 5 and a height of 4. When I tried it with other rectangles I saw that the width was always one more than the height."



Help students to see that the height of a rectangle formed in this fashion is n and the width is n+1. The area of the rectangle is $n(n+1)$, but since the actual figure is only half of the rectangle, the function that describes the number of blocks in each figure is: $f(n)=\frac{1}{2}n(n+1)$.

Conclude by going back to the table and the first difference. Ask students what type of function appears to be formed by the first difference. This will bring up the second difference, the rate of change of the first difference. In this case the second difference is always 1, which means that the first difference is linear. Tell students that functions with a linear rate of change are called quadratic functions.

In addition, tell students that because the domain is restricted in this case, we are only seeing half the graph. Display the graph of $f(x)=\frac{1}{2}x(x+1)$, demonstrating the typical quadratic graph. Ask students what they notice about the graph. Ask students why the equation would give a graph that decreases, then increases, and is symmetrical. Students will work with work extensively with the graphs of quadratic equations in module 2, but it is important that they notice many of the features of the graph as they are developing an understanding of quadratic relationships in module 1.

Aligned Ready, Set, Go: *Quadratic Functions 1.1*

Name: Quadratic Functions | 1.1

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Ready, Set, Go!

Ready

Topic: Distributive Property

Simplify the following expressions

1. $3(2x + 7)$
2. $-12(5x - 4)$
3. $5a(-3a + 13)$
4. $9x(6x - 2)$
5. $\frac{2x}{3}(12x + 18)$
6. $\frac{4a}{5}(10a - 25b)$
7. $\frac{-4x}{11}(121x + 22)$

Set

Topic: Recognizing linear, exponential, and quadratic equations.

In each set of 3 functions, one will be linear and one will be exponential. One of the three will be a new category of function. List the characteristics in each table that helped you to identify the linear and the exponential functions. What are some characteristics of the new function? Find an explicit and recursive equation for each.

8. Linear, exponential, or a new kind of function.

a.

x	$f(x)$
6	64
7	128
8	256
9	512
10	1024

Type and characteristics?

Explicit equation:

Recursive equation:

b.

x	$f(x)$
6	36
7	49
8	64
9	81
10	100

Type and characteristics?

Explicit equation:

Recursive equation:

c.

x	$f(x)$
6	11
7	13
8	15
9	17
10	19

Type and characteristics?

Explicit equation:

Recursive equation:

9. Linear, exponential, or a new kind of function?

d.

x	$f(x)$
-2	-17
-1	-12
0	-7
1	-2
2	3

Type and characteristics?

Explicit equation:

Recursive equation:

e.

x	$f(x)$
-2	1/25
-1	1/5
0	1
1	5
2	25

Type and characteristics?

Explicit equation:

Recursive equation:

f.

x	$f(x)$
-2	9
-1	6
0	5
1	6
2	9

Type and characteristics?

Explicit equation:

Recursive equation:

10. Graph the functions from the tables in #8. Add any additional characteristics you notice from the graph. Place your axes so that you can show all 5 points. Identify your scale. Write your *explicit* equation above the graph.

a. Equation:

b. Equation:

c. Equation:

d. Equation:

e. Equation:

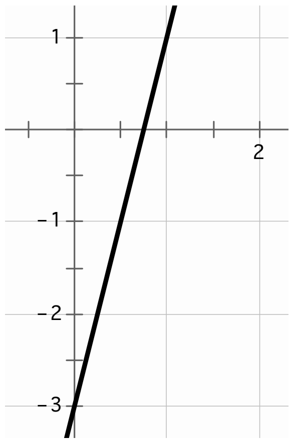
f. Equation:

Go

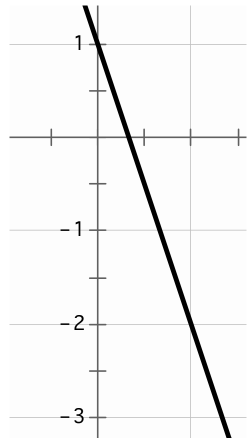
Topic: rates of change

Identify the rate of change in each of the representations below.

11.



12.

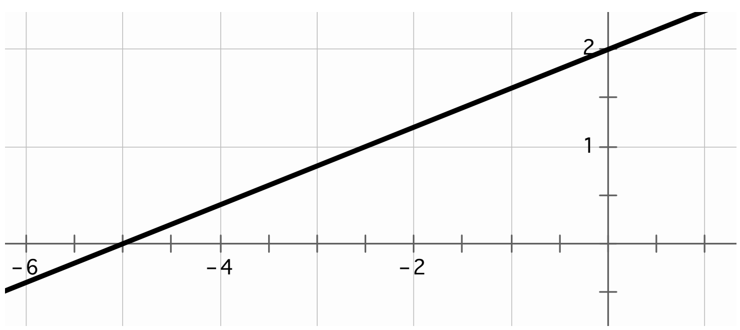


13.

x	$f(x)$
25	65
26	68
27	71
28	74

14.
$f(0) = 7; f(n+1) = f(n) + 5$

15.



16.
Slope of $\overleftrightarrow{AB}$
A(-3, 12) B(-11, -16)

17. George is loading freight into an elevator. He notices that the weight limit for the elevator is 1000 lbs. He knows that he weighs 210 lbs. He has loaded 15 boxes into the elevator. Each box weighs 50 lbs. Identify the rate of change for this situation.

18.

Independent variable	4	5	6	7	8
Dependent variable	5	5.5	6	6.5	7

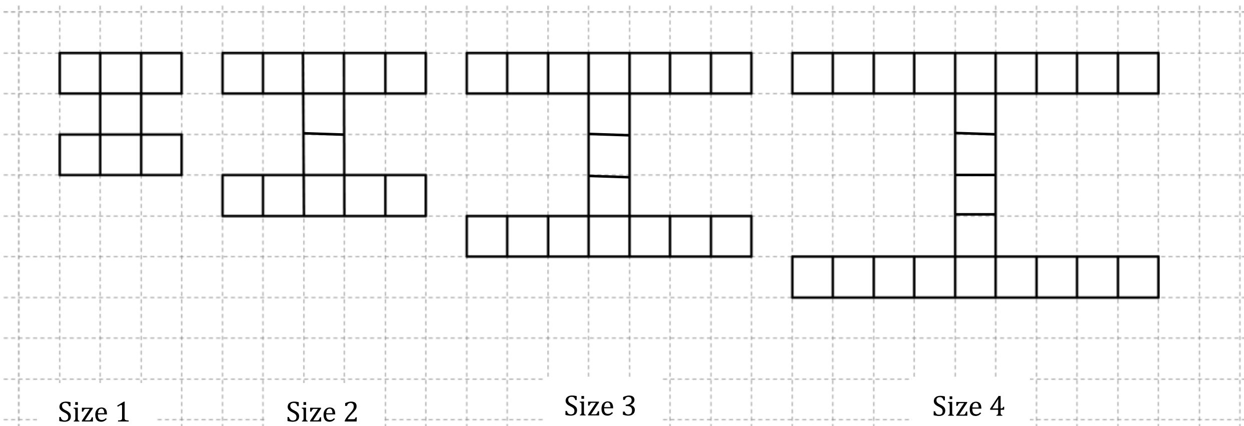
19.
$f(-4) = 24$ and $f(6) = -36$

1.2 I Rule!

A Solidify Understanding Task

Marco has started a new blog about sports at Imagination High School (mascot: the fighting unicorns) that he has decided to call "I Site". He created a logo for the web site that looks like this:

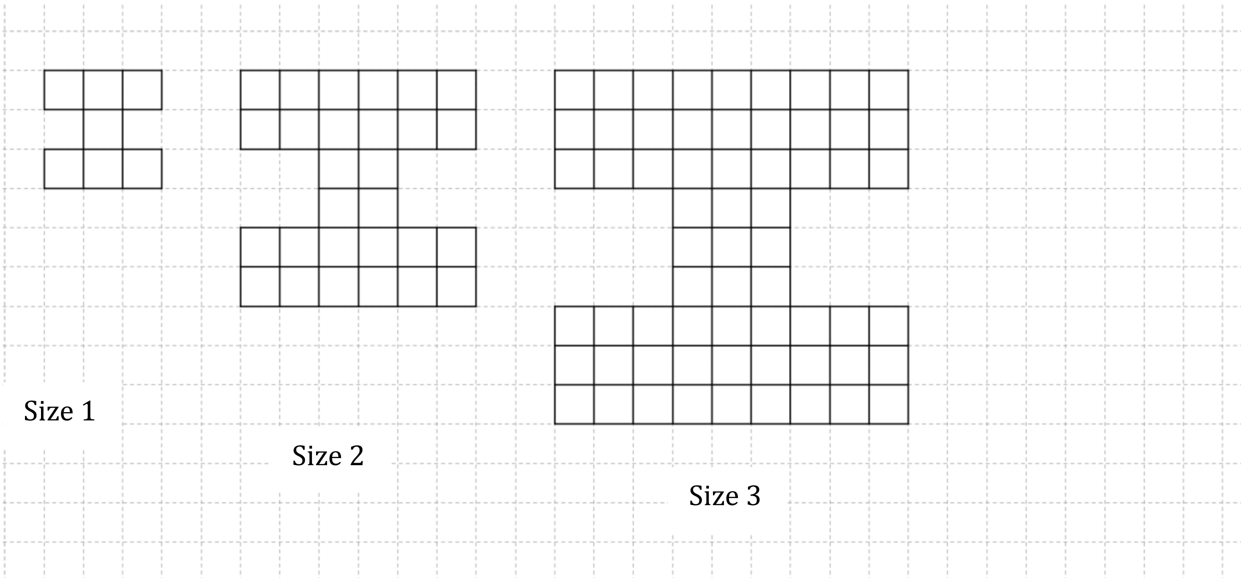
He is working on creating the logo in various sizes to be placed on different pages on the website. Marco developed the following designs:



1. How many squares will be needed to create the size 100 logo?

2. Develop a mathematical model for the number of squares in the logo for size *n*.

Marco decides to experiment with making his logo "blockier" so that it looks stronger. Here's what he came up with:



3. Assuming that Marco continues with the pattern as it has begun, draw the next figure, size 4, and find the number of blocks in the figure.

4. Develop a mathematical model for the number of blocks in a logo of size n.

5. Compare the models that you developed for the first set of logos to the second set of logos. In what ways are they similar? In what ways are they different?

I Rule! – Teacher Notes

A Solidify Understanding Task

Purpose: The purpose of this task is to solidify students' understanding of quadratic functions and their representations, by providing both an example and a non-example of a quadratic function. The task provides an opportunity for students to compare the growth of linear functions to the growth of quadratic functions. Equations, both recursive and explicit, graphs, and tables are used to describe the relationship between the number of blocks and the figure number in this task.

Core Standards Focus:

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Analyze functions using different representations.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

*Focus on situations that exhibit a quadratic or exponential relationship.

Construct and compare linear, quadratic, and exponential models and solve problems.

Compare linear and exponential growth studied in Secondary Mathematics I to quadratic growth.

Related Standards:

Launch (Whole Class):
Begin by having students read the task and understanding the context. Ask them to compare the two sets of logos in the first and second parts of the task and share what they notice about the mathematical/geometric attributes of the figures. Have students get started on questions 1 and 2.

Explore (Small Group):
Part 1: Monitor students as they work, looking for their use of tables, graphs, and equations. If students are stuck, ask how they see the figures changing. It may be useful to provide colored pencils to help them keep track of what changes and what stays the same in each figure. Encourage students to use as many representations as possible for their mathematical model.

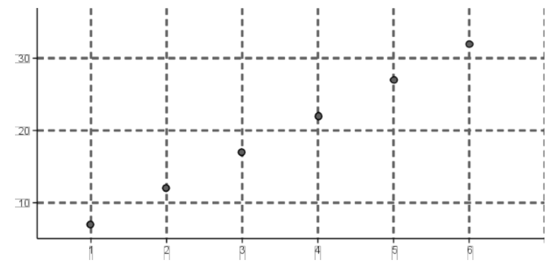
Discuss (Whole Class):

Part 1:

Start the discussion with students presenting a table and a graph. Ask students what type of function this is and how they know. Using the first difference, highlight the ideas that the rate of change is always 5.

Size No.	Number of Squares
1	7
2	12
3	17
4	22
5	27
n	

Number of Squares

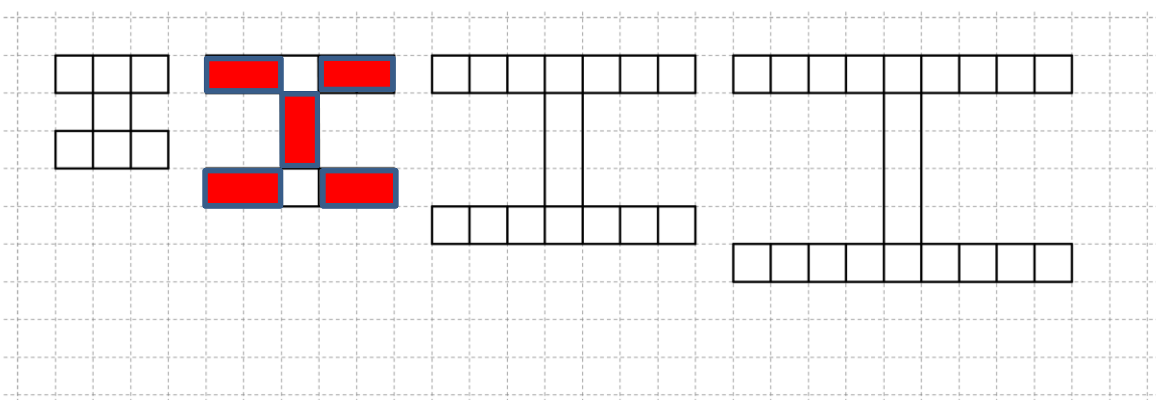


Size No.

Ask students to show how they see the change of 5 in each of figures. Then, ask a student to present a recursive equation and connect it to the figures. The recursive formula is:

$$f(1) = 7, f(n) = f(n-1) + 5$$

Ask a student to show how they used the diagram to find an explicit formula. A possible explanation is, "I noticed that for n =2 there were 5 groups of 2 blocks (shown in red) and then 2 more blocks left over. When I tried it on the other sizes it worked the same way so I decided the equation is: $f(n) = 5n + 2$."



At this point, ask students to work on the second part of the task with the "blockier" logos.

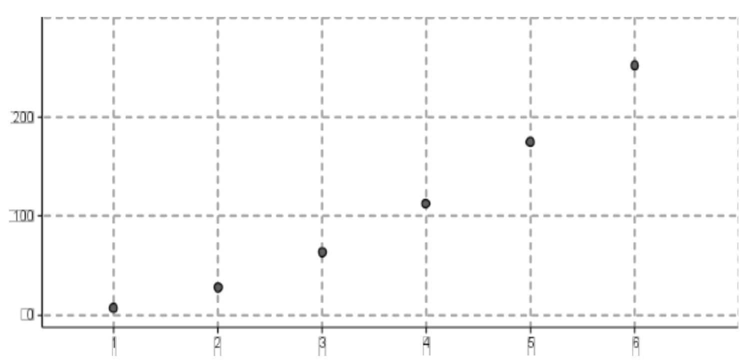
Explore (Small Group):
As you monitor student work, ensure that students are able to correctly draw the next figure. Listen for how they are using the diagram to think about how the figures are changing. Encourage students to use all the representations including getting both an explicit and recursive equation if time permits.

Discuss (Whole Group):
Part 2. Begin the discussion much like Part 1. Have students show a table and graph, making connections to the figures. Direct students to look at the first differences in the table and how they see the growing differences on the graph. Much as in the discussion in "Something to Talk About", help students to see that the first differences are linear, making this a quadratic function.

Size No.	Number of Squares
1	7
2	28
3	63
4	112
5	175
6	252
n	

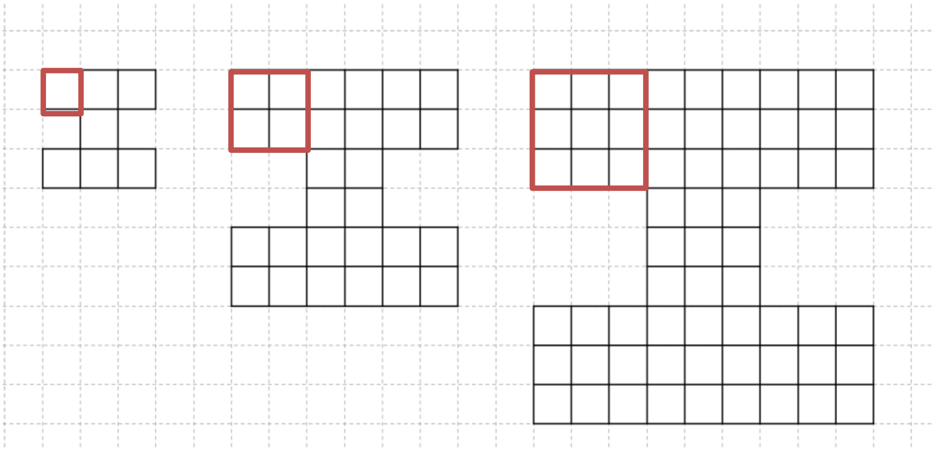
First Difference	Second Difference
21	
35	14
49	14
63	14
77	14

Number of squares

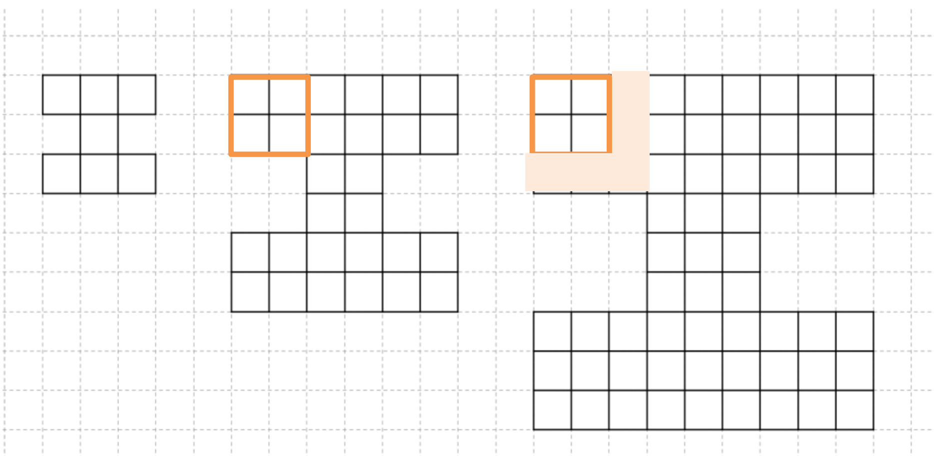


Size number

Ask students to present an explicit equation and to show how they used the diagram to get the equation. One possible way to think about it is show below, with a student saying, "I noticed that each time there are seven large squares. For n = 2, the squares are made up of 4 smaller squares, for n = 3, the squares are made up of 9 smaller squares, and so on, giving the equation: $f(n) = 7n^2$.



Ask students to share their thinking about a recursive equation. They have used the diagram a similar way, noticing that each time the big squares are wrapped with an "L" shape that adds $2n-1$ squares each time as shown below:



This thinking yields the equation:

$$f(1) = 7, f(n) = f(n-1) + 7(2n-1)$$

Ask students how they see the rate of change in the table showing up in the recursive equation. They may notice that the part added on to the previous term is the change, which for a quadratic function will be a linear expression.

End the discussion with a comparison of linear and quadratic functions. Lead the class in a discussion to complete a table such as:

	Linear	**Exponential**
Rate of change	Constant	Linear
Graph	Line	Parabola
Equation	Highest powered term is x	Highest powered term is x^2 term

Aligned Ready, Set, Go: ***Quadratic Functions 1.2***

Ready, Set, Go!

Ready

Topic: Adding and multiplying binomials

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Simplify the following expressions

1a. $(2x + 7) + (5x + 3)$ b. $(2x + 7)(5x + 3)$

2a. $(6x - 1) + (x - 10)$ b. $(6x - 1)(x - 10)$

3a. $(8x + 3) + (3x - 4)$ b. $(8x + 3)(3x - 4)$

4a. $(-5x + 2) + (7x - 13)$ b. $(-5x + 2)(7x - 13)$

5a. $(12x + 3) + (-4x + 3)$ b. $(12x + 3)(-4x + 3)$

6. $(x + 5)(x - 5)$

7. Compare your answers in 1 – 5 *part a* to your answers in #1 - #5 *part b* respectively. Look for a pattern in the answers. How are they different?

8. The answer to #6 is a different "shape" than the other *part b* answers, even though you were still multiplying. Explain how it is different from the other products. Try to explain *why* it is different. Think of 2 more examples of multiplication of two binomials that would do the same thing as #6.

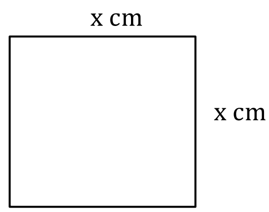
9. Try adding the two binomials in #6. $(x + 5) + (x - 5) =$_______________ Is this answer a different "shape" than the other *part a* answers? Explain.

Set

Topic: Comparing area and perimeter

Calculate the *perimeter* and the *area* of the figures below. (You answers will contain a variable.)

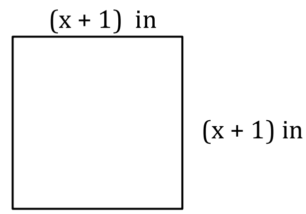
10.



a. Perimeter: ____________________

b. Area: _________________________

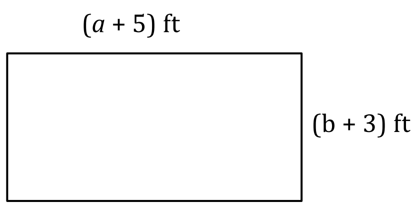
11.



a. Perimeter: ____________________

b. Area: _________________________

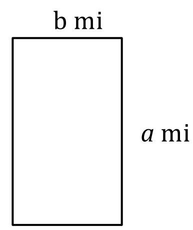
12.



a. Perimeter: ____________________

b. Area: _________________________

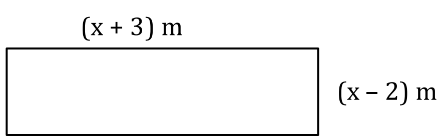
13.



a. Perimeter: ____________________

b. Area: _________________________

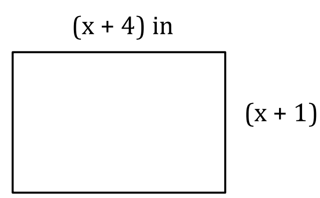
14.



a. Perimeter: ____________________

b. Area: _________________________

15.



a. Perimeter: ____________________

b. Area: _________________________

16. Compare the perimeter to the area in each of problems (10-15).

a. What do the perimeters and areas have in common?

b. In what way are the numbers and units in the perimeters and areas different?

Go

Topic: Greatest Common Factor (GCF)

Find the GCF of the given numbers.

17. $15abc^2$ and $25a^3bc$

18. $12x^5y$ and $32x^6y$

19. $17pqr$ and $51pqr^3$

20. $7x^2$ and $21x$

21. $6x^2$, $18x$, and -12

22. $4x^2$ and $9x$

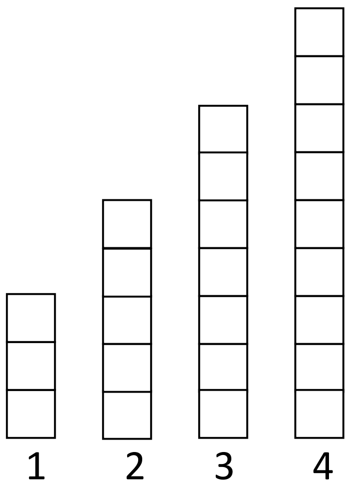
23. $11x^2y^2$, $33x^2y$, and $3xy^2$

24. $16a^2b$, $24ab$, and $16b$

25. $49s^2t^2$ and $36s^2t^2$

1.3 Scott's Macho March
A Solidify Understanding Task

After looking in the mirror and feeling flabby, Scott decided that he really needs to get in shape. He joined a gym and added push-ups to his daily exercise routine. He started keeping track of the number of push-ups he completed each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott was certain he can continue this pattern of increasing the number of push-ups for at least a few months.



1. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

Scott's gym is sponsoring a "Macho March" promotion. The goal of "Macho March" is to raise money for charity by doing push-ups. Scott has decided to participate and has sponsors that will donate money to the charity if he can do a total of at least 500 push-ups, and they will donate an additional $10 for every 100 push-ups he can do beyond that.

2. Estimate the total number of push-ups that Scott will do in a month if he continues to increase the number of push-ups he does each day in the pattern shown above.

3. How many push-ups will Scott have done after a week?

4. Model the total number of push-ups that Scott has completed on any given day during "Macho March". Include both recursive and explicit equations.

5. Will Scott meet his goal and earn the donation for the charity? Will he get a bonus? If so, how much? Explain.

Scott's Macho March – Teacher Notes

Purpose: The purpose of this task is to solidify student understanding of quadratic functions by giving another opportunity to create a quadratic model for a context. This task introduces the idea that quadratic functions are models for the sum of a linear function, which obviously creates a linear rate of change. Again, students have the opportunity to use algebraic, numeric, and graphical representations to model a story context with a visual model.

Standards Focus:
F-BF: Build a function that models a relationship between to quantities.
1: Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models
Construct and compare linear, quadratic and exponential models and solve problems.

A-CED: Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

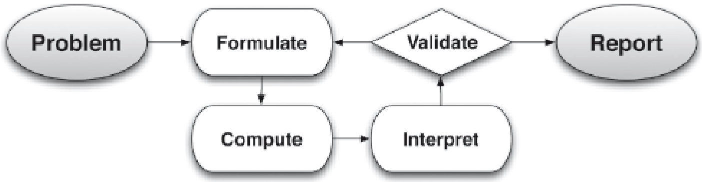
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF: Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

This task also follows the structure suggested in the Modeling standard:



Launch (Whole Class):
Begin the task by clarifying the context to be sure that students understand that in the first part of the task they are looking at the number of pushups that Scott does each day and in the second part of the task, they are looking at the sum of the number of pushups that Scott has completed on a particular day. Question 1 may be familiar for students that did the "Scott's Workout" task in the previous year.

Explore (Small Group): Monitor student thinking as they work. Since students should be very familiar with representations for linear functions, don't allow any group to spend too much time on question #1. Encourage students to use tables, graphs, and recursive and explicit equations as they work on the task. Listen to students and identify different groups to present and explain their work on one representation each. If students are having difficulty writing the equation, ask them to be sure that they have the other representations first. Like the previous tasks, it will be helpful to use the visual model to develop the explicit equation for the quadratic.

Discuss (Whole Class): When the various groups are prepared to present, start the discussion with a table for problem #1. Be sure that the columns of the table are labeled. After students have presented their table, ask students to identify the difference between consecutive terms and the type of function (linear). Have students present both an explicit and recursive equation and to connect their equations to the geometric representation and the table. Ask students to identify the domain and range of the function.

n Days	$f(n)$ Push-ups
1	3
2	5
3	7
4	9
5	11
…	…
n	$3+2(n-1)$

> 2
> 2
> 2

Recursive equation: $f(1)=3, f(n)=f(n-1)+2$

Explicit equation: $f(n)=3+2(n-1)$

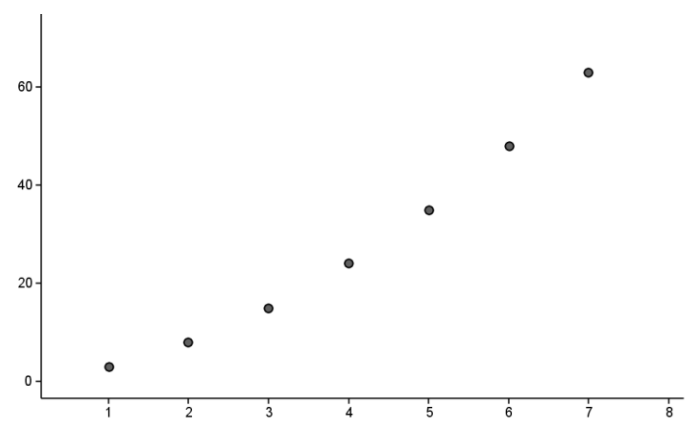
Turn the discussion to question #4. Extend the table presented earlier to include the sum of the pushups.

n Days	$f(n)$ Push-ups each day	$M(n)$ Total number of pushups in the month	*First Difference in $M(n)$*	*Second Difference in $M(n)$*
1	3	3		
2	5	8		
3	7	15		
4	9	24		
5	11	35		
…	…			
n	$3+2(n-1)$			

Begin to complete the first difference table and ask students what they notice. They should see that the first difference is the same as the number of pushups each day. Ask students why this turns out to be the case. At this point, identify that the function is quadratic, based on the linear rate of change.

Ask students to present a graph.

Total number of pushups



Number of days

Depending on the scale selected, their graphs may look nearly linear. Connect the graph to the type of growth shown in the table to explain why the graph isn't really linear.

Turn the discussion to the recursive equation. Ask the class to be prepared to connect their equations with the visual model in the problem. Students that have noticed that each figure is composed of the previous figure plus that day's pushups will write an equation similar to:

$$M(1)=3, M(n)=M(n-1)+(2n+1)$$

Compare the equation for the linear and quadratic function. Ask students to identify similarities and differences. The important thing to notice is that the change in the function is the expression added to the end of the recursive formula. If the change is a single number, as in the linear function, that shows a constant rate of change. If the change is a linear expression, then the function is quadratic.

Have students present how they thought about the visual representation to write an explicit formula. Students may use a strategy similar to “Something to Talk About” by copying and reflecting the figure on top of itself to form a rectangle. In this case, they will get an equation that is: $M(n) = \frac{n(2n+1)+3}{2}$, with the numerator of the fraction representing the area of the rectangle and then dividing by 2 to account for the fact that the actual figure is only half of the area of the rectangle. Other students may have used other strategies to find an explicit equation, but they should all simplify to be $M(n) = n(n+2)$. Ask students for the domain and range.

Conclude the discussion with a summary of what the class knows about quadratic functions thus far. The discussion should include features such as:

- The first difference in linear and the second difference is constant.
- They are models for the sum of the terms in a linear function.
- They model situations with change in 2 dimensions (like area)
- The equations are the product of 2 linear factors.
- The equations have a squared term.
- The graphs curve. (At this point, students have not experienced a full parabola graph because of the limited domains.)

Aligned Ready, Set, Go Homework: *Quadratic Functions 1.3*

Name: Quadratic Functions | 1.3

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Ready, Set, Go!

Ready

Topic: Fundamental Theorem of Arithmetic

The prime factorization of a number is given. Multiply each number to find the whole number that each factorization represents.

1. $2^4 \times 3^1 \times 5^2$
2. $3^4 \times 5^2 \times 7^2$
3. $5^2 \times 11^2 \times 13^1$

The following problems are factorizations of numerical expressions called quadratics. Given the factors, multiply to find the quadratic expression. Add the like terms. Write the x^2 term first, the x-term second, and the constant term last. (*Example:* $ax^2 + bx + c$.)

4. $(x+5)(x-7)$
5. $(x+8)(x+3)$
6. $2(x-9)(x-4)$
7. $3(x+1)(x-4)$
8. $2(3x-5)(x-1)$
9. $2(5x-7)(3x+1)$

Set

Use first and second differences to identify the pattern in the tables as *linear, quadratic*, or *neither*. Write the recursive equation for the patterns that are linear or quadratic.

10.

x	y
-3	-23
-2	-17
-1	-11
0	-5
1	1
2	7
3	13

a. Pattern:
b. Recursive equation:

11.

x	y
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

a. Pattern:
b. Recursive equation:

12.

x	y
-3	-15
-2	-10
-1	-5
0	0
1	5
2	10
3	15

a. Pattern:
b. Recursive equation:

13.

x	y
-3	24
-2	22
-1	20
0	18
1	16
2	14
3	12

a. Pattern:
b. Recursive equation:

14.

x	y
-3	48
-2	22
-1	6
0	0
1	4
2	18
3	42

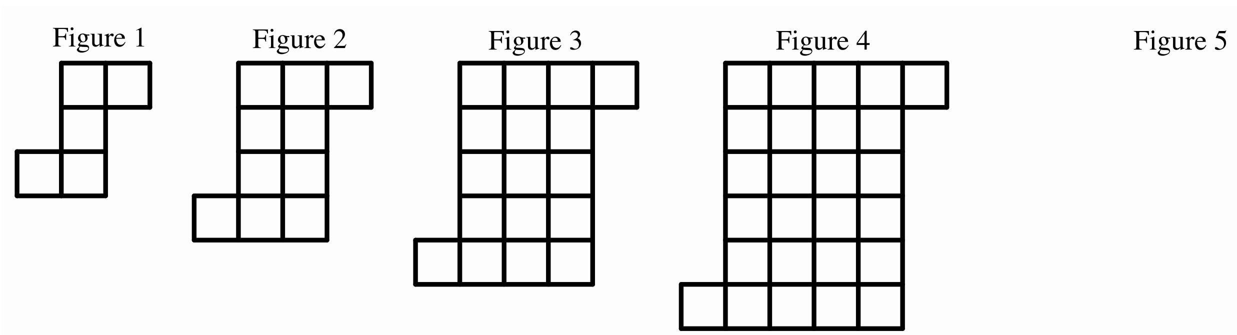
a. Pattern:
b. Recursive equation:

15.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

a. Pattern:
b. Recursive equation:

16.



a. Draw figure 5.
b. Predict the number of squares in figure 30. Show what you did to get your prediction.

Go

Topic: Interpreting recursive equations to write a sequence

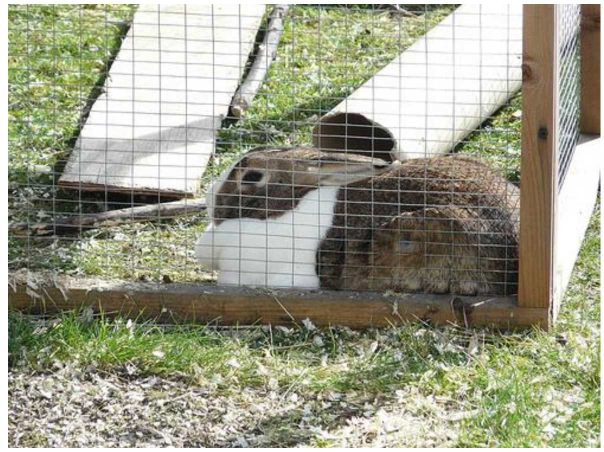
Write the first five terms of the sequence.

17. $f(0) = -5; f(n+1) = f(n) + 8$

18. $f(0) = 24; f(n+1) = f(n) - 5$

19. $f(0) = 25; f(n+1) = 3f(n)$

20. $f(0) = 6; f(n+1) = 2f(n)$



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1.4 Rabbit Run

A Solidify Understanding Task

Misha has a new rabbit that she named "Wascal". She wants to build Wascal a pen so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.

1. If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?

2. If Misha wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

3. Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.

4. What kind of function is this? Why?

5. How does this function compare to the second type of block **I** logos in *I Rule*?

Rabbit Run – Teacher Notes
A Solidify Understanding Task

Purpose: The purpose of this task is to solidify and extend student thinking about quadratic functions to include those with a maximum point. Students will use the graph of the function to discuss the domain and range of a continuous quadratic function in addition to identifying the maximum value and finding the intervals on which the function is increasing and decreasing.

Core Standards Focus:

F.BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms factors, and coefficients.

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

Related Standards: F.IF.4

Note: Graphing technology (either calculators or computer software such as Geogebra) would be useful for this task.

Launch (Whole Class):
Begin the task by familiarizing students with the context of building a rectangular rabbit pen that can be surrounded by 72 feet of fencing. Ask students to think of some possible dimensions for the pen. Be sure that they are thinking about perimeters, not areas, so that they know that a rectangle with dimensions of 8 x 9 won't work. After they have successfully found one or two possible dimensions for the pen, ask if all the different rectangles will have the same area. Compare the areas of a couple of the rectangles that they have found and tell students that the one of their jobs in this task will be to find the rectangle with the most area because that will give the rabbit the most room to move around.

Explore (Small Group):
As students are working, circulate to see that they are trying various values for the dimensions of the rectangle that have a perimeter of 72. It may help them to be systematic and organized in thinking about the possibilities for the dimensions so they can use the patterns they observe to find the relationship between the length and width and write the equation for the area.

Discuss (Whole Class):
Start the discussion with a table such as:

Length x	Width	Area $A(x)$
1	35	35
2	34	68
3	33	99
4	32	128
5	31	155
6	30	180
x	$36 - x$	$x(36 - x)$

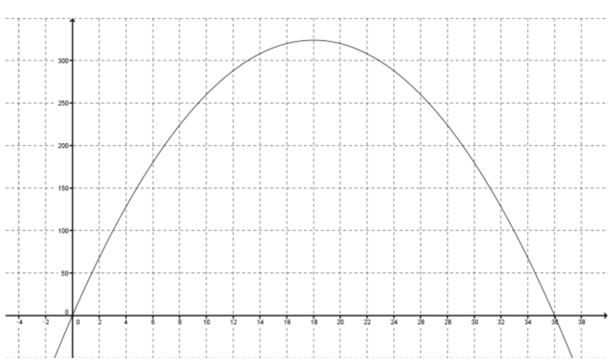
At this point, it may be useful to display the rest of the table using technology so that students can see a possible maximum value in the table. Students should notice that once they reach a length of 18, they begin to get the same rectangles, but with the length and width switched (different orientation).

Discuss the domain and range of the *A(x).* This is the first continuous context, which students may not immediately recognize. They should be able to write the domain in interval notation or set builder notation. Ask if this is a quadratic function and how they know.

Ask a student to present an equation for the function: $A(x) = x(36 - x)$. Ask students how this equation is connected to the table and how it fits what they know about quadratic functions. Ask what is different about this equation compared to other quadratics that they have seen. Since it is important for them to recognize that the x^2 term is negative, you may ask them to distribute the x to see another form of the equation.

Turn the discussion to the graph (using technology). Ask students about the features of the graph that they notice. They should come up with a list like:

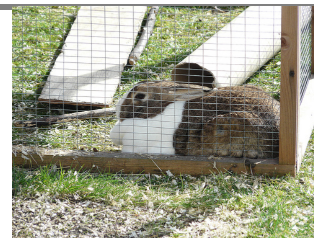
- The function is continuous.
- The function increases in the interval (0,18) and decreases in (18,36).
- The function has a maximum at (18, 324).



Ask how this graph compares with the graphs of other quadratic functions that they have seen. Since this is the first full parabola they have seen, you will need to tell them that because of the limited domains that were used in previous contexts, they have only seen part of the parabolas that are representative of quadratic functions. This one opens down and has a maximum, graphs of quadratic functions can also open upward and have a minimum.

Aligned Ready, Set, Go: ***Quadratic Functions 1.4***

Name: Quadratic Functions | 1.4



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Ready, Set, Go!

Ready

Topic: applying the slope formula

Calculate the slope of the line between the given points. Use your answer to indicate which line is the steepest.

1. A (-3, 7) B (-5, 17)

2. H (12, -37) K (4, -3)

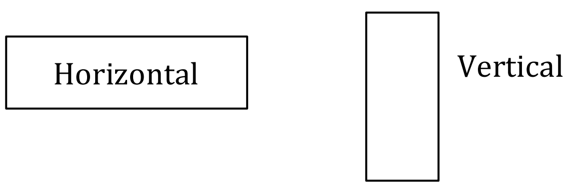
3. P (-11, -24) Q (21, 40)

4. R (55, -75) W(-15, -40)

Set

Topic: Investigating perimeters and areas

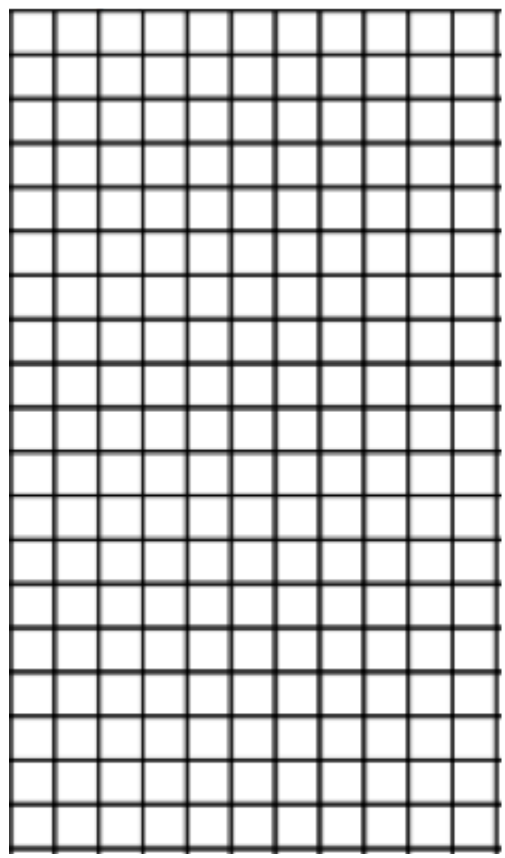
Adam and his brother are responsible for feeding their horses. In the spring and summer the horses graze in an unfenced pasture. The brothers have erected a portable fence to corral the horses in a grazing area. Each day the horses eat all of the grass inside the fence. Then the boys move it to a new area where the grass is long and green. The porta-fence consists of 16 separate pieces of fencing each 10 feet long. The brothers have always arranged the fence in a long rectangle with one length of fence on each end and 7 pieces on each side making the grazing area 700 sq. ft. Adam has learned in his math class that a rectangle can have the same perimeter but different areas. He is beginning to wonder if he can make his daily job easier by rearranging the fence so that the horses have a bigger grazing area. He begins by making a table of values. He lists all of the possible areas of a rectangle with a perimeter of 160 ft., while keeping in mind that he is restricted by the lengths of his fencing units. He realizes that a rectangle that is oriented horizontally in the pasture will cover a different section of grass than one that is oriented vertically. So he is considering the two rectangles as different in his table. Use this information to answer questions 5 – 9 on the next page.



5. Fill in Adam's table with all of the arrangements for the fence. (The first one is done for you.)

	Length in "fencing" units	Width in "fencing" units	Length in ft.	Width in ft.	Perimeter (ft)	Area $(ft)^2$
	1 unit	7 units	10 ft	70 ft	160 ft	700 ft^2
a.	2 units				160 ft	
b.	3 units				160 ft	
c.	4 units				160 ft	
d.	5 units				160 ft	
e.	6 units				160 ft	
f.	7 units				160 ft	

6. Discuss Adam's findings. Explain how you would rearrange the sections of the porta-fence so that Adam will be able to do less work.



7. Make a graph of Adam's investigation. Let length be the independent variable and area be the dependent variable. Label the scale.

8. What is the shape of your graph?

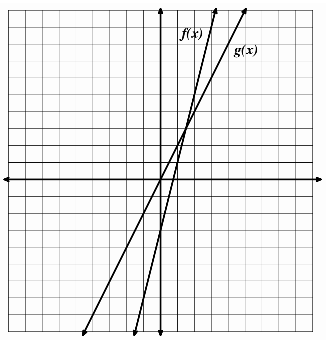
9. Explain what makes this function be a quadratic.

Go

Topic: Comparing linear and exponential rates of change.

Indicate which function is changing faster.

10.



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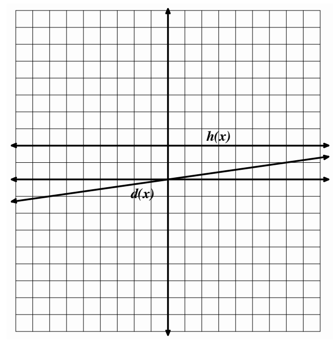

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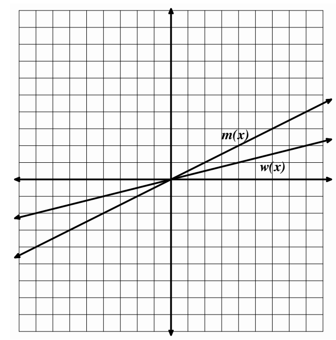

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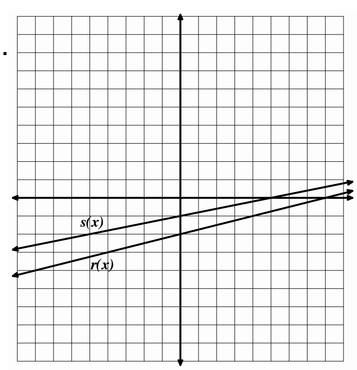

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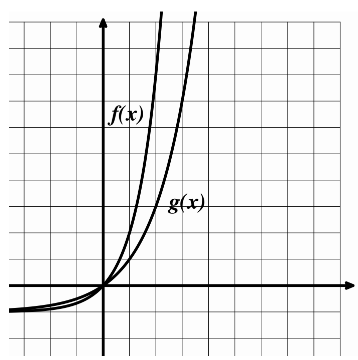

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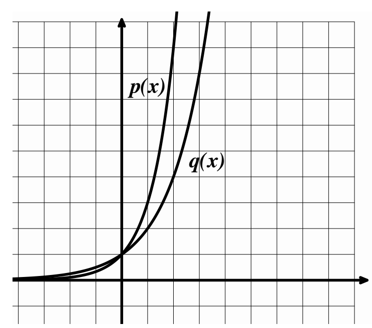
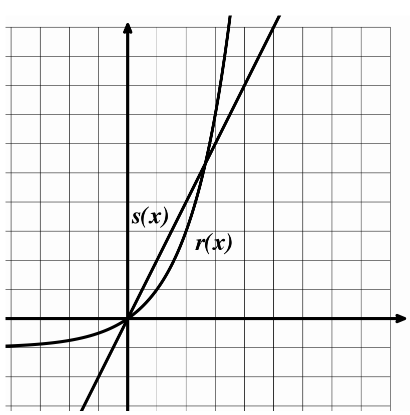




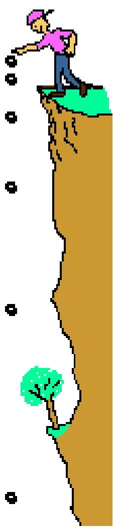
16a. Examine the graph at the left from 0 to 1. Which graph do you think is growing faster?

b. Now look at the graph from 2 to 3. Which graph is growing faster in this interval?

1.5 Look Out Below!
A Solidify Understanding Task

What happens when you drop a ball? It falls to the ground.

That question sounds as silly as "Why did the chicken cross the road?" (To get to the other side.) Seriously, it took scientists until the sixteenth and seventeenth centuries to fully understand the physics and mathematics of falling bodies. We now know that gravity acts on the object that is falling in a way that causes it to accelerate as it falls. That means that if there is no air resistance, it falls faster and faster, covering more distance in each second as it falls. If you could slow the process down so that you could see the position of the object as it falls, it would look something like the picture below.



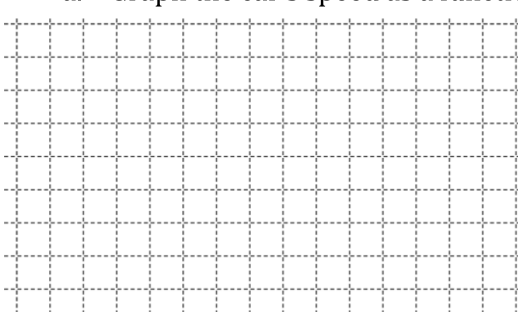
To be more precise, objects fall at a constant rate of acceleration on earth of about 32 feet per second per second. . The simplest case occurs when the object starts from rest, that is, when its speed is zero when $t = 0$. In this case, the object's instantaneous speed after 1 second is 32 feet per second; after 2 seconds, its instantaneous speed is $2(32) = 64$ feet per second; and so on. Other planets and moons each have a different rate of acceleration, but the basic principal remains the same. If the acceleration on a particular planet is g, then the object's instantaneous speed after 1 second is g units per second; after 2 seconds, its instantaneous speed is $2g$ units per second; and so on.

In this task, we will explore the mathematics of falling objects, but before we start thinking about falling objects we need to begin with a little work on the relationship between speed, time, and distance.

Part 1: Average speed and distance travelled

Consider a car that is traveling at a steady rate of 30 feet per second. At time $t = 0$, the driver of the car starts to increase his speed (accelerate) in order to pass a slow moving vehicle. The speed increases at a constant rate so that 20 seconds later, the car is traveling at a rate of 40 feet per second.

a. Graph the car's speed as a function of time for this 20-second time interval.



b. Calculate the average speed of the car for this 20-second time interval.

c. Find the total distance the car travels during this 20-second time interval.

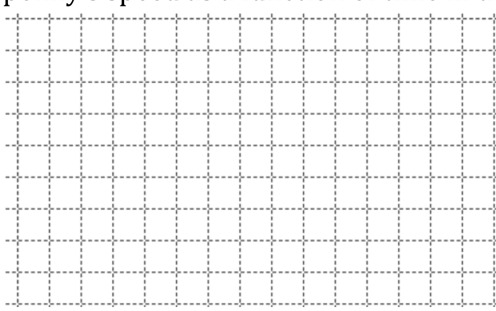
d. Explain how to use area to find the total distance the car travels during this 20-second interval.

This problem illustrates an important principle: ***If an object is traveling with constant acceleration, then its average speed over any time interval is the average of its beginning speed and its final speed during that time interval.***

Let's apply this idea to a penny that is dropped (initial speed is 0 when $t = 0$) from the top of the Empire State Building.

1. What will its speed be after 1 second?

2. Graph the penny's speed as a function of time in the 1 second interval.



3. What is the average speed of the penny in the 1-second interval?

4. What is the total distance that the penny fell in the 1-second interval?

Part 2: Falling, Falling, Falling

When the astronauts went to the moon, they performed Galileo's experiment to test the idea that any two objects, no matter their mass, will fall at the same rate if there is no air resistance (like on the moon). Because the moon doesn't have air resistance, we are going to pretend like we're the astronauts dropping moon rocks and thinking about what happens. On the surface of the moon the constant acceleration increases the speed of a falling object by 6 feet per second each second. That is, if an object is dropped near the surface of the moon (e.g., its initial speed is zero when $t = 0$), then the object's instantaneous speed after 1 second is 6 feet per second, after 2 seconds, its instantaneous speed is 12 feet per second, and so on.

1. Using this information, create a table for the speed of an object that is dropped from a height of 200 feet above the surface of the moon as a function of the elapsed time (in seconds) since it was dropped.

2. Add another column to your table to keep track of the distance the object has fallen as a function of elapsed time. Explain how you are finding these distances.

3. Approximately how long will it take for the object to hit the surface of the moon?

4. Write an equation for the distance the object has fallen as a function of elapsed time *t*.

5. Write an equation for the height of the object above the surface of the moon as a function of elapsed time *t*.

6. Suppose the object was not dropped, but was thrown downward from a height of 250 feet above the surface of the moon with an initial speed of 10 feet per second. Rewrite your equation for the height of the object above the surface of the moon as a function of elapsed time *t* to take into account this initial speed.

7. How is your work on these *falling objects problems* related to your work with the *rabbit runs?*

8. Why are the “distance fallen” and “height above the ground” functions quadratic?

Look Out Below – Teacher Notes
A Solidify Understanding Task

Special Note to Teachers: Graphing calculators are useful for this task.

Purpose: The purpose of this task is to solidify and extend students understanding to a physical context. In the task, students develop a formula for the distance that an object falls in a given time *t*. The tasks begins with the idea that the distance travelled in an interval where an object is moving with constant acceleration can be obtained by multiplying the average velocity by the time, making an argument for this based upon the area under the curve of the graph. Students use this idea to construct a table of the distance an object falls in a given interval of time, based upon calculating the average velocity in the interval. This is another example of a quadratic function being the product of two linear factors (the time and the average velocity). The task also asks students to also create an equation to model the height of the object at a given time after it is dropped. This leads to an opportunity to contrast quadratic functions with graphs that open upward and downward. An extension of the ideas is offered at the end of task when students are asked to use the same process as earlier to write an equation for an object that is thrown with an initial velocity.

Core Standards Focus:

F.BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

F.IF Interpret functions that arise in applications in terms of a context.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Launch (Whole Class):
Begin the task by helping students with vocabulary like acceleration, speed, initial speed, and at rest. The use of these words in the task may be unfamiliar and should be front-loaded. Have students draw the graph for #1. They will probably assume that the graph is linear and connect the points at the beginning and end of the interval with a line, but have them explain why this is

justifiable. Have them calculate the average speed in the interval and mark that point on the graph. In this case it is a simple average of the endpoints (which is not true if the acceleration is not constant). Walk through the rest of the problem together, highlighting the idea that the distance travelled is the average speed multiplied by the time. Use the area under the curve to justify this conclusion.

Have students work on the penny problems, noticing that this situation works the same way as the car. Make sure that students have made sense of what they are doing because this is the same process they will be using to build an equation later in the task.

Explore (Small Group):
After a thorough launch students should be able to enter the task. They may not think to construct a table initially, but will find that it helps them to organize their work. By the time they are finished, these may be more complicated tables than they have made in the past. They should end up with columns for time, speed at time t, average speed in the interval, distance travelled in the interval. It will also be helpful for them to be careful with their labels and to differentiate between the speed at time *t* and the average speed in the interval.

Discuss (Whole Class):
Begin the discussion by having different students share each column of the table, ending with a table that looks like this:

Time (t)	Speed at time t	Average speed in the interval from t_{n-1} to t_n	Distance travelled in the interval from t_{n-1} to t_n
0	0	0	0
1	6	3	3
2	12	6	12
3	18	9	27
4	24	12	48
5	30	15	75
6	36	18	108

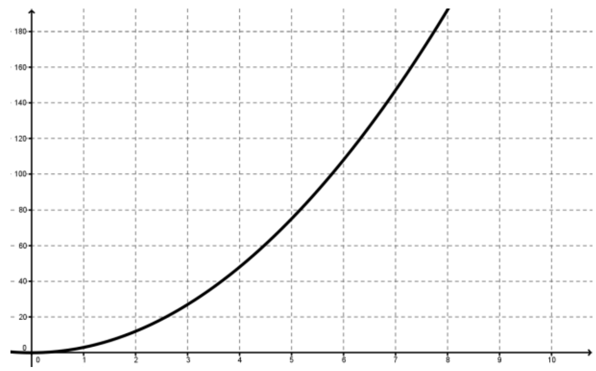
Ask students for observations about the table. They should notice that the object is falling faster and faster (speed increasing each interval) and that it is falling further in each interval. Ask students what kind of function this seems to be and then complete the first difference to see that it is linear, making the function quadratic.

Ask a student to write an explicit equation, which shouldn't be too challenging for most students. The average speed in the interval, $\frac{6t}{2}$ is multiplied by the time t to get the distance travelled, thus:

$d(t) = \frac{6t}{2} \cdot t = 3t^2$ (In a physics class they would use the formula $d = \frac{1}{2}at^2$.)

This equation should confirm that the function is a quadratic. At this point, either ask students to predict what the graph will look like or have a student that has created a graph present it.

Distance travelled (in feet)



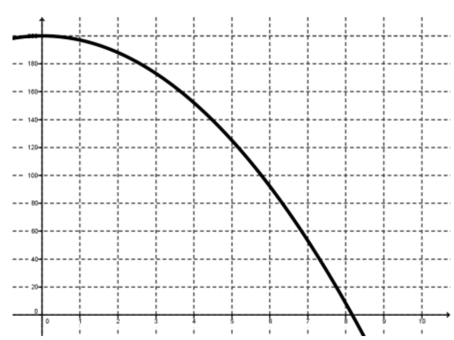
Time (in seconds)

Students often think that this graph is supposed to be the path of the object, so it will be useful to ask several questions to press on their interpretation of the graph. Ask why the graph is going up. Ask them how they could use the graph to predict about when the object hits the ground.

Next, ask a student to present his/her thinking about the equation for the height of the object above the ground. They should be able to explain that they took the distance travelled from the earlier equation and subtracted it from 200 ft, the height at which the ball was dropped, giving the equation: $h(t) = 200 - 3t^2$.

Discuss the graph of *h(t)* and ask where this graph shows the object hitting the ground. Why is this so different from the graph of *d(t)*?

Height above the ground (in feet)



Time (in seconds)

Conclude the lesson by discussing question #8, comparing and contrasting the d(t) and h(t) and why they are both quadratic functions.

Aligned Ready, Set, Go: *Quadratic Functions 1.5*

Name: Quadratic Functions | 1.5

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Ready, Set, Go!

Ready

Topic: Evaluating exponential functions

Find the indicated value of the function for each value of *x*. $x = \{-2, -1, 0, 1, 2, 3\}$

1. $f(x) = 3^x$

2. $g(x) = 5^x$

3. $h(x) = 10^x$

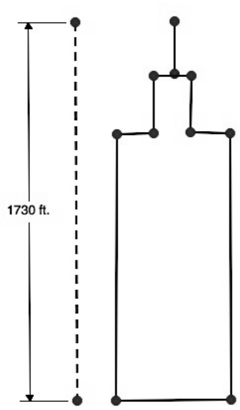
4. $k(x) = \left(\frac{1}{2}\right)^x$

5. $m(x) = \left(\frac{1}{3}\right)^x$

Set

The Sears Tower in Chicago is 1730 feet tall. If a penny were let go from the top of the tower, the position above the ground *s(t)* of the penny at any given time *t* would be $s(t) = -16t^2 + 1730$.

6. Fill in the missing positions in the chart. Then add to get the distance fallen.



Distance from ground

a. _______ 1 sec
b. _______ 2 sec
c. _______ 3 sec
d. _______ 4 sec
e. _______ 5 sec
f. _______ 6 sec
g. _______ 7 sec
h. _______ 8 sec
i. _______ 9 sec
j. _______ 10 sec

7. How far above the ground is the penny when 7 seconds have passed?

8. How far has it fallen when 7 seconds have passed?

9. Has the penny hit the ground at 10 seconds? Justify your answer.

The average rate of change of an object is given by the formula $r = \frac{d}{t}$ *, where r is the rate of change, d is the distance traveled, and t is the time it took to travel the given distance. We often use some form of this formula when we are trying to calculate how long a trip may take.*

10. If our destination is 225 miles away and we can average 75 mph, then we should arrive in 3 hours. $\left[\frac{225\ mile}{75\ mph} = 3\ hours\right]$ In this case you would be rearranging the formula so that $t = \frac{d}{r}$.
    However, if your mother finds out that the trip only took 2 ½ hours, she will be upset. Use the rate formula to explain why.

11. How is the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ like the formula for rate?

For the following questions, refer back to the penny spoken about in questions 6 – 9.

12. Find the average rate of change for the penny on the interval [0, 1] seconds.

13. Find the average rate of change for the penny on the interval [6, 7] seconds.

14. Explain why the penny's average speed is different from 0 to 1 second than between the 6th and 7th seconds.

15. What is the average speed of the penny from [0,10] seconds?

16. What is the average speed of the penny from [9,10] seconds?

17. Find the first differences on the table where you recorded the position of the penny at each second. What do these differences tell you?

18. Take the difference of the first differences. (This would be called the 2nd difference.) Did your answer surprise you? What do you think this means?

Go

Topic: Evaluating functions

19. *Find* $f(9)$ *given that* $f(x) = x^2 + 10$.

20. *Find* $g(-3)$ *given that* $g(x) = x^2 + 2x + 4$.

21. *Find* $h(-11)$ *given that* $h(x) = 2x^2 + 9x - 43$.

22. *Find* $r(-1)$ *given that* $r(x) = -5x^2 - 3x + 9$.

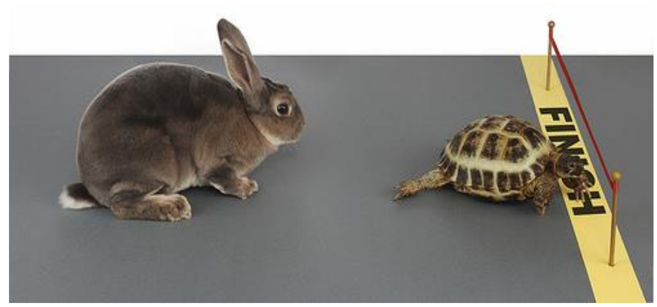
23. *Find* $s\left(\frac{1}{2}\right)$ *given that* $s(x) = x^2 + \frac{5}{4}x - \frac{1}{2}$.

24. *Find* $p(3)$ *given that* $p(x) = 5^x + 2x$.

25. *Find* $q(2)$ *given that* $q(x) = 7^x + 11x$

1.6 The Tortoise and the Hare

A Solidify Understanding Task



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In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$d = t^2$ (d in meters and t in seconds)

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d = 2^t$ (d in meters and t in seconds)

1. At what time does the hare catch up to the tortoise?

2. If the race course is very long, who wins: the tortoise or the hare? Why?

3. At what time(s) are they tied?

4. If the race course were 15 meters long who wins, the tortoise or the hare? Why?

5. Use the properties $d = 2^t$ and $d = t^2$ to explain the **speeds** of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$
[0, 2)		
[2, 4)		
[4 ,∞)		

The Tortoise and the Hare – Teacher Notes

A Solidify Understanding Task

Special Note to Teachers: Graphing technology is necessary for this task.

Purpose: The purpose of this task is to compare quadratic and exponential functions by examining tables and graphs for each. They will consider rates of change for each function type in various intervals and ultimately, see that an increasing exponential function will exceed a quadratic function.

Core Standards Focus:

F.BF.1 Write a function that describes a relationship between two quantities. *

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Focus on situations that exhibit a quadratic or exponential relationship.

F.LE: Construct and compare linear, quadratic and exponential models and solve problems.

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Launch (Whole Class):
Begin the task by simply making sure that students understand the problem situation and the functions that have been given to model the distance traveled by the tortoise and the hare. To avoid confusion, it may be useful to clarify that the hare gave the tortoise a one meter head start, which in this case means that at t = 0, the tortoise was at 1 meter. The equation $d = 2^t$ accounts for the head start because at t = 0, the equation yields 1.

The other possible source of confusion is found in the table in #5. It is asking students to describe the speed (the prompt doesn't indicate average or instantaneous in hopes that students may describe both), not simply the distance traveled in each interval.

Explore (Small Group):
Observe students as they work. The task doesn't prescribe a particular method for thinking about the behavior of each of the two functions, so tables, graphs, and equations should all be expected. It is good mathematical thinking to write an equation by setting the two functions equal to each other, but students may get stuck laboring over an algebraic solution. Encourage them to use their thinking about equality with a different representation. You may need to prompt students to change the windows on their to graphs display particular pieces of the graphs that are interesting.

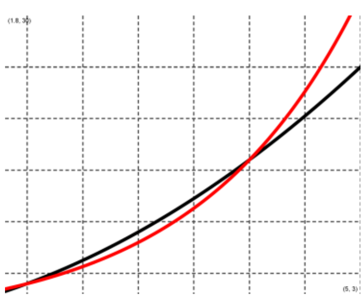
Discuss (Whole Class):
Begin the discussion of questions #1 and 3 with a student that wrote an equation that sets the two functions equal. Have the class explain why this strategy makes sense, but tell the class that this is a case where the algebraic solution to the equation is not accessible. Then ask a student to use a graph to find the intersections how this strategy relates to the equation that was written. Project a graph of the two functions on the same axes for discussion, possibly beginning with one like this, which is probably where most students started:

Distance (meters)

Time (seconds)

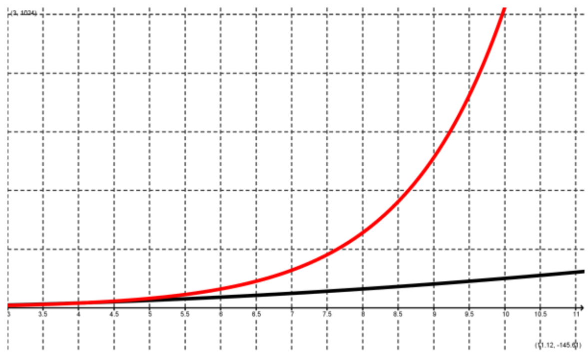
Briefly discuss the domain of the two functions in this context begins at t = 0. On the graph show, the tortoise is in red. Ask students what they notice about the graph, at what time the hare caught up with the tortoise and who looks like they will win in this view. Ask students which character was the fastest (had the greatest speed or rate of change) in the interval. Since they were both 4 meters from the starting line after 2 seconds, the hare must have been going faster. Students should notice the steeper slope of the hare's graph in this interval. Ask a student that used a table to show how to see the greater speed in the table.

Ask a student to present the two graphs in the interval from t = 2 to t = 4.



Ask students what they can say about the speed of the hare vs the speed of the tortoise in this interval. Ask them to calculate the average speed of both characters and compare that to what they described earlier. Tell them the difference between instantaneous speed at time t and the average speed in an interval. Finding the instantaneous speed can be done in Calculus (spoiler alert), but it can be estimated by finding the average speed in a small interval near the point of interest. We often use the first difference in a table as an approximation of the instantaneous rate of change, although it is really the average rate of change in the interval from one row of the table to the next.

Now ask, which character wins the race and why. Have a student show a graph similar to the one below: (Window $3 < t < 10, 8 < d < 1024$)

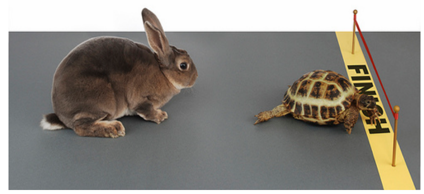


Ask students to discuss the rates of change in this interval. Why does the graph of the quadratic seem so flat compared to the exponential function? Make the point that an increasing exponential function will eventually exceed any quadratic function because the quadratic is increasing linearly, but the exponential is increasing exponentially. Confirm the idea by sharing tables for each of the two functions, looking at the first difference for each.

Aligned Ready, Set, Go: *Quadratic Functions 1.6*

Name:

Ready, Set, Go!



Ready

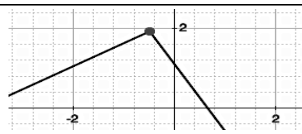
Topic: Recognizing functions

Identify which of the following representations are functions. If it is NOT a function state how you would fix it so it was.

1. D = {(4,-1) (3, -6) (2, -1) (1, 2) (0, 4) (2, 5)}

2. The number of calories you have burned since midnight at any time during the day.

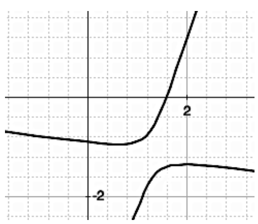
3.



4.

x	-12	-8	-6	-4
$f(x)$	25	25	25	25

5.

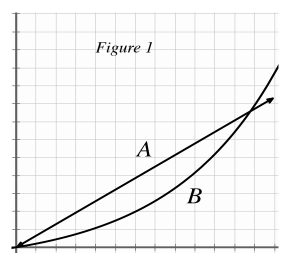


6.

Set

Topic: Comparing rates of change in linear, quadratic, and exponential functions

The graph at the right shows a time vs. distance graph of two cars traveling in the same direction along the freeway.



7. Which car has the cruise control on? How do you know?

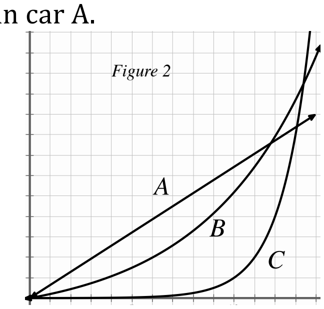
8. Which car is accelerating? How do you know?

9. Identify the interval in *figure 1* where car A seems to be going faster than car B.

10. Identify the interval in *figure 1* where car B seems to be going faster than car A.

11. What in the graph indicates the speed of the cars?

12. A third car *C* is now shown in the graph (*see figure 2*). All 3 cars have the same destination. If the destination is a distance of 12 units from the origin, which car do you predict will arrive first? Justify your answer.

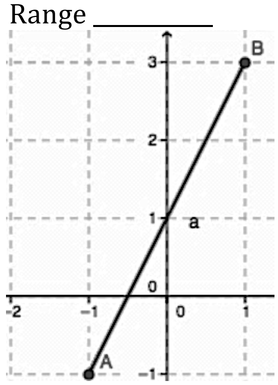


Go

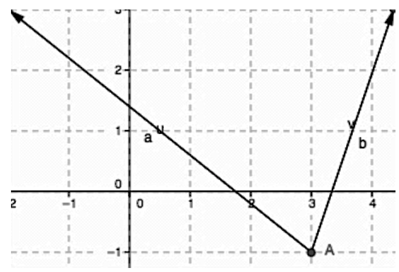
Topic: Identifying domain and range from a graph.

State the domain and range of each graph. Use interval notation where appropriate.

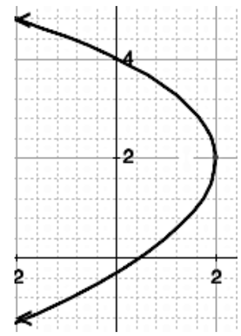
13a. Domain _________
b. Range __________



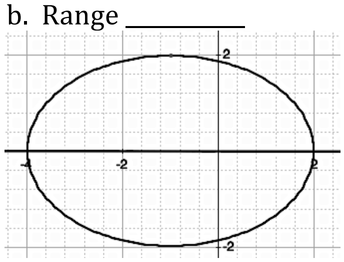
14a. Domain _________
b. Range __________



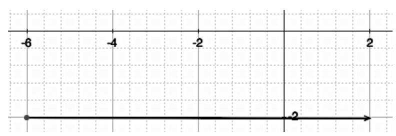
15a. Domain _________
b. Range __________



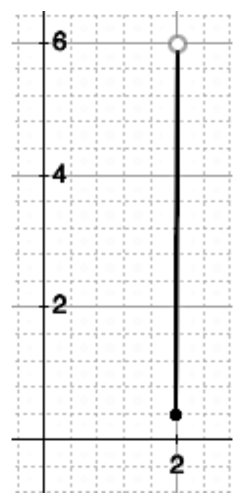
16a. Domain _________
b. Range __________



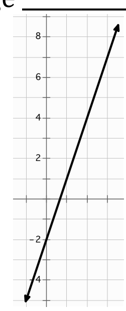
17a. Domain _________
b. Range __________



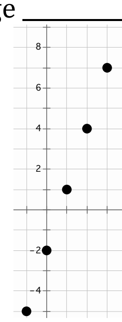
18a. Domain _________
b. Range __________



19a. Domain _________
b. Range __________



20a. Domain _________
b. Range __________



21. Are the domains of #19 and #20 the same? Explain.

1.7 How Does It Grow?

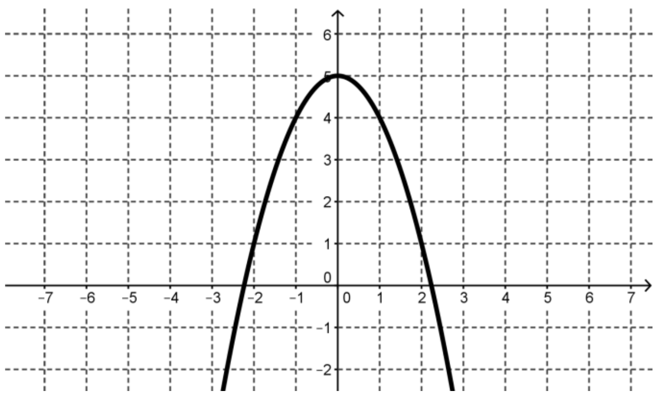
A Practice Understanding Task

For each relation given:

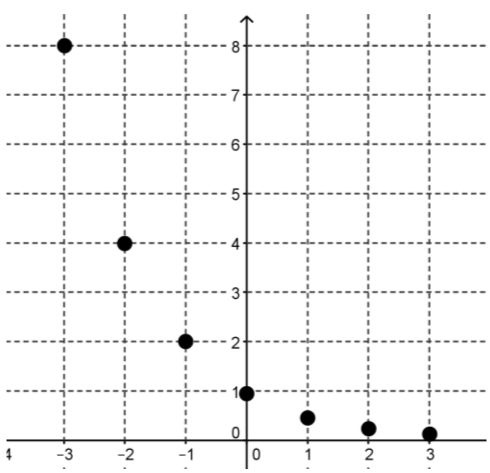
a. Identify whether or not the relation is a function;
b. Determine if the function is linear, exponential, quadratic or neither;
c. Describe the type of growth
d. Create one more representation for the relation.

1. A plumber charges a base fee of $55 for a service call plus $35 per hour for each hour worked during the service call. The relationship between the total price of the service call and the number of hours worked.

2.

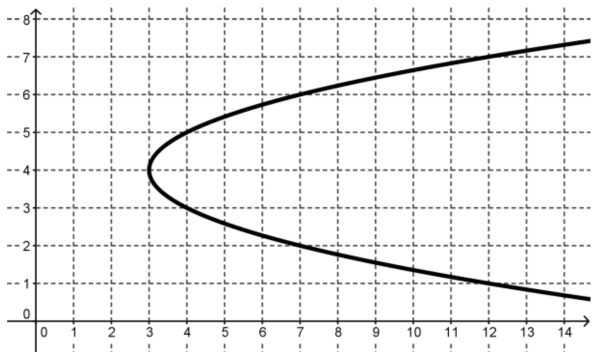


3.



4. $y = \frac{1}{3}(x-2)^2 + 4$

5.

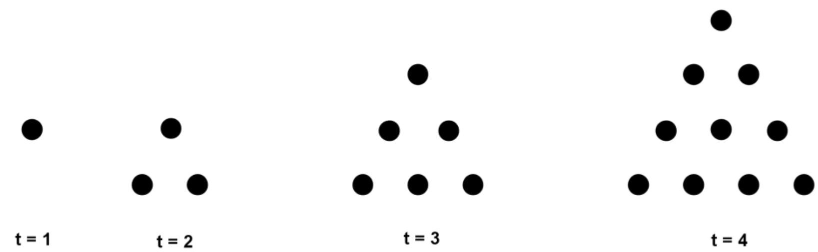


6. $y = \frac{1}{3}(x-2) + 4$

7. The relationship between the speed of a car and the distance it takes to stop when travelling at that speed.

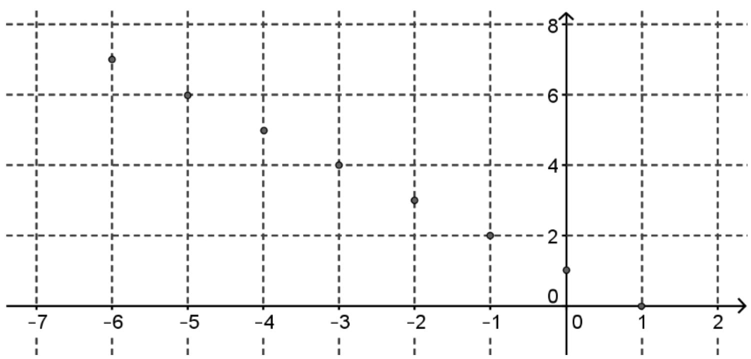
Speed (mph)	**Stopping Distance (ft)**
10	12.5
20	36.0
30	69.5
40	114.0
50	169.5
60	249.0
70	325.5

8. The relationship between the number of dots in the figure and the time, *t.*



9. The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. The relationship between the amount of caffeine in the bloodstream and the number of hours from the time the adult drinks the caffeinated beverage.

10.



11. $y = (4x + 3)(x - 6)$

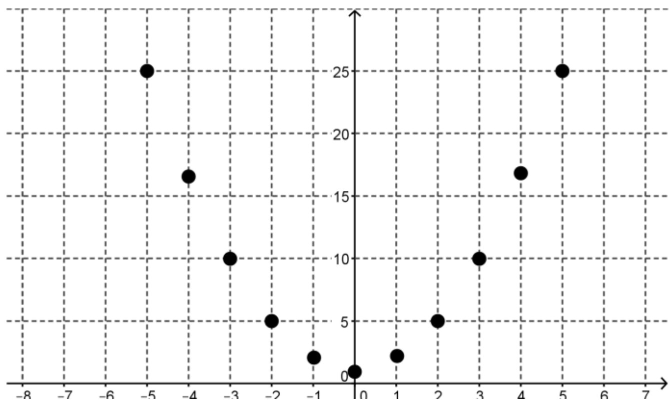
12. Mary Contrary wants to build a rectangular flower garden surrounded by a walkway 4 meters wide. The flower garden will be 6 meters longer than it is wide.

    a. The relationship between the width of the garden and the perimeter of the walkway.

    b. The relationship between the width of the garden and area of the walkway.

13. $y = \left(\frac{1}{3}\right)^{x-2} + 4$

14.



How Does It Grow? – Teacher Notes

A Practice Understanding Task

Purpose: The purpose of this task is to refine student understanding of quadratic functions by distinguishing between relationships that are quadratic, linear, exponential or neither. Examples include relationships given with tables, graphs, equations, visuals, and story context. Students are asked to draw upon their understanding of representations to determine the type of change shown and to create a second representation for the relationships given.

Core Standards Focus:

F.IF: Analyze functions using different representations.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

*Focus on situations that exhibit a quadratic or exponential relationship.

F.LE: Construct and compare linear, quadratic, and exponential models and solve problems.

Launch (Whole Class):

To activate student background knowledge, ask students to take the first 3 minutes to write down what they know about linear, quadratic, and exponential functions. Ask students to share a few of their ideas with the class without recording their answers. Go over the instructions for the task to ensure that students know what they are to do.

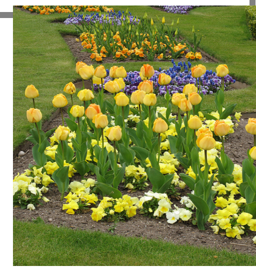
Explore (Small Group):

Monitor students as they work, looking for problems that are generating discussion, student misconceptions, or important ideas for the discussion. Watch for discussions of whether or not all the various function types can be continuous or discrete. Listen for how they deal with problem #5, which is a parabola, but not a function. (Such cases will be explored in a later module.) Since most of the quadratics that students have worked with so far have domains that begin at 0, students may not immediately that graphs like #14 are quadratic. (Graphing quadratics is a major topic in module 2). Many of the equations in this task are in unfamiliar forms which will push students to build tables or graphs to identify the type of change. The focus for this work should be on rates of change and representations.

Discuss (Whole Class):

Begin the discussion by asking which relationships they classified as quadratic functions. (Answer: 2, 4, 7, 8, 11,12b, and 14). For each of these, have a student show a second representation and talk about how they identified the type of change in their two representations. When this is complete, ask a student to identify one exponential function and share the second representation and their justification in deciding the function is exponential. (Problems 3, 9, and 13 are exponential.) Repeat the process with a linear functions. Close the class by asking students to write for 3 more minutes to add to their list of things they know about linear, quadratic, and exponential functions.

Aligned Ready, Set, Go: *Quadratic Functions 1.7*



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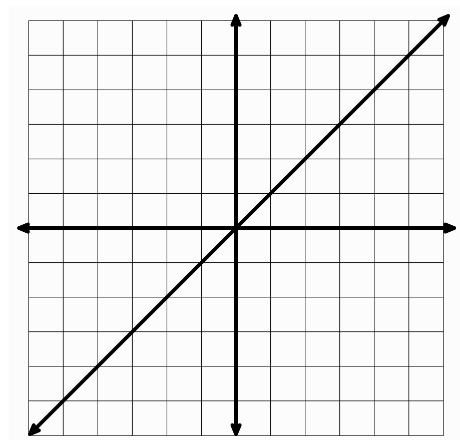
Ready, Set, Go!

Ready

Topic: transforming lines

1. Graph the following linear equations on the grid. The equation y = x has been graphed for you. For each new equation explain what the number 3 does to the graph of y = x. Pay attention to the y-intercept, the x-intercept, and the slope. Identify what changes in the graph and what stays the same.

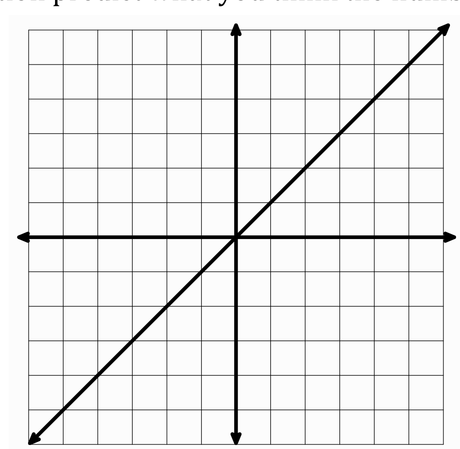


   a. $y = x + 3$

   b. $y = x - 3$

   c. $y = 3x$

2. The graph of y = x is given. (*See figure 2.*) For each equation predict what you think the number -2 will do to the graph. Then graph the equation.



   a. $y = x + (-2)$
   Prediction:

   b. $y = x - (-2)$
   Prediction:

   c. $y = -2x$
   Prediction:

Set

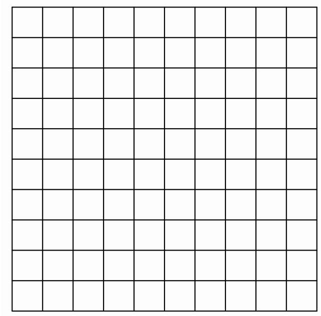
Topic: Distinguishing between linear, exponential, and quadratic functions

For each relation given:

- **a. Identify whether or not the relation is a function. (If it's not a function, skip b – d.)**
- **b. Determine if the function is Linear, Exponential, Quadratic or Neither.**
- **c. Describe the type of growth.**
- **d. Express the relation in the indicated form.**

3. I had 81 freckles on my nose before I began using vanishing cream. After the first week I had 27, the next week 9, then 3 . . .

a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Make a graph. Label your axes and the scale Show all 4 points.

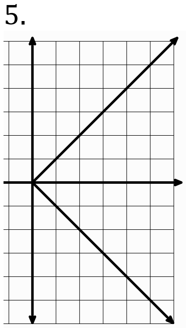


4.

x	y
0	80
1	$80\frac{2}{3}$
2	$80\frac{1}{3}$
3	80
4	$79\frac{2}{3}$

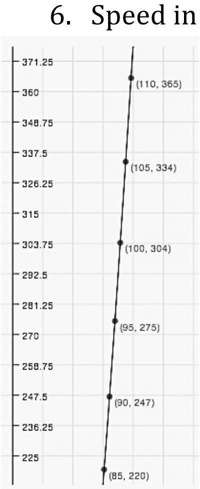
a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Write the explicit equation.

5.



a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Create a table

6. Speed in mph of a baseball vs. distance in ft.



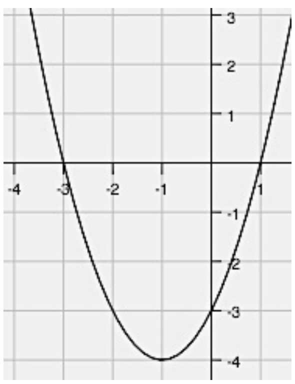
a. Function?
b. Linear, Exponential, Quadratic or Neither
c. How does it grow?
d. Predict the distance the baseball flies if it leaves the bat at a speed of 115 mph.

Go

Match the function on the left with the equivalent function on the right.

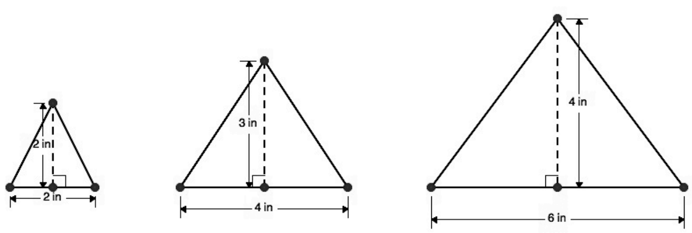
_____ 7. $f(x) = -2x + 5$

_____8.



_____9. I put $7000 in a savings account that pays 3% interest compounded annually. I plan to leave it in the bank for 20 years. The amount I will have then.

_____10. The area of the triangles below.



_____11. $f(0) = 5; f(n) = 2 * f(n-1)$

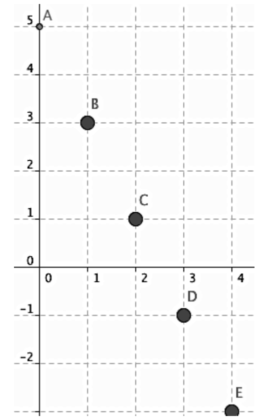
_____12. $f(0) = 5; f(n) = f(n-1) - 2$

_____13.

x	-7.75	-¼	½	11.6
f (x)	7.75	¼	-½	-11.6

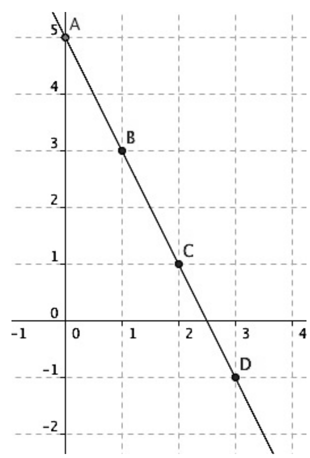
a. $f(x) = 5(2)^x$

b.



c. $f(1) = 2; \ f(n+1) = f(n) + 2n$

d.



e. y + x = 0

f. $y = (x - 1)(x + 3)$

g. $A = 7000(1.03)^{20}$