Secondary Two Mathematics: An Integrated Approach
Module 6
Similarity and Right Triangle Trigonometry

By

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In partnership with the Utah State Office of Education

Module 6 – Similarity & Right Triangle Trigonometry

Classroom Task: 6.1 Photocopy Faux Pas – A Develop Understanding Task
Describing the essential features of a dilation (**G.SRT.1**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.1

Classroom Task: 6.2 Triangle Dilations – A Solidify Understanding Task
Examining proportionality relationships in triangles that are know to be similar to each other based on dilations (**G.SRT.2, G.SRT.4**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.2

Classroom Task: 6.3 Similar Triangles and Other Figures – A Solidify Understanding Task
Comparing definitions of similarity based on dilations and relationships between corresponding sides and angles (**G.SRT.2, G.SRT.3**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.3

Classroom Task: 6.4 Cut by a Transversal – A Solidify Understanding Task
Examining proportional relationships of segments when two transversals intersect sets of parallel lines (**G.SRT.4**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.4

Classroom Task: 6.5 Measured Reasoning – A Practice Understanding Task
Applying theorems about lines, angles and proportional relationships when parallel lines are crossed by multiple transversals (**G.CO.9, G.CO.10, G.SRT.4, G.SRT.5**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.5

Classroom Task: 6.6 Yard Work in Segments – A Solidify Understanding Task
Applying understanding of similar and congruent triangles to find midpoint or any point on a line segment that partitions the segment in a given ratio. (**G.GPE.6**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.6

Classroom Task: 6.7 Pythagoras by Proportions – A Practice Understanding Task
Using similar triangles to prove the Pythagorean theorem and theorems about geometric means in right triangles (**G.SRT.4, G.SRT.5**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.7

Classroom Task: 6.8 Are Relationships Predictable? – A Develop Understanding Task
Developing and understanding of right triangle trigonometric relationships based on similar triangles (**G.SRT.6, G.SRT.8**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.8

Classroom Task: 6.9 Relationships with Meaning – A Solidify Understanding Task
Finding relationships between the sine and cosine ratios for right triangles, including the Pythagorean identity (**G.SRT.6, G.SRT.7, F.TF.8**)

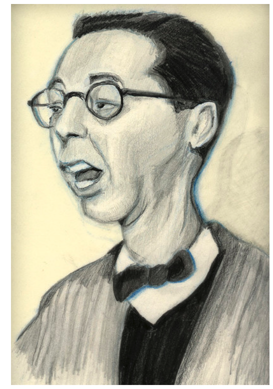
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.9

Classroom Task: 6.10 Finding the Value of a Relationship – A Solidify Understanding Task
Solving for unknown values in right triangles using trigonometric ratios (**G.SRT.7, G.SRT.8**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.10

Classroom Task: 6.11 Solving Right Triangles Using Trigonometric Relationships – A Practice Understanding Task
Practicing setting up and solving right triangles to model real world contexts (**G.SRT.6, G.SRT.7, F.TF.8**)
Ready, Set, Go Homework: Similarity & Right Triangle Trigonometry 6.11

6.1 Photocopy Faux Pas

A Develop Understanding Task



Burnell has a new job at a copy center helping people use the photocopy machines. Burnell thinks he knows everything about making photocopies, and so he didn't complete his assignment to read the training manual.

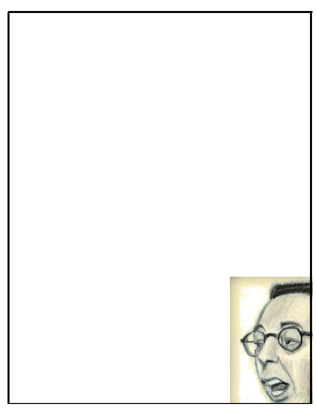
Mr. and Mrs. Donahue are making a scrapbook for Mr. Donahue's grandfather's 75th birthday party, and they want to enlarge a sketch of their grandfather which was drawn when he was in college. They have purchased some very expensive scrapbook paper, and they would like this image to be centered on the page. Because they are unfamiliar with the process of enlarging an image, they have come to Burnell for help.

"We would like to make a copy of this image that is twice as big, and centered in the middle of this very expensive scrapbook paper," Mrs. Donahue says. "Can you help us with that?"

"Certainly," says Burnell. "Glad to be of service."

Burnell taped the original image in the middle of a white piece of paper, placed it on the glass of the photocopy machine, inserted the expensive scrapbook paper into the paper tray, and set the enlargement feature at 200%.

In a moment, this image was produced.



"You've ruined our expensive paper," cried Mrs. Donahue. "Much of the image is off the paper instead of being centered."

"And this image is more than twice as big," Mr. Donahue complained. "One fourth of grandpa's picture is taking up as much space as the original."

In the diagram below, both the original image—which Burnell taped in the middle of a sheet of paper—and the copy of the image have been reproduced in the same figure.

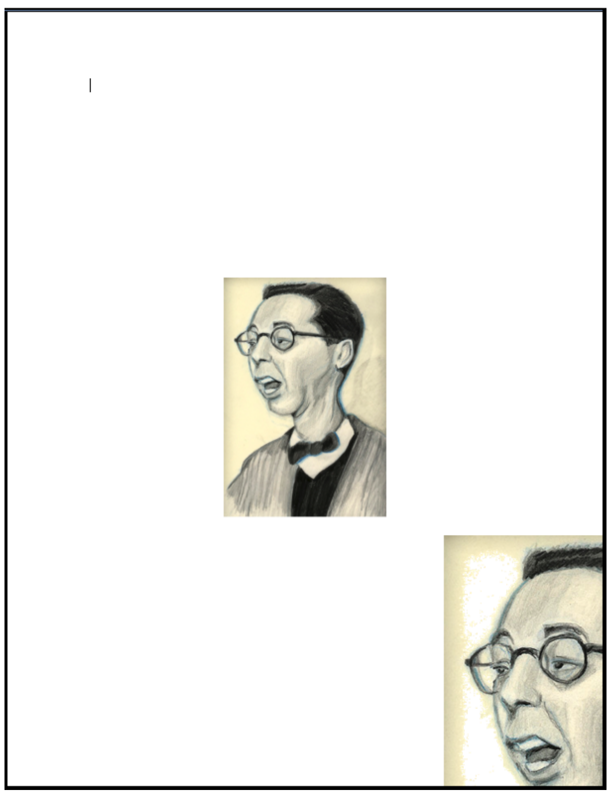
1. Explain how the photocopy machine produced the partial copy of the original image.

2. Using a "rubber band stretcher" finish the rest of the enlarged sketch.

3. Where should Burnell have placed the original image if he wanted the final image to be centered on the paper?

4. Mr. Donahue complained that the copy was four times bigger than the original. What do you think? Did Burnell double the image or quadruple it? What evidence would you use to support your claim?

5. Transforming a figure by shrinking or enlarging it in this way is a called a *dilation*. Based on your thinking about how the photocopy was produced, list all of the things you need to pay attention to when dilating an image.



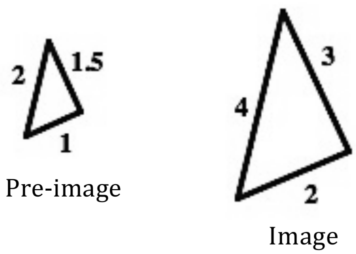
Ready, Set, Go!

Ready

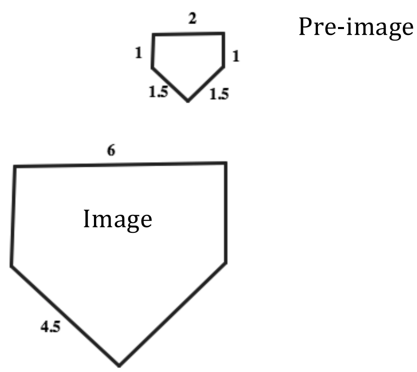
Topic: Scale factors for similar shapes.

Give the factor by which each pre-image was multiplied to create the image. Use the scale factor to fill in any missing lengths.

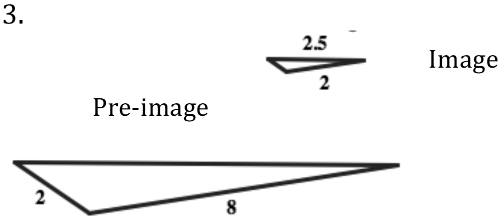
1.



2.



3.



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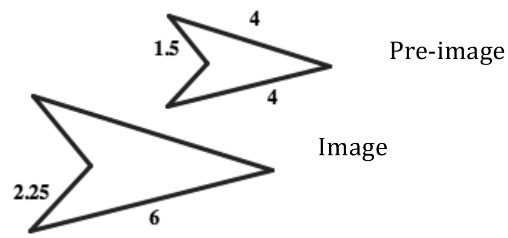
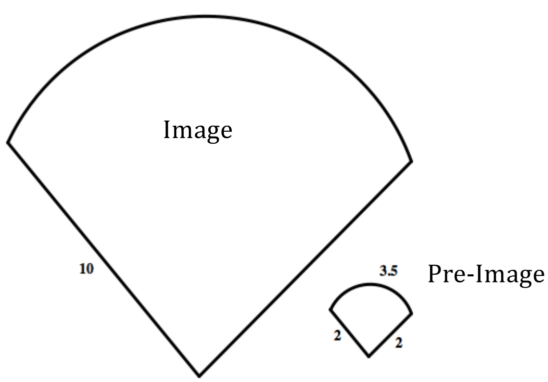


5.



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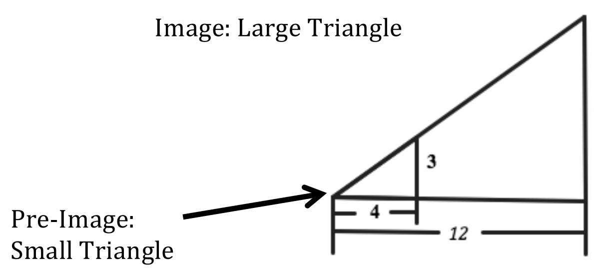


Set

Topic: Dilations in real world contexts

For each real-world context or circumstance determine the center of the dilation and the tool being used to do the dilation.

7.
Fran walks backward to a distance that will allow her family to all show up in the photo she is about to take.

8.
The theatre technician plays with the zoom in and out buttons in effort to fill the entire movie screen with the image.

9.
Melanie estimates the height of the waterfall by holding out her thumb and using it to see how many thumbs tall to the top of the waterfall from where she is standing. She then uses her thumb to see that a person at the base of the waterfall is half a thumb tall.

10.
A digital animator creates artistic works on her computer. She is currently doing an animation that has several telephone poles along a street that goes off into the distance.

11.
Ms. Sunshine is having her class do a project with a rubber-band tracing device that uses three rubber bands.

12.
A copy machine is set at 300% for making a photo copy.

Go

Topic: Rates of change related to linear, exponential and quadratic functions

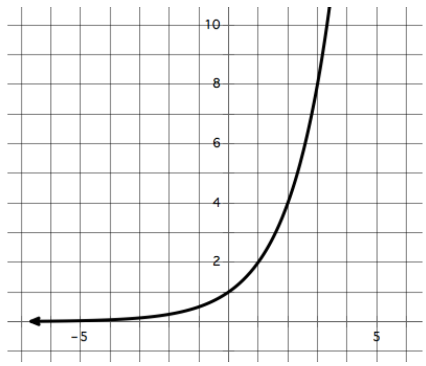
Determine whether the given representation is representative of a linear, exponential or quadratic function, classify as such and justify your reasoning.

13.

X	Y
2	7
3	12
4	19
5	28

Type of function:
Justification:

14.

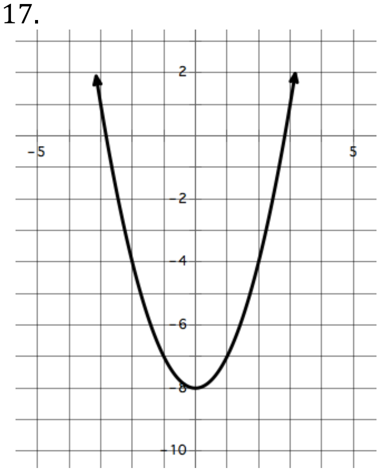


Type of function:
Justification:

15. $y = 3x^2 + 3x$
Type of function:
Justification:

16. $y = 7x - 10$
Type of function:
Justification:

17.



Type of function:

Justification:

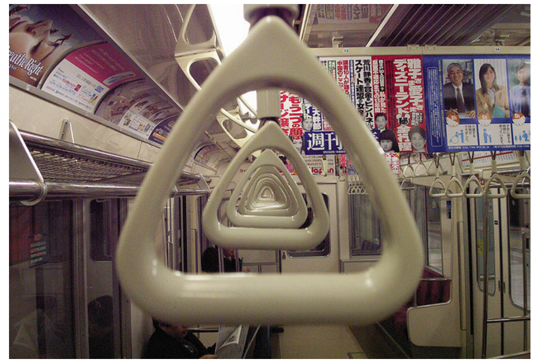
18.

X	Y
2	-5
7	5
14	19
25	41

Type of function:

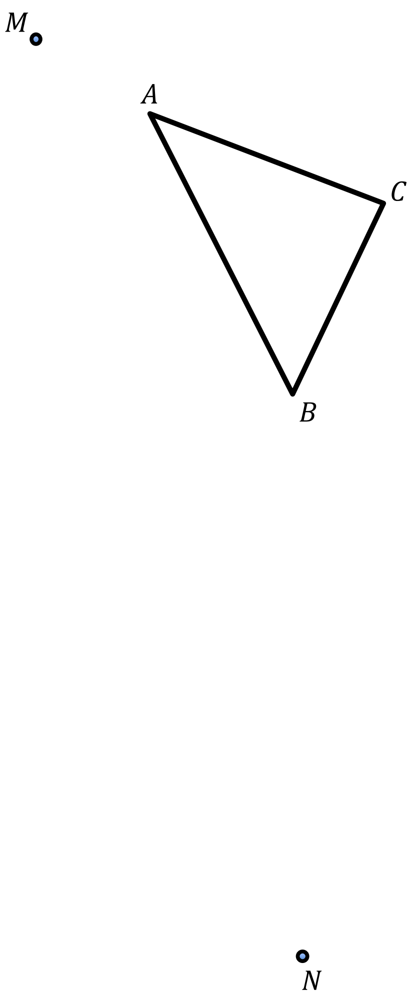
Justification:

6.2 Triangle Dilations
A Solidify Understanding Task



1. Given ΔABC, use point M as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of ΔABC.

2. Now use point N as the center of a dilation to locate the vertices of a triangle that has side lengths that are one-half the length of the sides of ΔABC.



3. Label the vertices in the two triangles you created in the diagram above. Based on this diagram, write several proportionality statements you believe are true. First write your proportionality statements using the names of the sides of the triangles in your ratios. Then verify that the proportions are true by replacing the side names with their measurements, measured to the nearest millimeter.

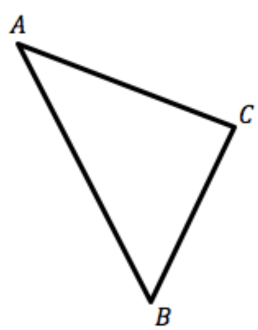
   My list of proportions: (try to find at least 10 proportionality statements you believe are true)

4. Based on your work above, under what conditions are the corresponding line segments in an image and its pre-image parallel after a dilation? That is, which word best completes this statement?

   After a dilation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

5. Give reasons for your answer. If you choose "sometimes", be very clear in your explanation how to tell when the corresponding line segments before and after the dilation are parallel and when they are not.

Given ΔABC, use point A as the center of a dilation to locate the vertices of a triangle that has side lengths that are twice as long as the sides of ΔABC.



6. Explain how the diagram you created above can be used to prove the following theorem:

   The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.

Name: Similarity & Right Triangle Trigonometry | 6.2

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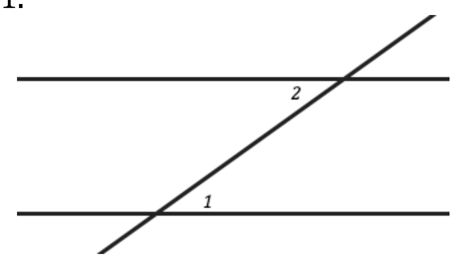
Ready, Set, Go!

Ready

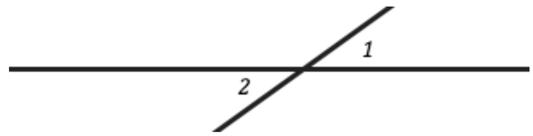
Topic: Angle relationships

Match the diagrams below with the best name or phrase that describes the angles.

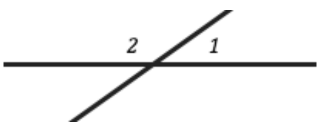
_____1.



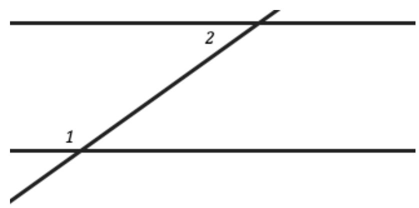
______2.



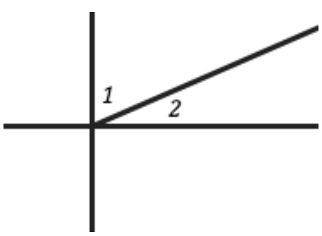
_____3.



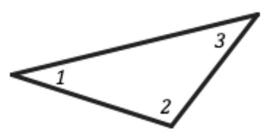
_____4.



____5.



_____6.



a. Alternate Interior Angles b. Vertical Angles c. Complementary Angles

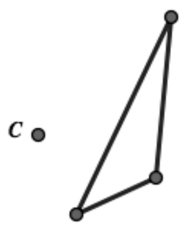
d. Triangle Sum Theorem e. Linear Pair f. Same Side Interior Angles

Set

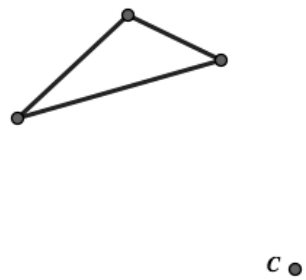
Topic: Creating dilations and examining their parts.

Use the given pre-image and point *C* as the center of dilation to perform the dilation that is indicated below.

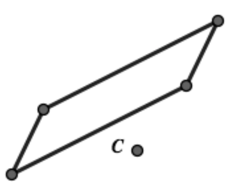
7. Create an image with side lengths twice the size of the given triangle.



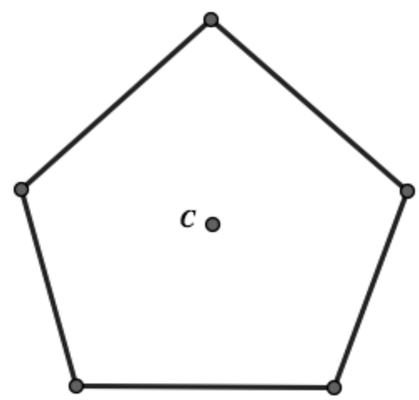
8. Create an image with side lengths half the size of the given triangle.



9. Create an image with side lengths three times the size of the given parallelogram.

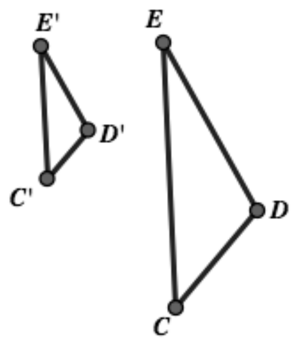


10. Create an image with side length one fourth the size of the given pentagon.

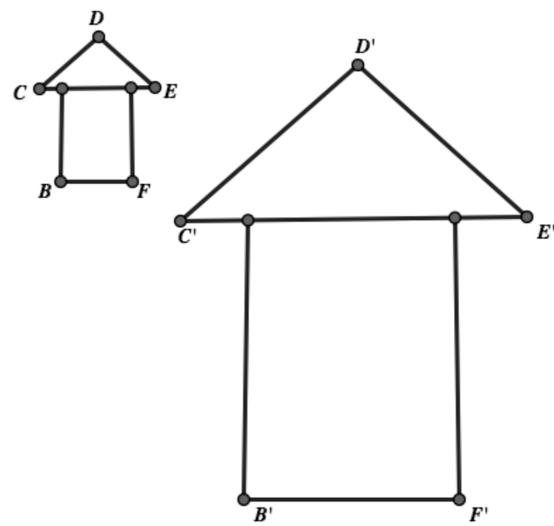


Use the given pre-image and image in each diagram to define the dilation that occurred. Include as many details as possible such as the center of the dilation and the ratio.

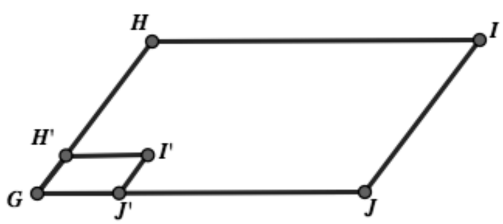
11.



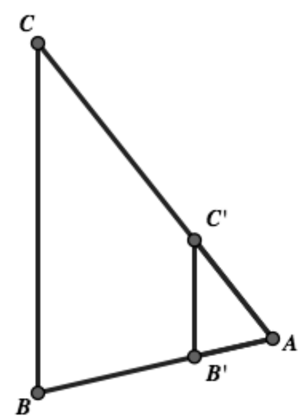
12.



13.



14.



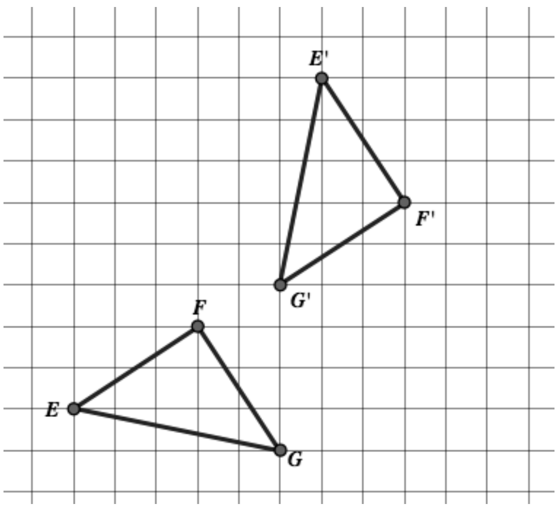
Go

Topic: Classify the transformation and define it.

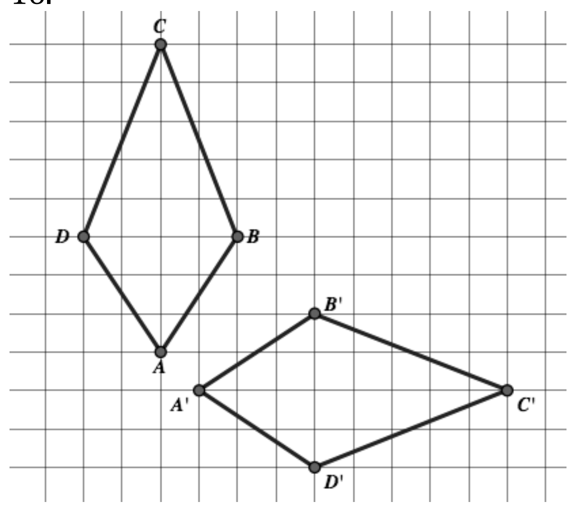
Based on the given image and pre-image determine the transformation that occurred. Further, prove that the transformation occurred by showing evidence of some kind.

(*For example, if the transformation was a reflection show the line of reflection exists and prove that it is the perpendicular bisector of all segments that connect corresponding points from the image and pre-image. Do likewise for rotations, translations and dilations.*)

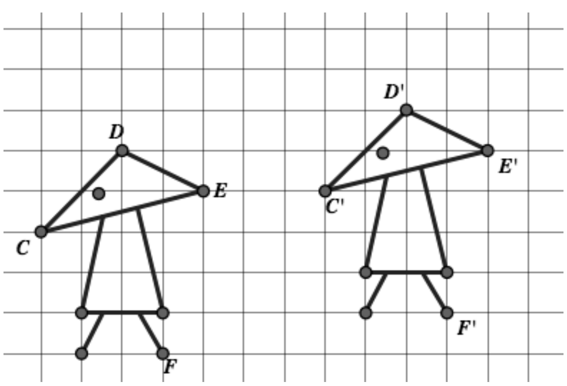
15.



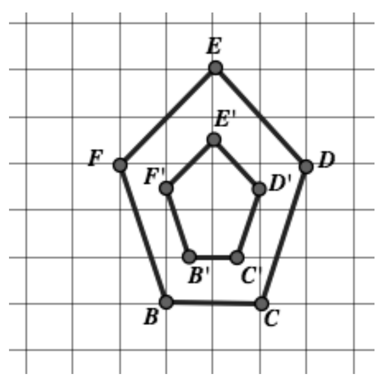
16.



17.



18.



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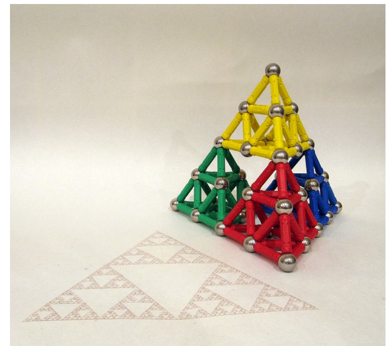
6.3 Similar Triangles and Other Figures

A Solidify Understanding Task



Two figures are said to be *congruent* if the second can be obtained from the first by a sequence of rotations, reflections, and translations. In Mathematics I we found that we only needed three pieces of information to guarantee that two triangles were congruent: SSS, ASA or SAS.

What about AAA? Are two triangles congruent if all three pairs of corresponding angles are congruent? In this task we will consider what is true about such triangles.

Part 1

Definition of Similarity: Two figures are *similar* if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

Mason and Mia are testing out conjectures about similar polygons. Here is a list of their conjectures.

Conjecture 1: *All rectangles are similar.*

Conjecture 2: *All equilateral triangles are similar.*

Conjecture 3: *All isosceles triangles are similar.*

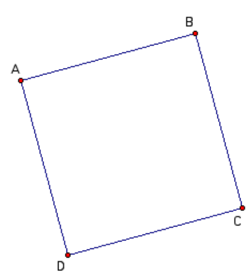
Conjecture 4: *All rhombuses are similar.*

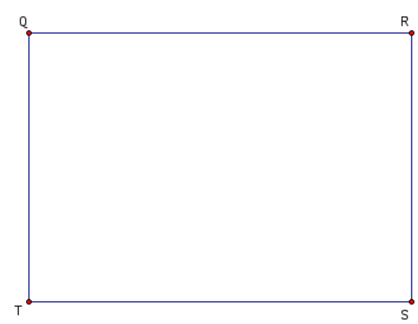
Conjecture 5: *All squares are similar.*

1. Which of these conjectures do you think are true? Why?

Mason is explaining to Mia why he thinks conjecture 1 is true using the diagram given below.

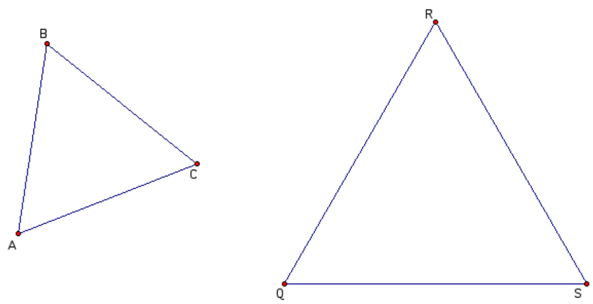
"All rectangles have four right angles. I can translate and rotate rectangle *ABCD* until vertex *A* coincides with vertex *Q* in rectangle *QRST*. Since $\angle A$ and $\angle Q$ are both right angles, side *AB* will lie on top of side *QR*, and side *AD* will lie on top of side *QT*. I can then dilate rectangle *ABCD* with point *A* as the center of dilation, until points *B*, *C*, and *D* coincide with points *R*, *S*, and *T*.





2. Does Mason's explanation convince you that rectangle *ABCD* is similar to rectangle *QRST* based on the definition of similarity given above? Does his explanation convince you that *all rectangles are similar*? Why or why not?

Mia is explaining to Mason why she thinks conjecture 2 is true using the diagram given below.



"All equilateral triangles have three 60° angles. I can translate and rotate ΔABC until vertex A coincides with vertex Q in ΔQRS. Since $\angle A$ and $\angle Q$ are both 60° angles, side AB will lie on top of side QR, and side AC will lie on top of side QS. I can then dilate ΔABC with point A as the center of dilation, until points B and C coincide with points R and S."

3. Does Mia's explanation convince you that ΔABC is similar to ΔQRS based on the definition of similarity given above? Does her explanation convince you that *all equilateral triangles are similar*? Why or why not?

4. For each of the other three conjectures, write an argument like Mason's and Mia's to convince someone that the conjecture is true, or explain why you think it is not always true.

   a. Conjecture 3: *All isosceles triangles are similar.*

   b. Conjecture 4: *All rhombuses are similar.*

   c. Conjecture 5: *All squares are similar.*

While the definition of similarity given at the beginning of the task works for all similar figures, an alternative definition of similarity can be given for polygons: **Two polygons are similar if all corresponding angles are congruent and all corresponding pairs of sides are proportional.**

5. How does this definition help you find the error in Mason's thinking about conjecture 1?

6. How does this definition help confirm Mia's thinking about conjecture 2?

7. How might this definition help you think about the other three conjectures?

    a. Conjecture 3: *All isosceles triangles are similar.*

    b. Conjecture 4: *All rhombuses are similar.*

    c. Conjecture 5: *All squares are similar.*

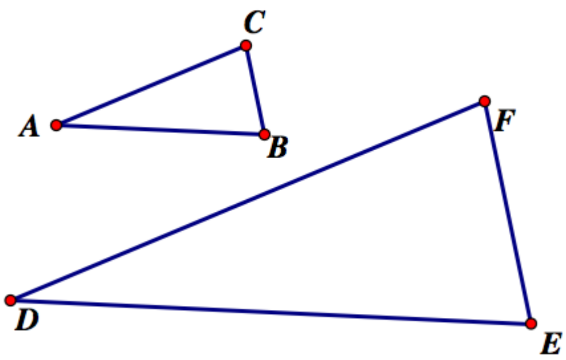
Part 2 (AAA Similarity)

From our work above with rectangles it is obvious that knowing that all rectangles have four right angles (an example of AAAA for quadrilaterals) is not enough to claim that all rectangles are similar. What about triangles? In general, are two triangles similar if all three pairs of corresponding angles are congruent?

8. Decide if you think the following conjecture is true.

    Conjecture: *Two triangles are similar if their corresponding angles are congruent.*

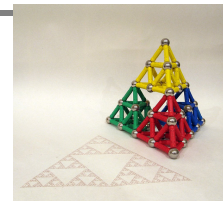
9. Explain why you think the conjecture—*two triangles are similar if their corresponding angles are congruent*—is true. Use the following diagram to support your reasoning, Remember to start by marking what you are given to be true (AAA) in the diagram.



Hint: *If you translate A to D, where do points B and C end up?*

10. Mia thinks the following conjecture is true. She calls it "*AA Similarity for Triangles*." What do you think? Is it true? Why?

Conjecture: *Two triangles are similar if they have two pair of corresponding congruent angles.*



Ready, Set, Go!

Ready

Topic: Solving proportions

Solve each proportion. Show your work and check your solution.

1. $\frac{3}{4} = \frac{x}{20}$

2. $\frac{x}{7} = \frac{18}{21}$

3. $\frac{3}{6} = \frac{8}{x}$

4. $\frac{9}{c} = \frac{6}{10}$

5. $\frac{3}{4} = \frac{b+3}{20}$

6. $\frac{7}{12} = \frac{a}{24}$

7. $\frac{a}{2} = \frac{13}{20}$

8. $\frac{3}{b+2} = \frac{6}{5}$

9. $\frac{\sqrt{3}}{2} = \frac{\sqrt{12}}{c}$

Set

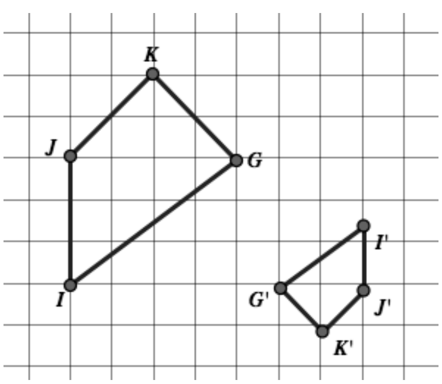
Topic: Proving similarity

Provide an argument to prove each conjecture, or provide a counterexample to disprove it.

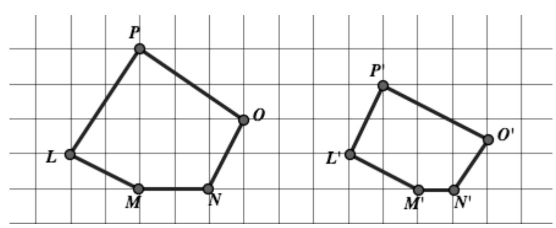
10. All right triangles are similar

11. All regular polygons are similar to other regular polygons with the same number of sides.

12. The polygons on the grid below are similar.

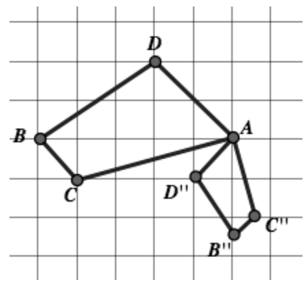


13. The polygons on the grid below are similar.



A sequence of two transformations occurred to create the two similar polygons. Justify each transformation and be as specific as you can about how the pre-image is transformed to create the image.

14.



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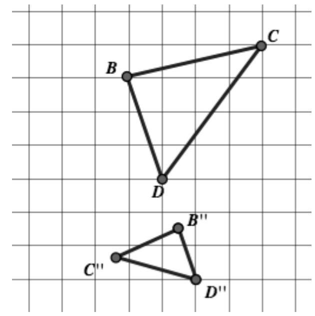

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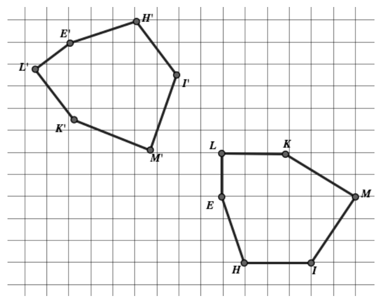

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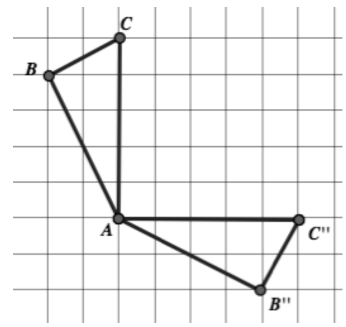
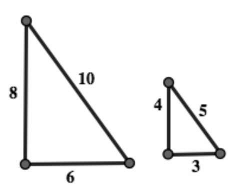


Go

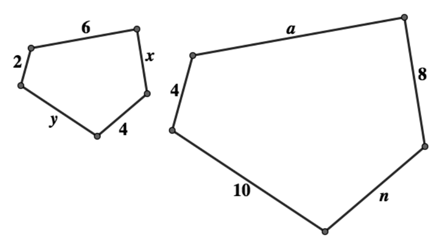
Topic: Ratios in dilated polygons

For each pair of similar polygons give three ratios that would be equivalent.

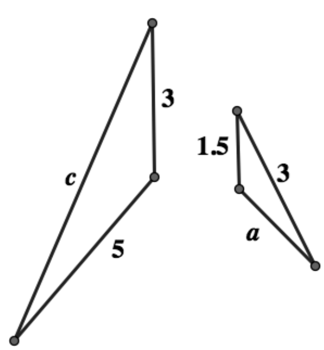
18.



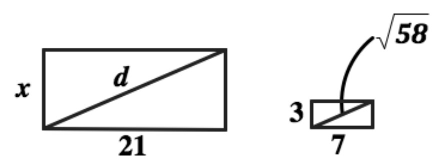
19.



20.

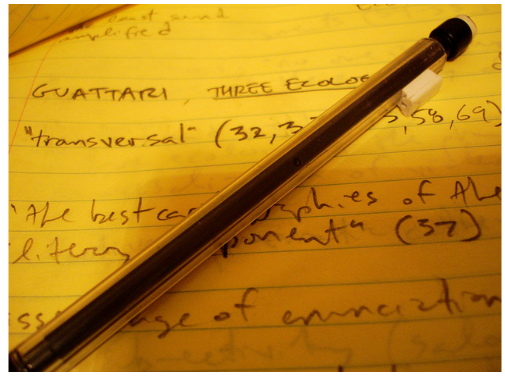


21.



6.4 Cut By a Transversal

A Solidify Understanding Task



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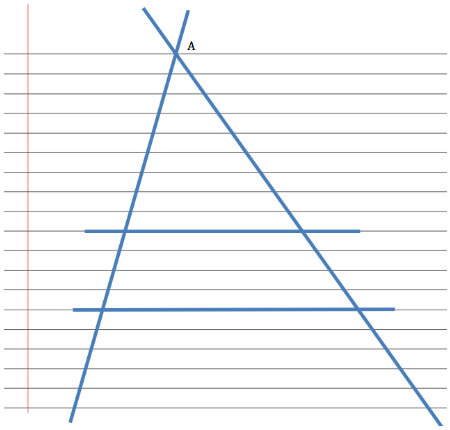
Draw two intersecting transversals on a sheet of lined paper, as in the following diagram. Label the point of intersection of the transversals *A.* Select any two of the horizontal lines to form the third side of two different triangles.



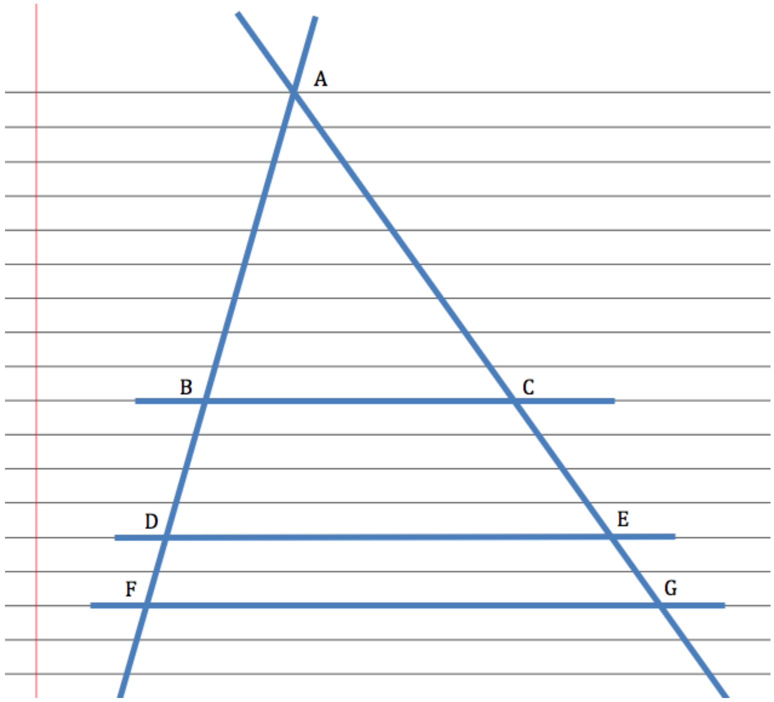
1. What convinces you that the two triangles formed by the transversals and the horizontal lines are similar?

2. Label the vertices of the triangles. Write some proportionality statements about the sides of the triangles and then verify the proportionality statements by measuring the sides of the triangles.

3. Select a third horizontal line segment to form a third triangle that is similar to the other two. Write some additional proportionality statements and verify them with measurements.

Tristan has written this proportion for question 3, based on his diagram: $\frac{BD}{AB}=\frac{CE}{AC}$

Tia thinks Tristan's proportion is wrong, because some of the segments in his proportion are not sides of a triangle.



4. Check out Tristan's idea using measurements of the segments in his diagram at the left.

5. Now check out this same idea using proportions of segments from your own diagram. Test at least two different proportions, including segments that do not have *A* as one of their endpoints.

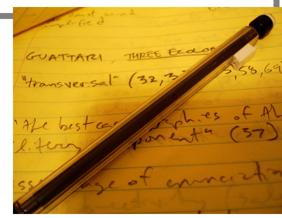
6. Based on your examples, do you think Tristan or Tia is correct?

Tia still isn't convinced, since Tristan is basing his work on a single diagram. She decides to start with a proportion she knows is true: $\frac{AD}{AB}=\frac{AE}{AC}$. (Why is this true?)

Tia realizes that she can rewrite this proportion as $\frac{AB+BD}{AB}=\frac{AC+CE}{AC}$ (Why is this true?)

Can you use Tia's proportion to prove algebraically that Tristan is correct?

Name: Similarity & Right Triangle Trigonometry | 6.4



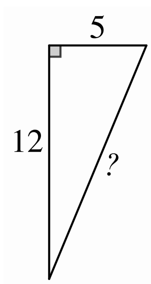
Ready, Set, Go!

Ready

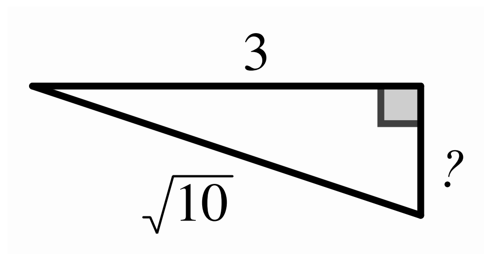
Topic: Pythagorean Theorem and ratios for similar triangles.

Find the missing side in each right triangle.

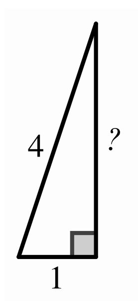
1.



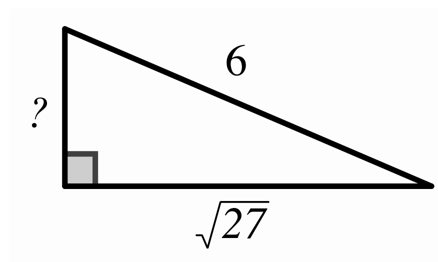
2.



3.

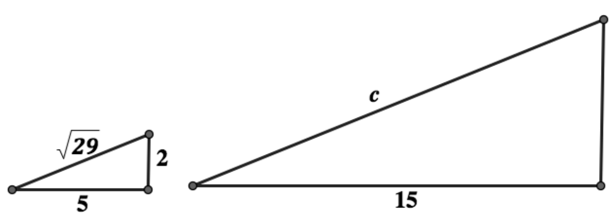


4.

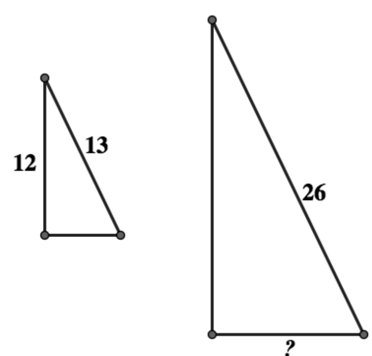


Create a proportion for each set of similar triangles. Then solve the proportion.

5.



6.

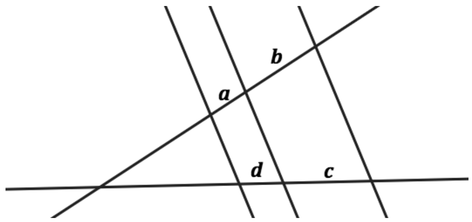


Set

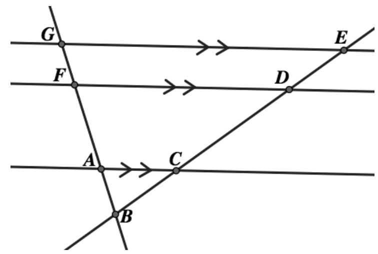
Topic: Proportionality of transversals across parallel lines.

For questions 7 and 8, write three equal ratios.

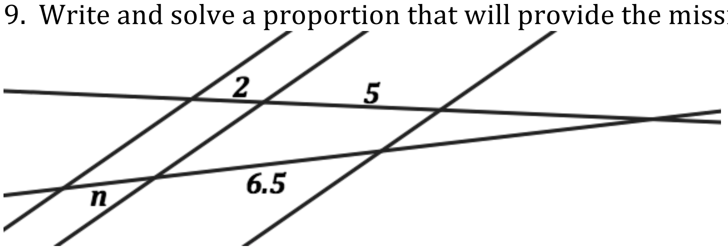
7. *The letters a,b,c and d represent lengths of line segments.*



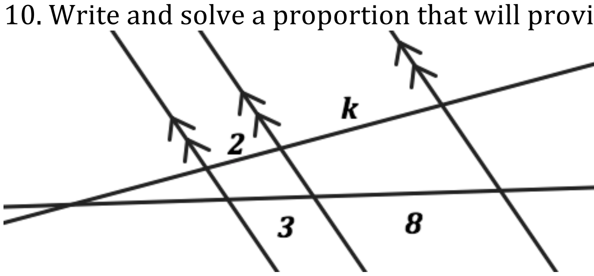
8.



9. Write and solve a proportion that will provide the missing length.

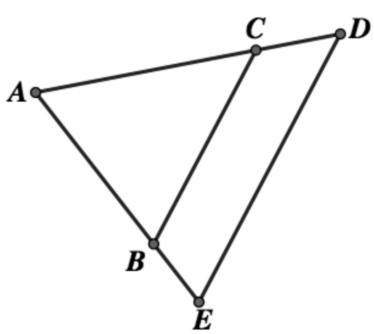


10. Write and solve a proportion that will provide the missing length.



For questions 11 – 14 find and label the parallel lines. (i.e. $\overline{AB} \parallel \overline{CD}$) Then write a similarity statement for the triangles that are similar. (i.e. $\Delta\, ABC \sim \Delta\, XYZ$)

11.



12.

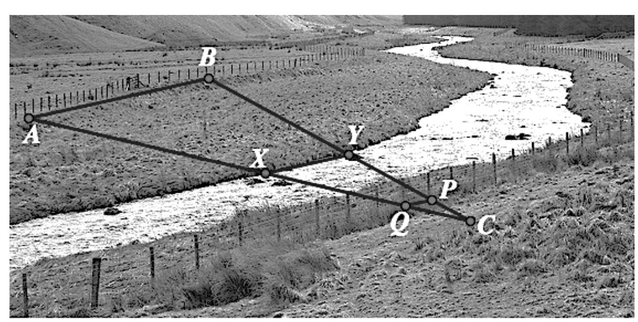


Photo of river
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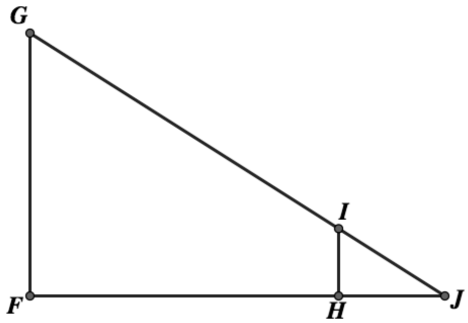
13.



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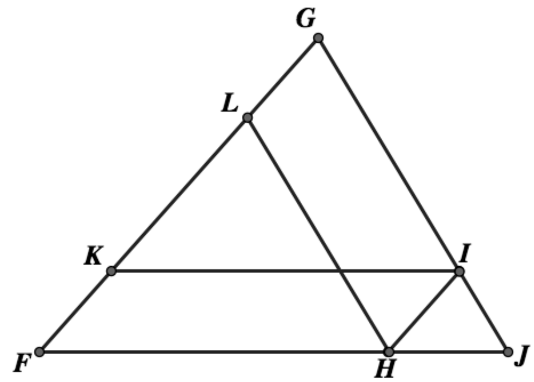
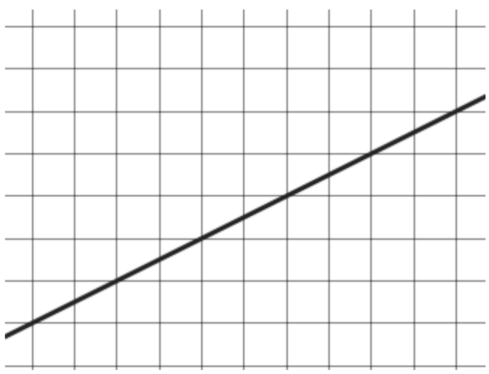


Go

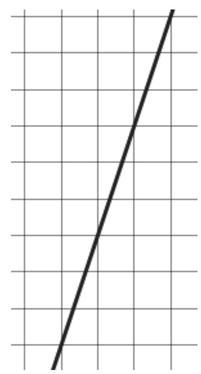
Topic: Similarity of slope triangles.

Each line below has several triangles that can be used to determine the slope. Draw in three slope defining triangles of different sizes for each line and then create the ratio of rise to run for each.

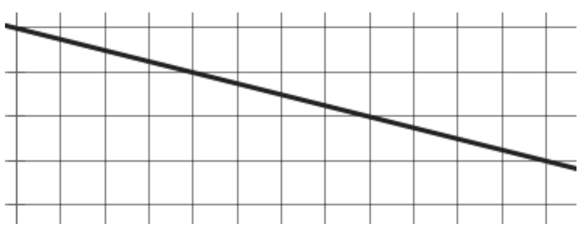
15.



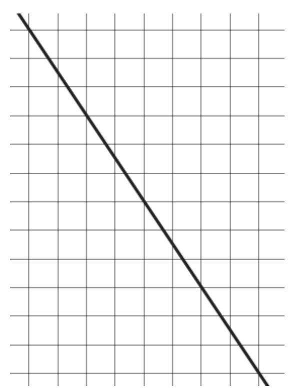
16.



17.



18.

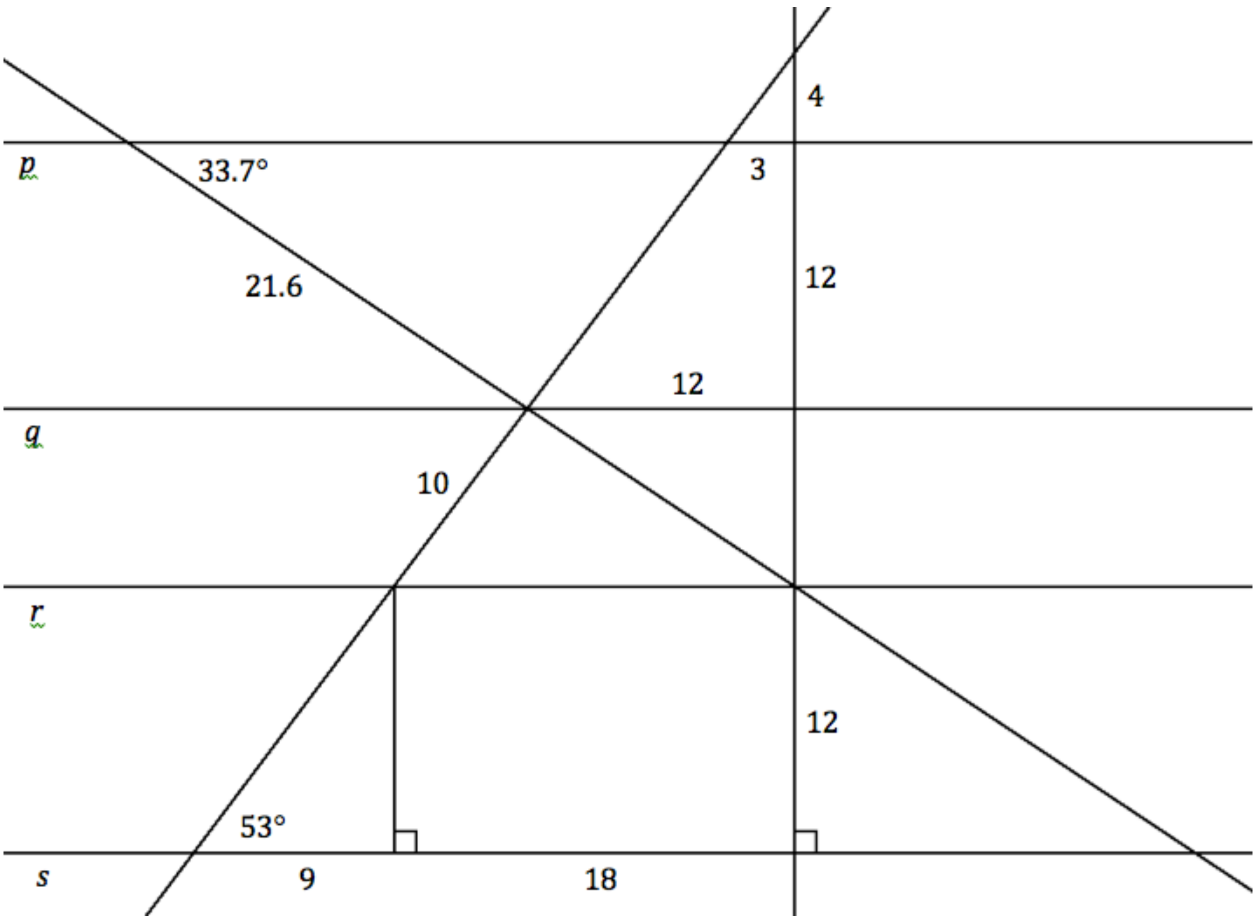


6.5 Measured Reasoning

A Practice Understanding Task

Find the measures of all missing sides and angles by using geometric reasoning, not rulers and protractors. If you think a measurement is impossible to find, identify what information you are missing.

Lines *p*, *q*, *r*, and *s* are all parallel.



1. Identify at least three different quadrilaterals in the diagram. Find the sum of the interior angles for each quadrilateral. Make a conjecture about the sum of the interior angles of a quadrilateral.

   Conjecture:

2. Identify at least three different pentagons in the diagram. Find the sum of the interior angles for each pentagon. Make a conjecture about the sum of the interior angles of a pentagon.

   Conjecture:

3. Do you see a pattern in the sum of the angles of a polygon as the number of sides increases? How can you describe this pattern symbolically?

4. How can you convince yourself that this pattern holds for all n-gons?

Name: Similarity & Right Triangle Trigonometry | 6.5

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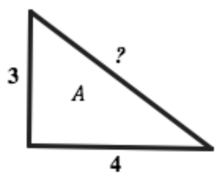
Ready, Set, Go!

Ready

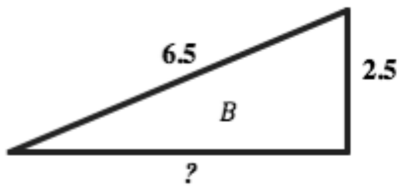
Topic: Pythagorean Theorem and ratios of similar triangles

Find the missing side in each right triangle. Triangles are not drawn to scale.

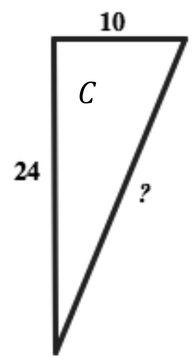
1.



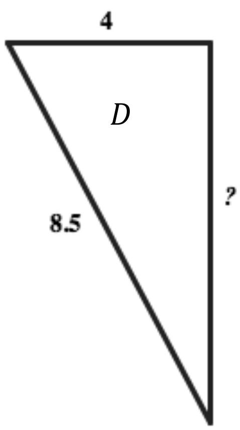
2.



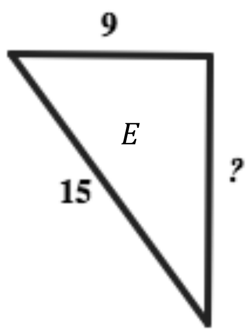
3.



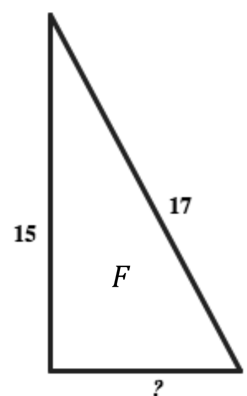
4.



5.



6.



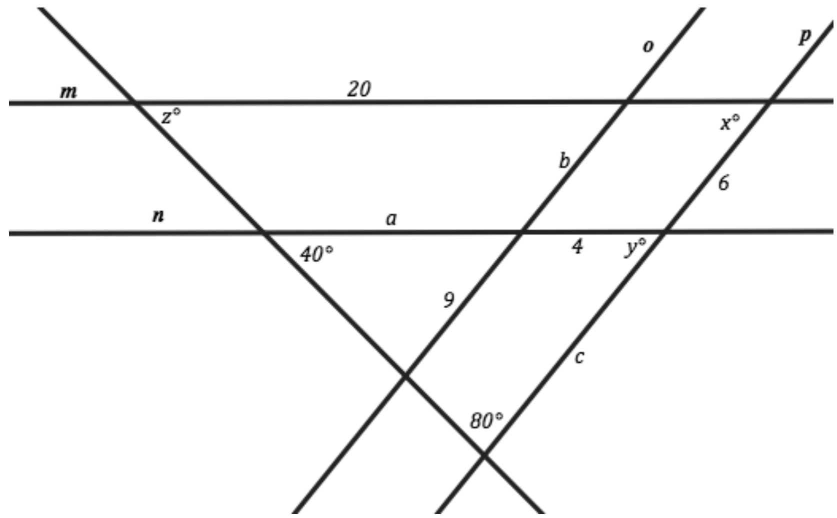
7. Based on ratios between side lengths, which of the right triangles above are mathematically similar to each other? Provide the letters of the triangles and the ratios.

Set

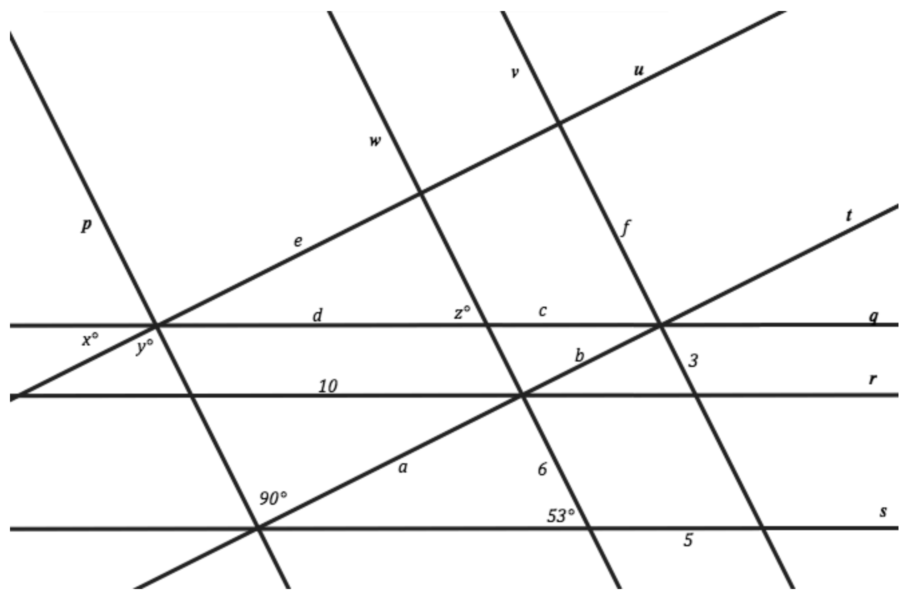
Topic: Using parallel lines, and angle relationships to find missing values.

In each of the diagrams use the given information provided to find the missing lengths and angle measurements.

8. Line $m \parallel n$ and $o \parallel p$, find the values of angles *x, y* and *z*. Also, find the lengths of *a, b* and *c*.



9. Line $q \parallel r \parallel s$ and $t \parallel u$ and $p \parallel w \parallel v$, find the values of angles *x, y* and *z*. Also, find the lengths of *a, b, c, d, e, f*.



Go

Topic: Solve equations including those including proportions

Solve each equation below.

10.

$$3x - 5 = 2x + 7$$

11.

$$\frac{5}{7} = \frac{x}{21}$$

12.

$$\frac{3}{x} = \frac{18}{5x + 2}$$

13.

$$\frac{1}{2}x - 7 = \frac{3}{4}x - 8$$

14.

$$17 + 3(x - 5) = 2(x + 3)$$

15.

$$\frac{x + 5}{6} = \frac{3(x + 2)}{9}$$

16.

$$x + 2 + 3x - 8 = 90$$

17.

$$\frac{5}{12} = \frac{x}{8}$$

18.

$$\frac{4}{5} = \frac{x + 2}{15}$$

6.6 Yard Work in Segments

A Solidify Understanding Task

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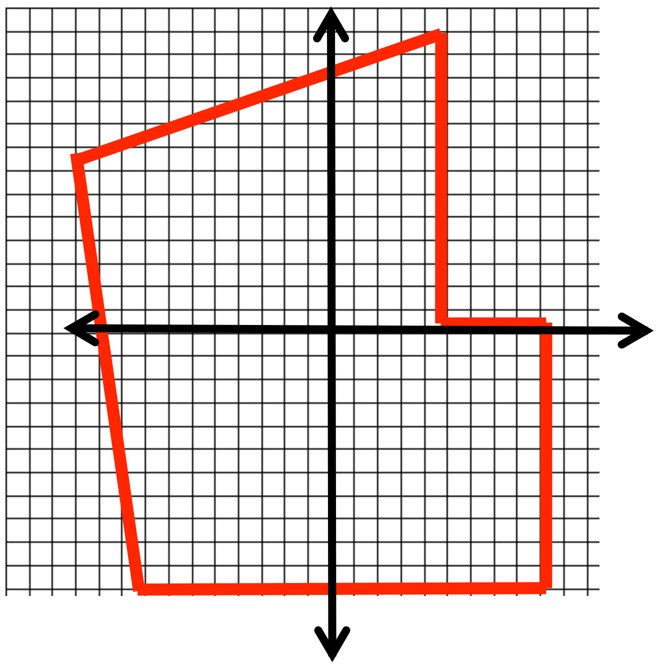
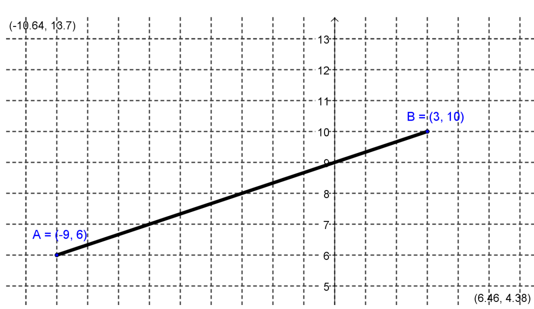
Malik's family has purchased a new house with an unfinished yard. They drew the following map of the back yard:

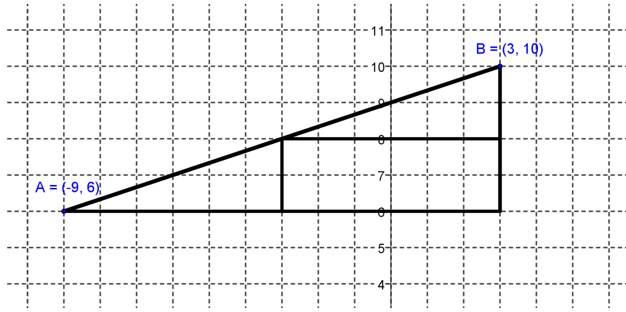


Malik and his family are using the map to set up gardens and patios for the yard. They plan to lay out the yard with stakes and strings so they know where to plant grass, flowers, or vegetables. They want to begin with a vegetable garden that will be parallel to the fence shown at the top of the map above.

1. They set the first stake at (-9, 6) and the stake at the end of the garden at (3, 10). They want to mark the middle of the garden with another stake. Where should the stake that is the midpoint of the segment between the two end stakes be located? Use a diagram to describe your strategy for finding this point.



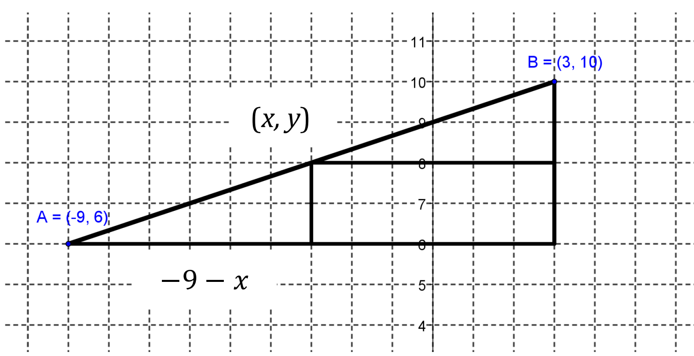
2. Malik figured out the midpoint by saying, "It makes sense to me that the midpoint is going to be halfway over and halfway up, so I drew a right triangle and cut the horizontal side in half and the vertical side in half like this:"



Malik continued, "That put me right at (-3, 8). The only thing that seems funny about that to me is that I know the base of the big triangle was 12 and the height of the triangle was 4, so I thought the midpoint might be (6, 2)."

Explain to Malik why the logic that made him think the midpoint was (6, 2) is almost right, and how to extend his thinking to use the coordinates of the endpoints to get the midpoint of (-3,6).

3. Malik's sister, Sapana, looked at his drawing and said, "Hey, I drew the same picture, but I noticed the two smaller triangles that were formed were congruent. Since I didn't know for sure what the midpoint was, I called it (x, y). Then I used that point to write an expression for the length of the sides of the small triangles. For instance, I figured that the base of the lower triangle was $-9 - x$."



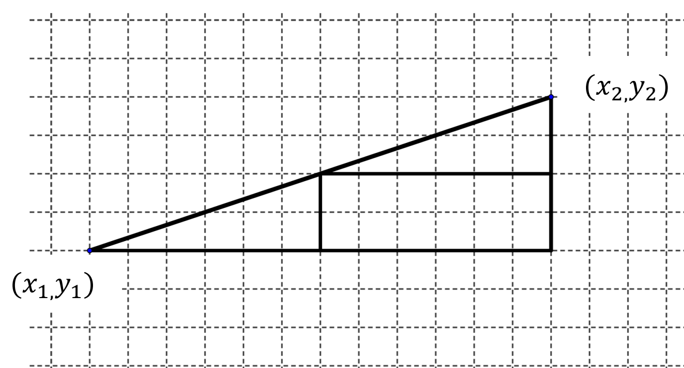
Label all of the other legs of the two smaller right triangles using Sapana's strategy.

Sapana continued, "Once I labeled the triangles, I wrote equations by making the bases equal and the heights equal."

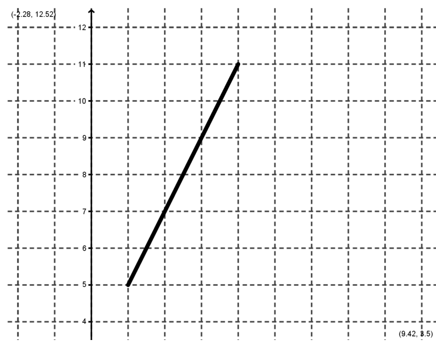
Does Sapana's strategy work? Show why or why not.

4. Choose a strategy and use it to find the midpoint of the segment with endpoints (-3, 4) and (2, 9).

5. Use either strategy to find the midpoint of the segment between (x_1, y_1) and (x_2, y_2).



6. The next area in the garden to be marked is for a flower garden. Malik's parents have the idea that part of the garden should contain a big rose bush and the rest of the garden with have smaller flowers like petunias. They want the section with the other flowers to be twice as long as the section with the rose bush. The stake on the endpoints of this garden will be at (1, 5) and (4, 11). Malik's dad says, "We'll need a stake that marks the end of the rose garden." Help Malik and Sapana figure out where the stake will be located.



7. There's only one more set of stakes to put in. This time the endpoint stakes are at (-8, 5) and (2,-10). Another stake needs to be set that partitions the segment into two parts so that the ratio of the lengths is 2:3. Where must the stake be located?

Name: Similarity & Right Triangle Trigonometry 6.6

Ready, Set, Go!

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Ready

Topic: Averages and center

For each set of numbers find the mean (average). Explain how the mean of the set compares to the values in the set.

1. 6, 12, 10, 8
2. 2, 7, 12
3. -13, 21
4. 3, -9, 15
5. 43, 52
6. 38, 64, 100

Find the value that is exactly half way between the two given values. Explain how you find this value.

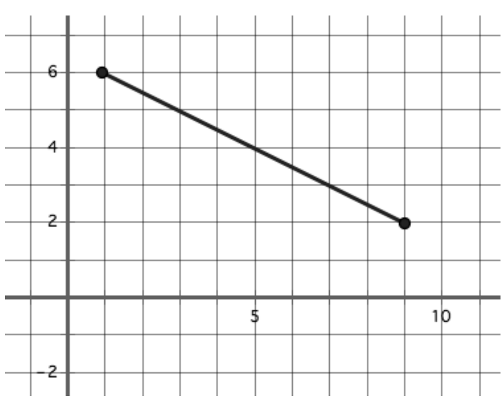
7. 5, 13
8. 26, 42
9. 57, 77
10. -34, -22
11. -45, 3
12. -12, 18

Set

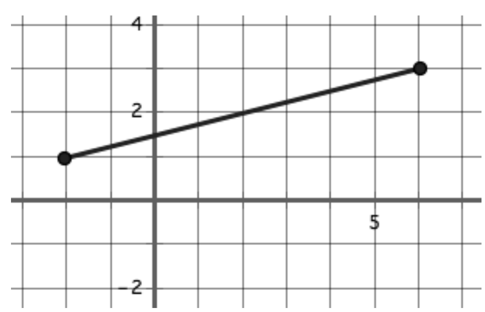
Topic: Midpoints of segments and proportionality of sides in embedded similar triangles

Find the coordinates of the midpoint of each line segment below. If multiple line segments are given then give the midpoints of all segments.

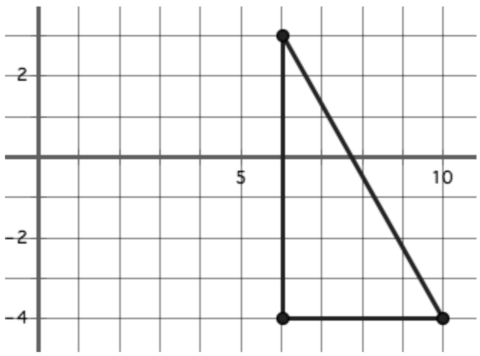
13.



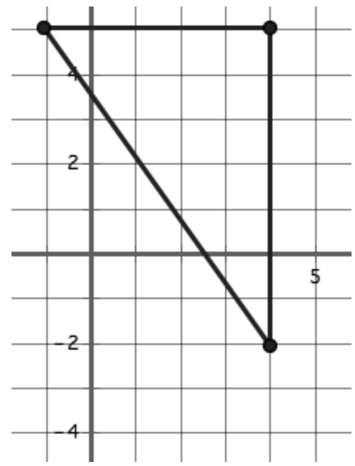
14.



15.



16.

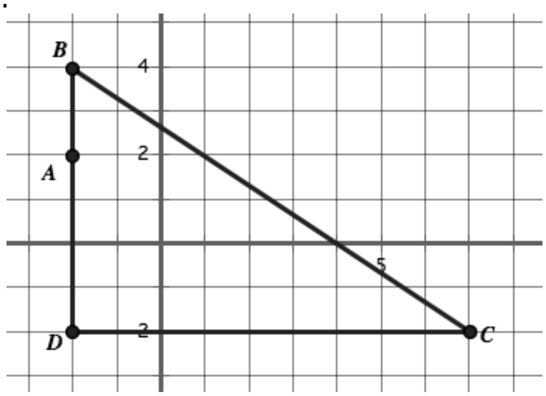


17. A line segment between (2, 3) and (10, 15)

18. A line segment between (-2, 7) and (3, -8)

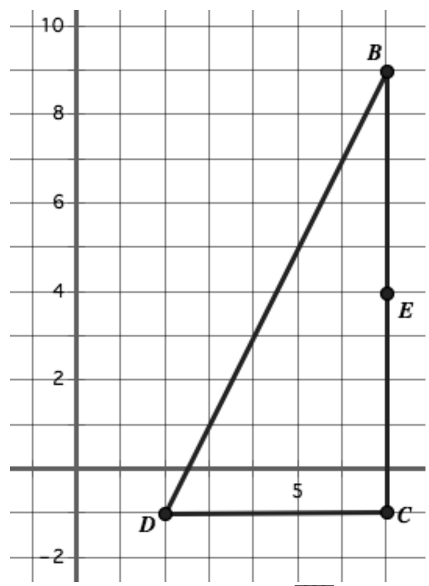
Use proportional relationships to find the desired values.

19.



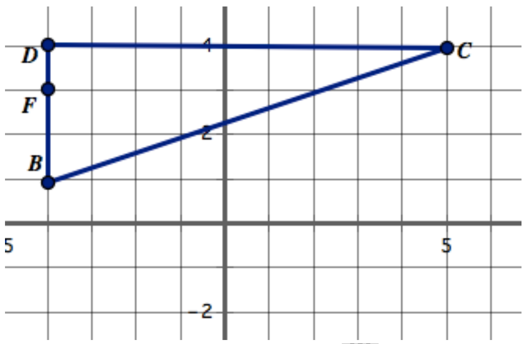
If a line is drawn parallel to $\overline{BC}$ and through point A. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

20.



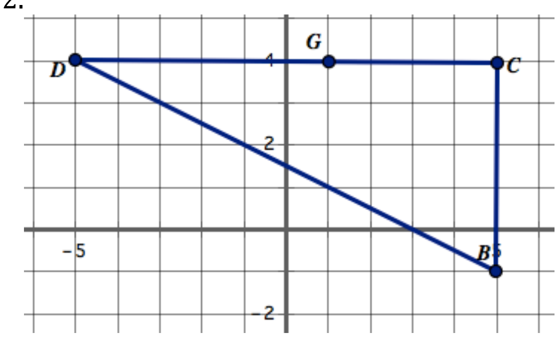
If a line is drawn parallel to $\overline{BD}$ and through point E. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

21.



If a line is drawn parallel to $\overline{BC}$ and through point F. At what coordinate will the intersection of this parallel line be with $\overline{DC}$?

22.



If a line is drawn parallel to $\overline{BD}$ and through point G. At what coordinate will the intersection of this parallel line be with $\overline{BC}$?

23. When a line is drawn parallel to one side of a triangle so that it intersects the other two sides of the triangle, how do the measures of the parts of the two intersected sides compare? Explain.

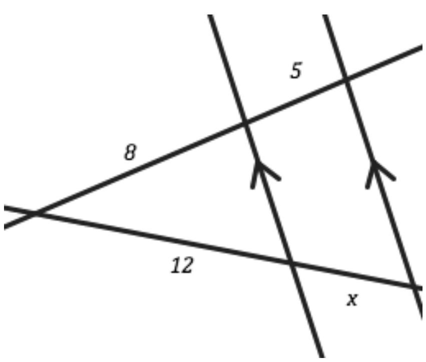
24. Problems 19-22 provided right triangles. Could a determination of the coordinates be made if they were not right triangles? Why or why not?

Go

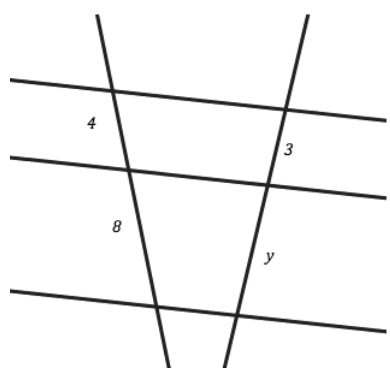
Topic: Proportionality with parallel lines.

Write a proportion for each of the diagrams below and solve for the missing value.

25.

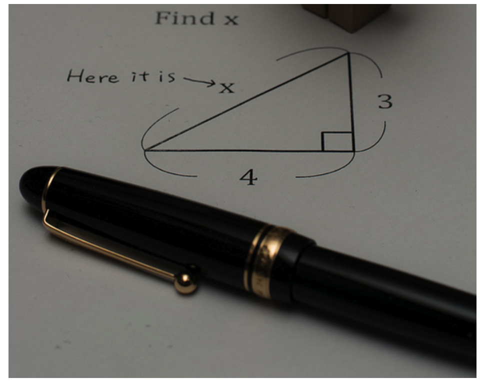


26.



6.7 Pythagoras by Proportions

A Practice Understanding Task



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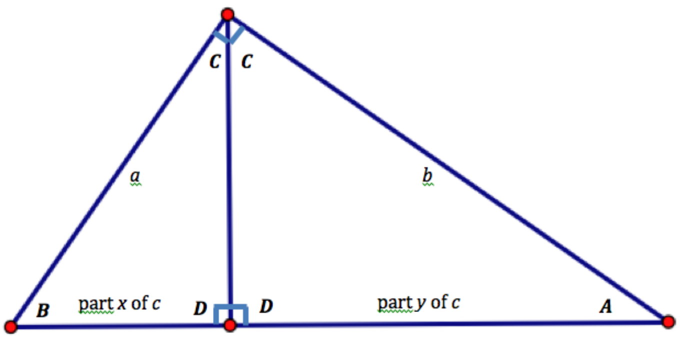
There are many different proofs of the Pythagorean Theorem. Here is one based on similar triangles.

Step 1: Cut a 4 × 6 index card along one of its diagonals to form two congruent right triangles.

Step 2: In each right triangle, draw an altitude from the right angle vertex to the hypotenuse.

Step 3: Label each triangle as shown in the following diagram. Flip each triangle over and label the matching sides and angles with the same names on the back as on the front.



Step 4: Cut one of the right triangles along the altitude to form two smaller right triangles.

Step 5: Arrange the three triangles in a way that convinces you that all three right triangles are similar. You may need to reflect and/or rotate one or more triangles to form this arrangement.

Step 6: Write proportionality statements to represent relationships between the labeled sides of the triangles.

Step 7: Solve one of your proportions for x and the other proportion for y. (If you have not written proportions that involve x and y, study your set of triangles until you can do so.)

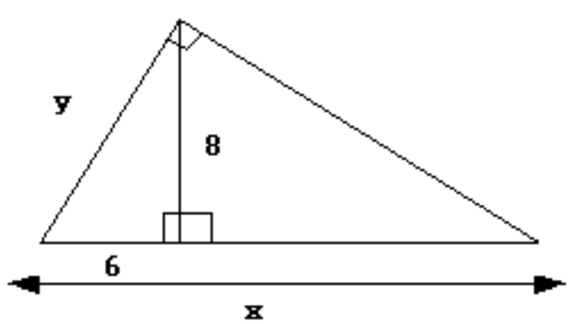
Step 8: Work with the equations you wrote in step 7 until you can show algebraically that $a^2 + b^2 = c^2$. (Remember, $x + y = c$.)

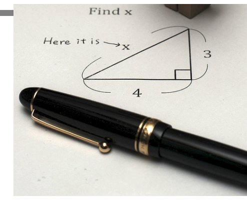
Use your set of triangles to help you prove the following two theorems algebraically. For this work, you will want to label the length of the altitude of the original right triangle *h*. The appropriate legs of the smaller right triangles should also be labeled *h.*

Right Triangle Altitude Theorem 1: If an altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the lengths of the two segments formed on the hypotenuse.

Right Triangle Altitude Theorem 2: If an altitude is drawn to the hypotenuse of a right triangle, the length of each leg of the right triangle is the geometric mean between the length of the hypotenuse and the length of the segment on the hypotenuse adjacent to the leg.

Use your set of triangles to help you find the values of *x* and *y* in the following diagram.





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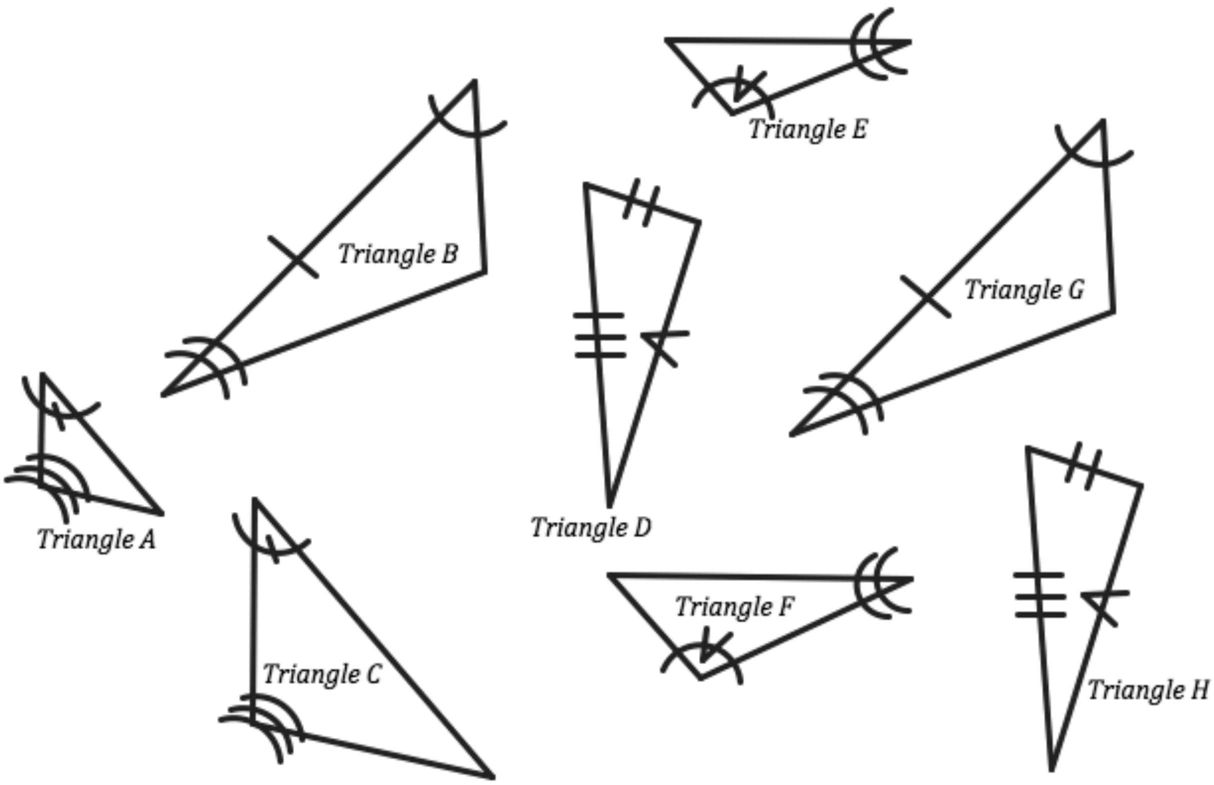
Ready, Set, Go!

Ready

Topic: Determining similarity and congruence in triangles.

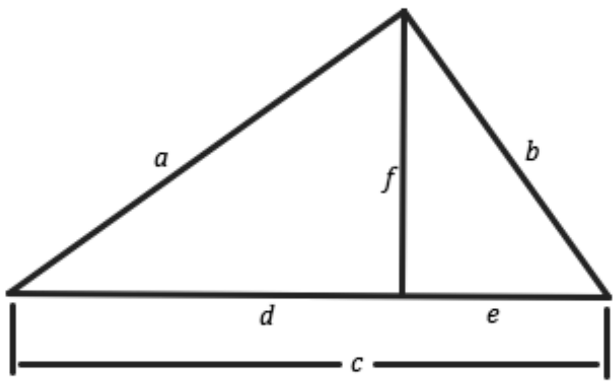
1. Determine which of the triangles below are similar and which are congruent. Justify your conclusions. Give your reasoning for the triangles you pick to be similar and congruent.



Set

Topic: Similarity in right triangles.

Use the given right triangles with altitudes drawn to the hypotenuse to correctly complete the proportions.



2. $\frac{a}{c} = \frac{f}{?}$

3. $\frac{a}{f} = \frac{c}{?}$

4. $\frac{a}{b} = \frac{f}{?}$

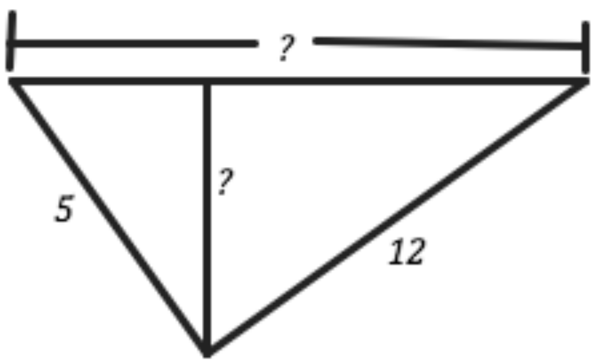
5. $\frac{a}{d} = \frac{c}{?}$

6. $\frac{f}{d} = \frac{e}{?}$

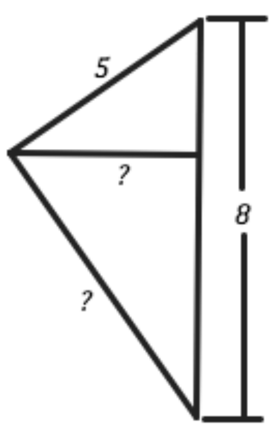
7. $\frac{b}{c} = \frac{e}{?}$

Find the missing value for each right triangle with altitude.

8.



9.

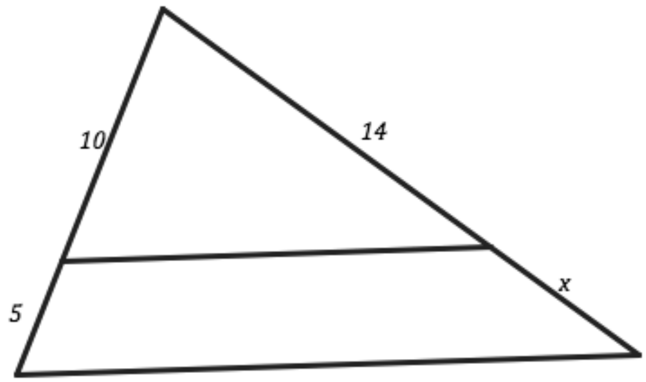


Go

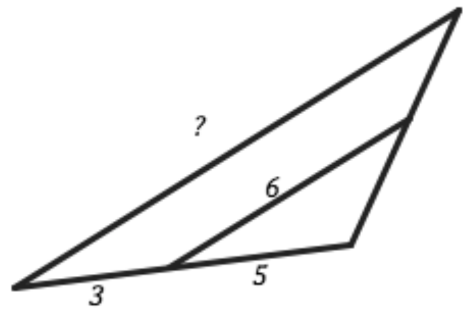
Topic: Using similarity and parallel lines to solve problems
Finding Geometric and Arithmetic Means

In each problem determine the desired values using the similar triangles parallel lines and proportional relationships. Write a proportion and solve.

10.



11.



Analyze each table below closely and determine the missing values based on the given information and values in the table.

12. An *Arithmetic* Sequence

Term	1	2	3	4
Value	7			22

13. A *Geometric* Sequence

Term	1	2	3	4
Value	7			56

14. An *Arithmetic* Sequence

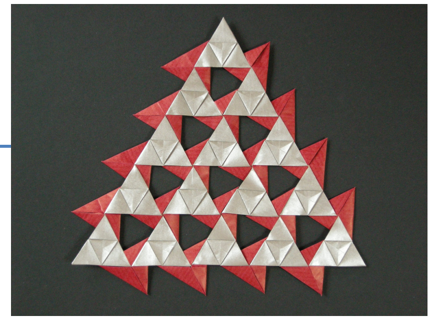
Term	5	6	7	8
Value	10			43

15. A *Geometric* Sequence

Term	7	8	9	10
Value	3			24

6.8 Are Relationships Predictable?

A Develop Understanding Task



1. In your notebook draw a right triangle with one angle of 60^0.
   Measure each side of your triangle as accurately as you can with a centimeter ruler.
   Using the 60^0 angle as the ***angle of reference*** list the measure for each of the following:

   Length of the ***adjacent*** side:

   Length of the ***opposite*** side:

   Length of the ***hypotenuse***:

2. Create the following ratios using your measurements:

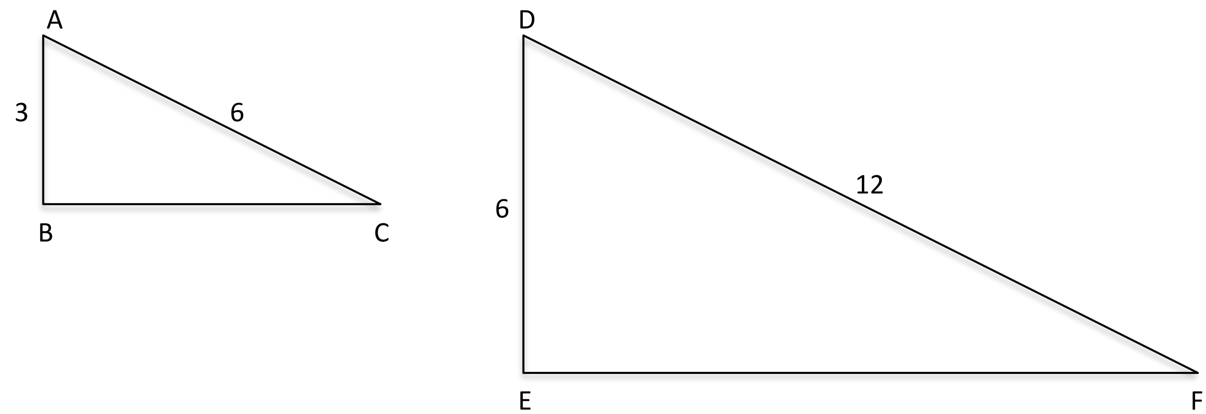
   $$\frac{opposite\ side}{hypotenuse} =$$

   $$\frac{adjacent\ side}{hypotenuse} =$$

   $$\frac{opposite\ side}{adjacent\ side} =$$

3. Compare your ratios with others that had a triangle of a different size. What do you notice? Explain any connections you find to others' work?

4. In the right triangles below find the missing side length and then create the desired ratios based on the angle of reference (angle A and angle D).



List the ratios for ΔABC using angle A as the angle of reference.

$$\frac{opposite\ side}{hypotenuse} =$$

$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

List the ratios for ΔDEF using angle D as the angle of reference.

$$\frac{opposite\ side}{hypotenuse} =$$

$$\frac{adjacent\ side}{hypotenuse} =$$

$$\frac{opposite\ side}{adjacent\ side} =$$

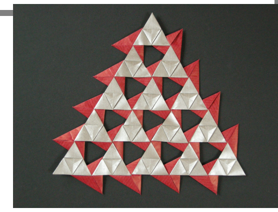
5. What do you notice about the ratios from the two given triangles? How do these ratios compare to the ratios from the triangle you made on the previous page?

6. What can you infer about the angle measures of ΔABC and ΔDEF ? Explain?

7. Why do the relationships you have noticed occur?

8. What can you conclude about the ratio of sides in a right triangle that has a 60^0? Would you think that right triangles with other angle measures would have a relationship among there ratios?

Name: Similarity & Right Triangle Trigonometry | 6.8



Ready, Set, Go!

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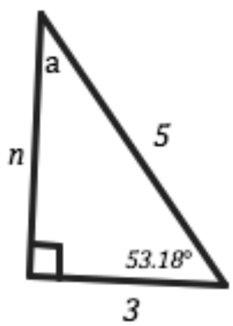
Ready

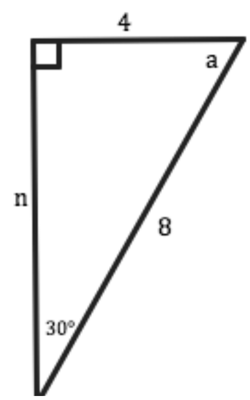
Topic: Properties of Right Triangles

For each right triangle below find the missing side *n* (Pythagorean Theorem could be helpful) and the missing angle, a (Angle Sum Theorem for Triangles could be useful).

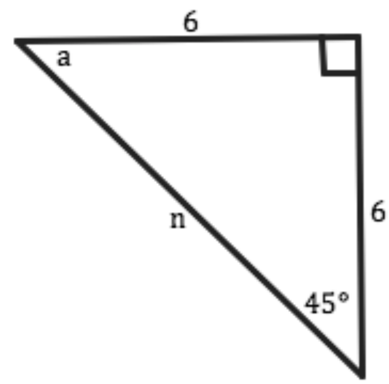
1.



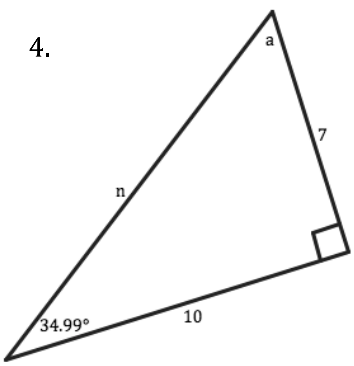
2.



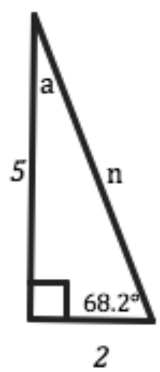
3.



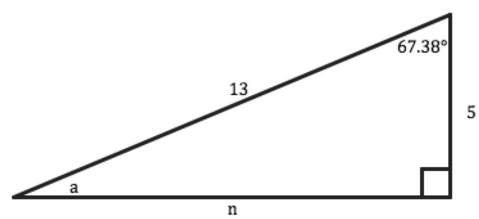
4.



5.



6.

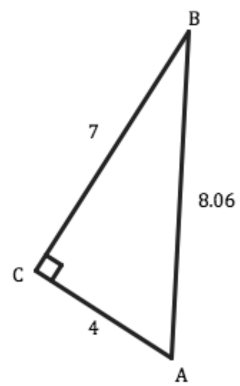


Set

Topic: Creating Trigonometric Ratios for Right Triangles

For each right triangle and the identified angle of reference create the desired trigonometric ratios. If any sides of the triangle are missing, find them before determining the ratio.

7.



a. *cos* (A)=

b. *sin* (A)=

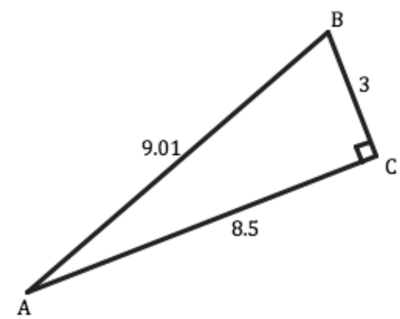
c. *tan* (A)=

d. *cos* (B)=

e. *sin* (B)=

f. *tan* (B)=

8.



a. *cos* (A)=

b. *sin* (A)=

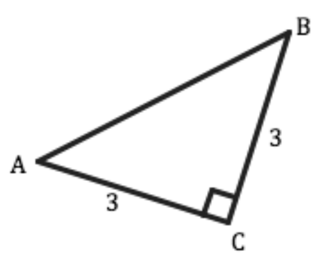
c. *tan* (A)=

d. *cos* (B)=

e. *sin* (B)=

f. *tan* (B)=

9.



a. *cos* (A)=

b. *sin* (A)=

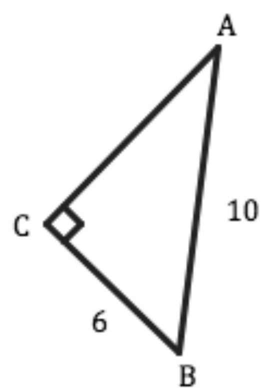
c. *tan* (A)=

d. *cos* (B)=

e. *sin* (B)=

f. *tan* (B)=

10.



a. *cos* (A)=

b. *sin* (A)=

c. *tan* (A)=

d. *cos* (B)=

e. *sin* (B)=

f. *tan* (B)=

Go

Topic: Factoring Quadratics

Write each of the quadratic functions in factored form and then determine both the x-intercepts as well as the y-intercept.

11. $f(x) = x^2 + 9x + 20$

a. Factored form:

b. x-intercepts:

c. y-intercept:

12. $g(x) = x^2 + 2x - 15$

a. Factored form:

b. x-intercepts:

c. y-intercept:

13. $h(x) = x^2 - 49$

a. Factored form:

b. x-intercepts:

c. y-intercept:

14. $r(x) = x^2 - 13x + 30$

a. Factored form:

b. x-intercepts:

c. y-intercept:

15. $f(x) = x^2 + 20x + 100$

a. Factored form:

b. x-intercepts:

c. y-intercept:

16. $g(x) = x^2 - 8x - 48$

a. Factored form:

b. x-intercepts:

c. y-intercept:

17. $h(x) = x^2 + 16x + 64$

a. Factored form:

b. x-intercepts:

c. y-intercept:

18. $k(x) = x^2 - 36$

a. Factored form:

b. x-intercepts:

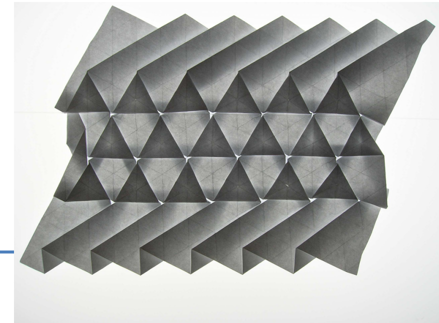
c. y-intercept:

19. $p(x) = x^2 - 2x - 24$

a. Factored form:

b. x-intercepts:

c. y-intercept:

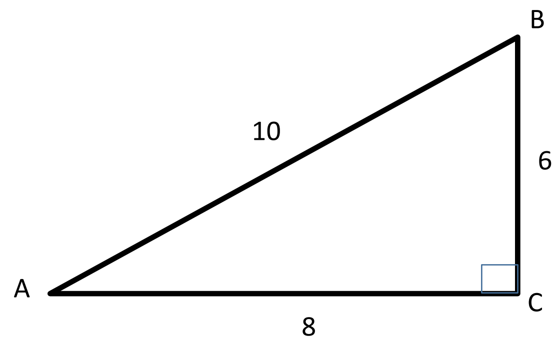


6.9 Relationships with Meaning

A Solidify Understanding Task

Part I

1. Use the information from the given triangle to write the following trigonometric ratios:



$sin\ (A) = \frac{opposite}{hypotenuse} =$

$cos\ (A) = \frac{adjacent}{hypotenuse} =$

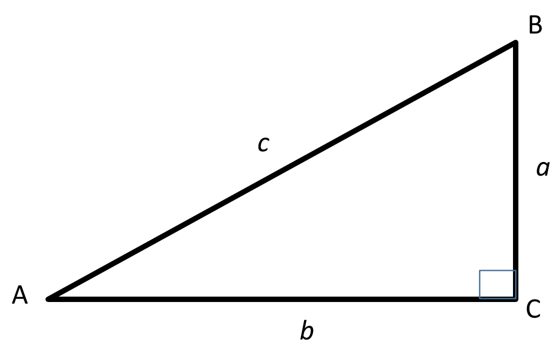
$tan\ (A) = \frac{opposite}{adjacent} =$

sin (B) =

cos (B) =

tan (B) =

2. Do the same for this triangle:



sin (A) =

cos (A) =

tan (A) =

sin (B) =

cos (B) =

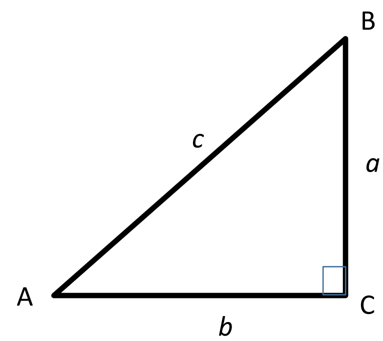
tan (B) =

3. Use the information above to write observations you notice about the relationships of trigonometric ratios.

4. Do you think these observations will always hold true? Why or why not?

Part 2

The following is a list of conjectures made by students about right triangles and trigonometric relationships. For each, state whether you think the conjecture is true or false. Justify your answer.



5. $\cos(A) = \sin(A)$

6. $\tan(A) = \frac{\sin(A)}{\cos(A)}$

7. $\sin(A) = \cos(90° - A)$

8. $\cos(A) = \sin(B)$

9. $\cos(B) = \sin(90° - A)$

10. $\tan(A) = \frac{1}{\tan(B)}$

Note the following convention used to write: $[\sin(A)]^2 = \sin^2(A)]$

11. $\sin^2(A) + cos^2(A) = 1$

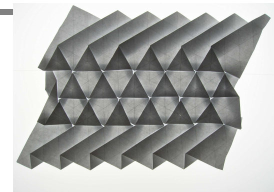
12. $1 - \sin(A)^2 = cos^2(A)$

13. $\sin^2(A) = \sin(A^2)$

Part III

14. Given: A right triangle with the following trigonometric ratio: *sin (30°)* = $\frac{1}{2}$, find all trigonometric ratios for this triangle. How do you know these values are always going to be true when given this angle?

Name: Similarity & Right Triangle Trigonometry | 6.9



Ready, Set, Go!

Ready

Topic: Solving equations and proportions.

Solve each equation below and justify your work.

1.

$8x - 10 = x + 11$	Justification

2.

$3x + 9 = 44 - 2x$	Justification

3.

$\frac{3}{5}x = 9$	Justification

4.

$\frac{2}{3} = \frac{x}{21}$	Justification

Set

Topic: Trigonometric Ratios and Connections between them.

Based on the given trigonometric ratio, sketch a triangle and find a possible value for the missing side as well as the other missing trig ratios. Angles A and B are the two non-right angles in a right triangle.

5. a. tan (A)= $\frac{3}{4}$
   b. sin (A)=
   c. cos (A)=
   d. tan (B)=
   e. sin (B)=
   f. cos (B)=

g. Sketch of Triangle:

6. a. tan (A)=
   b. sin (A)=
   c. cos (A)=
   d. tan (B)=
   e. sin (B)= $\frac{8}{17}$
   f. cos (B)=

g. Sketch of Triangle:

7. a. tan (A)=
   b. sin (A)=
   c. cos (A)=$\frac{12}{13}$
   d. tan (B)=
   e. sin (B)=
   f. cos (B)=

g. Sketch of Triangle:

8. a. tan (A)=
   b. sin (A)=
   c. cos (A)=
   d. tan (B)=
   e. sin (B)=$\frac{1}{\sqrt{2}}$
   f. cos (B)=

g. Sketch of Triangle:

Given a right triangle with angles A and B as the non-right angles. Determine if the statements below are true or false. Justify your reasoning and show your argument.

9. $\cos(A) = \frac{1}{\sin A}$

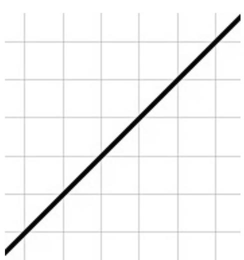
10. $\tan(B) = \tan(90° - A)$

11. $\tan(A) \cdot \cos(A) = \sin(A)$

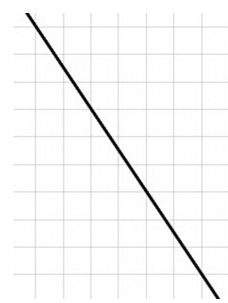
Go

Topic: Slope as a ratio

On each grid draw three slope defining triangles of different sizes and label the rise and run then write the slope of the line.

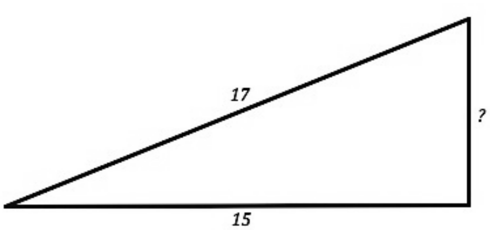
12. 

13. 

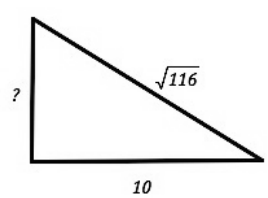
14. 

Find the missing length in each right triangle. Then determine the slope of the hypotenuse.

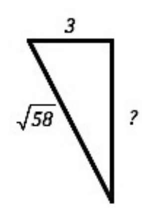
15.



16.

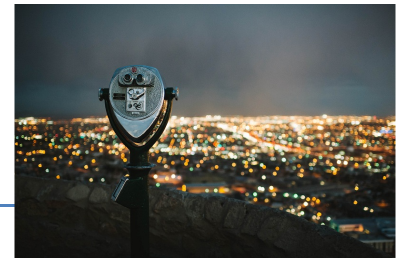


17.



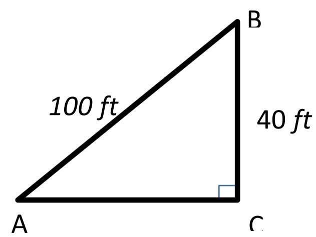
6.10 Finding the Value of a Relationship

A Solidify Understanding Task



Part I: What's your angle?

Andrea and Bonita are going for a walk straight up the side of a hill. Andrea decided to stretch before heading up the hill while Bonita thought this would be a good time to get a head start. Once Bonita was 100 feet away from Andrea, she stopped to take a break and looked at her GPS device that told her that she had walked 100 feet and had already increased her elevation by 40 feet. With a bit of time to waste, Bonita wrote down the trigonometric ratios for ∠A and for ∠B.



1. Name the trigonometric ratios for ∠A and for ∠B.

When Andrea caught up, she said "What about the unknown angle measures? When I was at the bottom and looked up to see you, I was thinking about the "upward" angle measure from me to you. Based on your picture, this would be ∠A." Bonita knows that she can solve equations involving variables by isolating the variable. She then wrote the following trig ratio she found: $\sin A = \frac{2}{5}$ and said "Now we just have to get 'A' by itself." Together, the girls talked about using *inverse trigonometric functions* to find unknown angle values. Bonita explained, "The inverse of sine is also written as sin^{-1}. To solve for ∠A, take the inverse of the trigonometric function on both sides to get ∠A by itself." Using Bonita's explanation, Andrea solved for ∠A using the following steps:

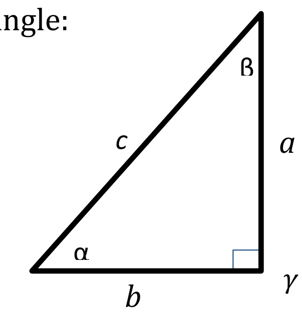
$$\sin A = \frac{2}{5}$$

$$sin^{-1}(\sin A) = sin^{-1}\left(\frac{2}{5}\right)$$

$$A \approx 23.578°$$

2. Use the trigonometric ratio you found for $\cos B$ to find the value of ∠B.

3. Find all unknown values for the following right triangle:
    a) $\angle\alpha =$ _____
    b) $\angle\beta =$_____
    c) $\angle\gamma =$ _90°_
    d) $a =$ _12 m_
    e) $b =$ _8 m_
    f) $c =$ ______



4. Bonita and Andrea started talking about all of the ways to find unknown values in right triangles and decided to make a list. What do you think should be on their list? Be specific and precise in your description. For example, 'trig ratios' is not specific enough. You may use the following sentence frame to assist with writing each item in your list:

    When given ____________________, you can find ____________________ by _______________.

Part II: Angle of elevation and angle of depression

During their hike, Andrea mentioned that she looked up to see Bonita. In mathematics, when you look straight ahead, we say your line of sight is a horizontal line. From the horizontal, if you look up, the angle from the horizontal to your line of sight is called the **angle of elevation.** Likewise, if you are looking down, the angle from the horizontal to your line of sight is called the **angle of depression.**

5. After looking at this description, Andrea mentioned that her angle of elevation to see Bonita was 23.5°. They both agreed. Bonita then said her angle of depression to Andrea was 66.5°. Andrea agreed that it was an angle of depression but said Bonita's angle of depression is 23.5°. Who do you think is correct? Use drawings and words to justify your conclusion.

6. What conclusion can you make regarding the angle of depression and the angle of elevation? Why?

Name: **Similarity & Right Triangle Trigonometry | 6.10**

Ready, Set, Go!

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Ready

Topic: Modeling contexts with visuals

For each story presented below *sketch a picture* of the situation *and label as much of the picture as possible.* No need to answer the question or find the missing values, simply represent the situation with a sketch.

1. Jill put a ladder up against the house to try and reach a light that is out and needs to be changed. She knows the ladder is 10 feet long and the distance from the base of the house to the bottom of the ladder is 4 feet.

2. Francis is a pilot of an airplane that if flying at an altitude of 3,000 feet when the plane begins its decent toward the ground. If the angle of decent of the plane is 15^0 how much farther will the plane fly before it is on the ground?

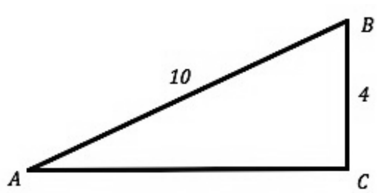
3. Abby is standing at the top of a very tall skyscraper and looking through a telescope at the scenery all around her. The angle of decline on the telescope says 35^0 and Abby knows she is 30 floors up and each floor is 15 feet tall. How far from the base of the building is the object that Abby is looking at?

Set

Topic: Solving triangles using Trigonometric Ratios

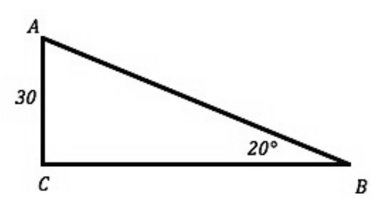
In each triangle find the missing angles and sides. In all questions $m\angle C = 90°$

4.



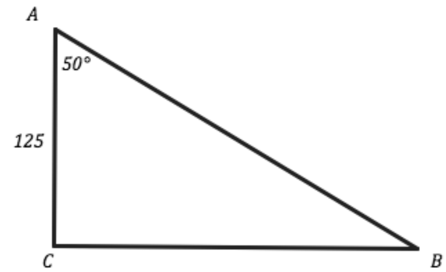
a. $m\angle A =$ b. $m\angle B =$ c. $AC =$

5.



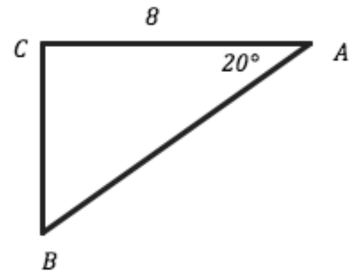
a. $m\angle A =$ b. $AB =$ c. $BC =$

6.



a. $m\angle B =$ b. $AB =$ c. $BC =$

7.



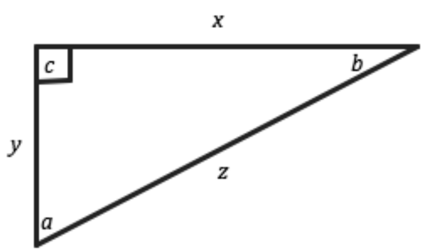
a. $m\angle B =$ b. $AB =$ c. $BC =$

Go

Topic: Trigonometric Ratios

Use the given right triangle to identify the trigonometric ratios. And angles were possible.

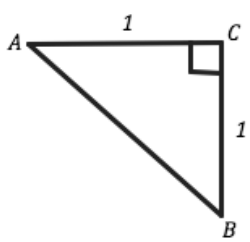
8.



a. $\sin(a) =$ b. $\cos(a) =$ c. $\tan(a) =$

d. $\sin(b) =$ e. $\cos(b) =$ f. $\tan(b) =$

9.

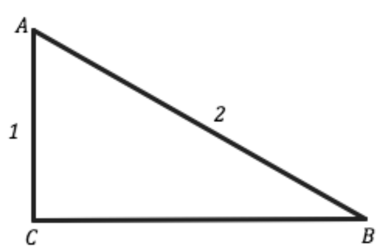


a. $\sin(A) =$ b. $\cos(A) =$ c. $\tan(A) =$

d. $\sin(B) =$ e. $\cos(B) =$ f. $\tan(B) =$

g. m∠A= h. m∠B=

10.



a. $\sin(A) =$ b. $\cos(A) =$ c. $\tan(A) =$

d. $\sin(B) =$ e. $\cos(B) =$ f. $\tan(B) =$

g. m∠A= h. m∠B= m∠C= 90°

6.11 Solving Right Triangles Using Trigonometric Relationships

A Practice Understanding Task

http://www.flickr.com/photos/whitnuld

I. For each problem:

- make a drawing
- write an equation
- solve (do not forget to include units of measure)

1. Carrie places a 10 foot ladder against a wall. If the ladder makes an angle of 65° with the level ground, how far up the wall is the top of the ladder?

2. A flagpole casts a shadow that is 15 feet long. The angle of elevation at this time is 40°. How tall is the flagpole?

3. In southern California, there is a six mile section of Interstate 5 that increases 2,500 feet in elevation. What is the angle of elevation?

4. A hot air balloon is 100 feet straight above where it is planning to land. Sarah is driving to meet the balloon when it lands. If the angle of elevation to the balloon is 35°, how far away is Sarah from where the balloon will land?

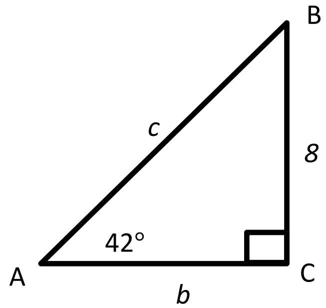
5. An airplane is descending as it approaches the airport. If the angle of depression from the plane to the ground is 7°, and the plane is 2,000 feet above the ground, what is the distance from the plane to the airport?

6. Michelle is 60 feet away from a building. The angle of elevation to the top of the building is 41°. How tall is the building?

7. A ramp is used for loading equipment from a dock to a ship. The ramp is 10 feet long and the ship is 6 feet higher than the dock. What is the angle of elevation of the ramp?

II. For each right triangle below, find all unknown side lengths and angle measures:

8.



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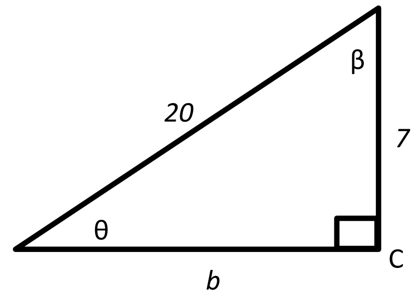

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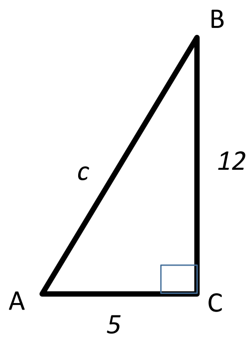
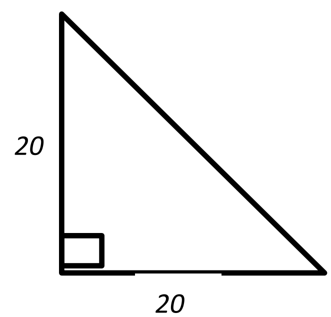


11.



12. Draw and find the missing angle measures of the right triangle whose sides measure 4, 6, and 8.

III. Determine the values of the two remaining trigonometric ratios when given one of the trigonometric ratios.

13. $\cos(\alpha) = \frac{3}{5}$

14. $\tan(\theta) = \frac{8}{3}$

15. $\sin(\beta) = \frac{4}{7}$

Name: Similarity & Right Triangle Trigonometry | 6.11

Ready, Set, Go!

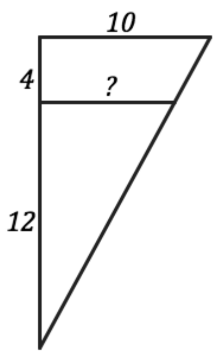
http://www.flickr.com/photos/whitnuld

Ready

Topic: Similar triangles and proportional relationships with parallels

Based on each set of triangles or parallel lines create a proportion and solve it to find the missing values.

1.



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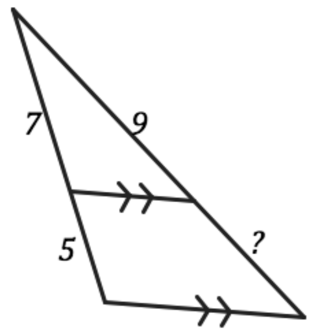

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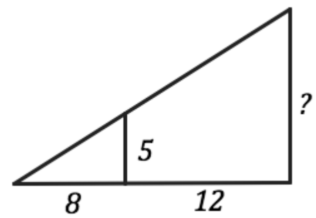

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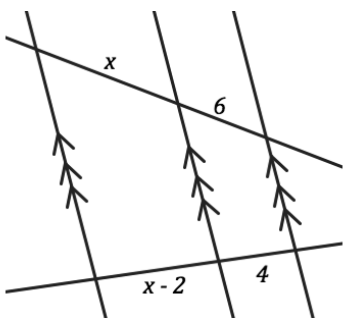

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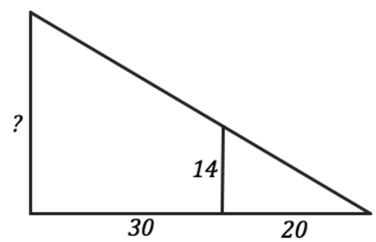

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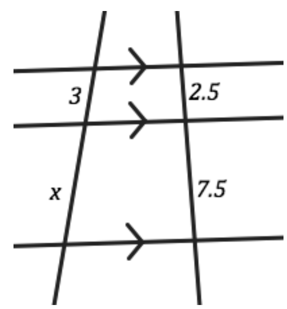
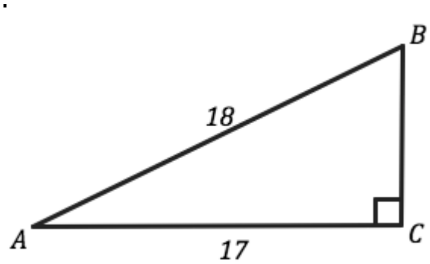


Set

Topic: Solving triangles with trigonometric ratios and Pythagorean Theorem

Solve each right triangle. Give any missing sides and missing angles.

7.



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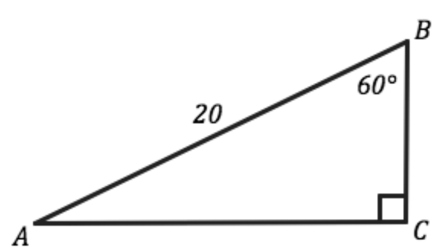

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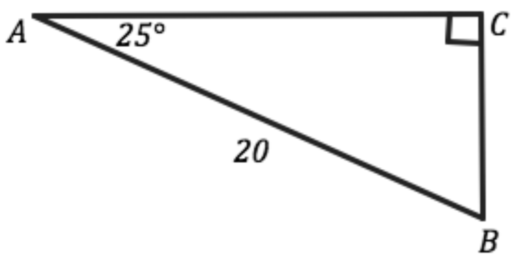

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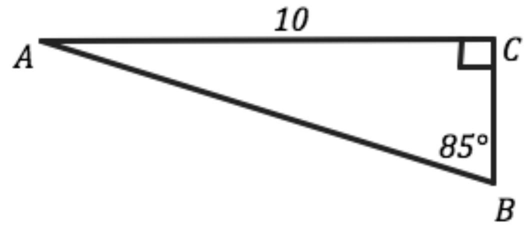

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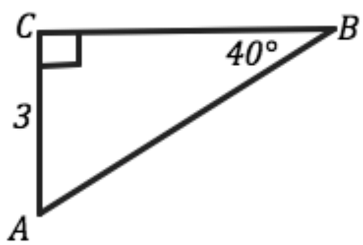

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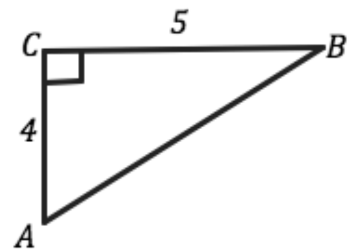


Use the given trigonometric ratio to sketch a right triangle and find the missing sides and angles.

13. $\sin(A) = \frac{1}{2}$

14. $\cos(B) = \frac{3}{5}$

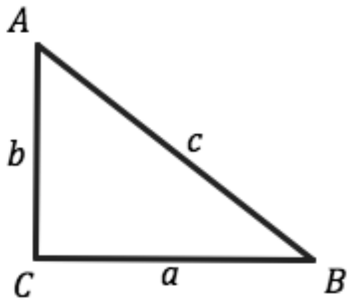
15. $\tan(B) = \frac{6}{7}$

16. $\sin(B) = \frac{7}{10}$

17. $\cos(A) = \frac{5}{8}$

18. $\tan(A) = \frac{4}{15}$

19. Use the right triangle below to determine which of the following are equivalent.



a. $\sin(A)$

b. $\cos(A)$

c. $\tan(A)$

d. $\sin(B)$

e. $\cos(B)$

f. $\tan(B)$

g. $\frac{\sin(A)}{\cos(A)}$

h. $\frac{1}{\tan(A)}$

i. 1

j. $a^2 + b^2$

k. c^2

l. $sin^2(b) + cos^2(b)$

Go

Topic: Applying trigonometric ratios and identities to solve problems.

Sketch a drawing of the situation. Solve each problem.

20. Mark is building his son a pitcher's mound so he can practice for his upcoming baseball season in the back yard. Mark knows that the league requires an incline of 12^o and an elevation of 8 inches in height. How long will the front of the pitcher's mound need to be?

21. Susan is designing a wheelchair ramp. Wheelchair ramps require a slope that is no more than 1-inch of rise for every 12-inches of ramp length. Susan wants to determine how much horizontal distance a ramp of 6-feet in length will span? She also wants to know the degree of incline from the base of the ramp to the ground.

22. Michael is designing a house with a roof pitch of 5. Roof pitch is the number of inches that a roof will rise for every 12 inches of run. What is the angle that will need to be used in building the trusses and supports for the roof? What is the angle of a roof with 5/12 pitch increase? At the peak of the roof what angle will there be when the front and the back of the roof come together?