Secondary Two Mathematics: An Integrated Approach
Module 8
Circles and Other Conics

By

The Mathematics Vision Project:
Scott Hendrickson, Joleigh Honey,
Barbara Kuehl, Travis Lemon, Janet Sutorius
www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 8 – Circles and Other Conics

Classroom Task: 8.1 Circling Triangles (or Triangulating Circles) – A Develop Understanding Task
Deriving the equation of a circle using the Pythagorean Theorem (**G.GPE.1**)
Ready, Set, Go Homework: Circles and Other Conics 8.1

Classroom Task: 8.2 Getting Centered – A Solidify Understanding Task
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Ready, Set, Go Homework: Circles and Other Conics 8.2

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To build understanding of the definition of a parabola as the set of all points equidistant from a given point (the focus) and a line (the directrix). (**G.GPE, G.GPE.2**)
Ready, Set, Go Homework: Circles and Other Conics 8.7H

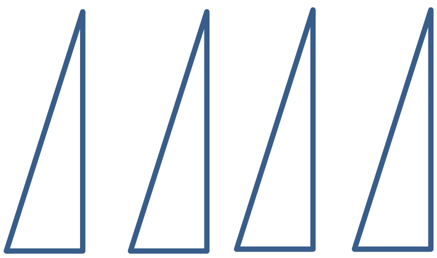
Classroom Task: 8.8H What Happens if … ? – A Solidify Understanding Task
To develop the definition of a hyperbola as the set of all points in a plane such that the difference between the distances from the point to each of the two foci is constant. (**G.GPE, G.GPE.3**)
Ready, Set, Go Homework: Circles and Other Conics 8.8H

8.1 Circling Triangles (or Triangulating Circles)

A Develop Understanding Task

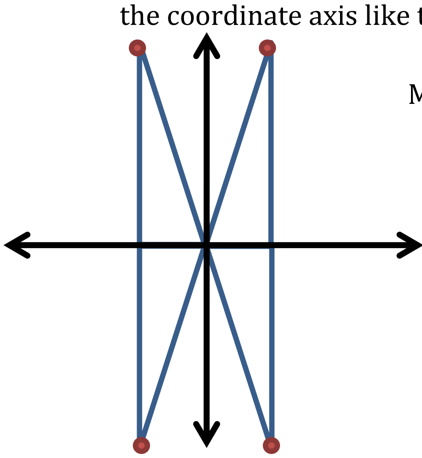
Using the corner of a piece of colored paper and a ruler, cut a right triangle with a 6" hypotenuse, like so:

Use this triangle as a pattern to cut three more just like it, so that you have a total of four congruent triangles.



1. Choose one of the legs of the first triangle and label it x and label the other leg y. What is the relationship between the three sides of the triangle?

2. When you are told to do so, take your triangles up to the board and place each of them on the coordinate axis like this:



Mark the point at the end of each hypotenuse with a pin.

3. What shape is formed by the pins after the class has posted all of their triangles? Why would this construction create this shape?

4. What are the coordinates of the pin that you placed in:
   a. the first quadrant?
   b. the second quadrant?
   c. the third quadrant?
   d. the fourth quadrant?

5. Now that the triangles have been placed on the coordinate plane, some of your triangles have sides that are of length –*x* or –*y*. Is the relationship $x^2 + y^2 = 6^2$ still true for these triangles? Why or why not?

6. What would be the equation of the graph that is the set on all points that are 6" away from the origin?

7. Is the point (0, -6) on the graph? How about the point (3, 5.193)? How can you tell?

8. If the graph is translated 3 units to the right and 2 units up, what would be the equation of the new graph? Explain how you found the equation.

Name:

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Ready, Set, Go!

Ready

Topic: Special products and factors

Factor the following as the difference of 2 squares or as a perfect square trinomial. Do not factor if they are neither.

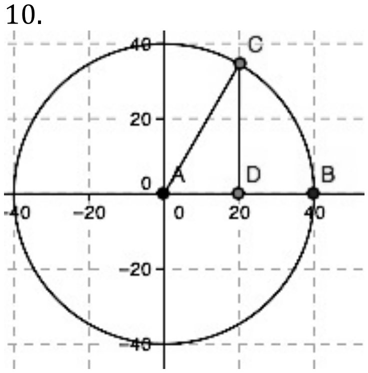
1. $25b^2 - 49y^2$
2. $100b^2 - 20b + 1$
3. $36b^2 + 30b + 25$
4. $x^2 + 6x - 9 - y^2$
5. $x^2 - 2xy + y^2 - 25$
6. $a^2 + 2ab + b^2 + 4a + 4b + 4$
7. $x^2 + 2xy + 12x + y^2 + 12y + 36$
8. $x^2 + 2cs + 2dx + c^2 + 2cd + d^2$
9. $144x^2 - 312xy + 169y^2$

Set

Topic: Writing the equations of circles.

Write the equation of each circle centered at the origin.

10.



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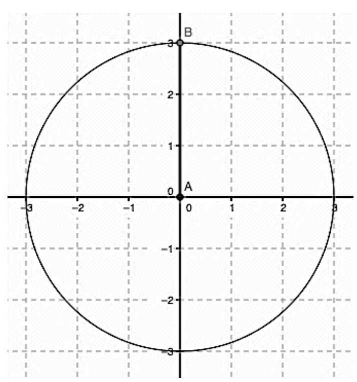

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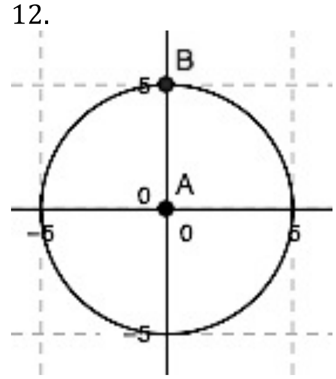

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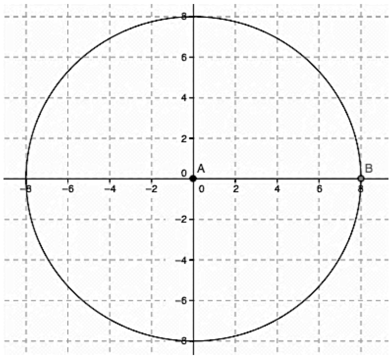

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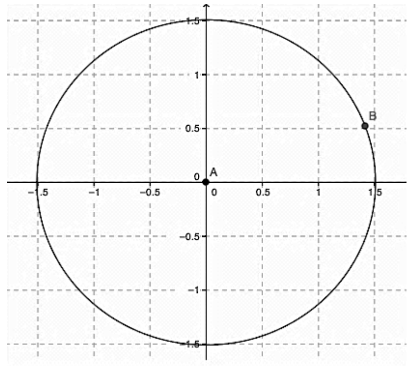

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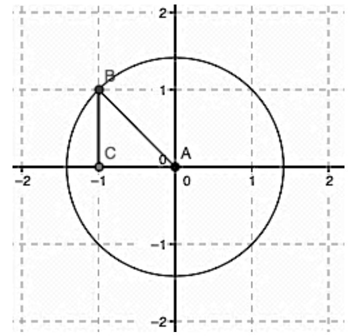

Go

Topic: Verifying Pythagorean triples.

Identify which sets of numbers could be the sides of a right triangle. Show your work.

16. $\{9, 12, 15\}$

17. $\{9, 10, \sqrt{19}\}$

18. $\{1, \sqrt{3}, 2\}$

19. $\{2, 4, 6\}$

20. $\{\sqrt{3}, 4, 5\}$

21. $\{10, 24, 26\}$

22. $\{\sqrt{2}, \sqrt{7}, 3\}$

23. $\{2\sqrt{2}, 5\sqrt{3}, 9\}$

24. $\{4ab^3\sqrt{10}, 6ab^3, 14ab^3\}$

8.2 Getting Centered

A Solidify Understanding Task

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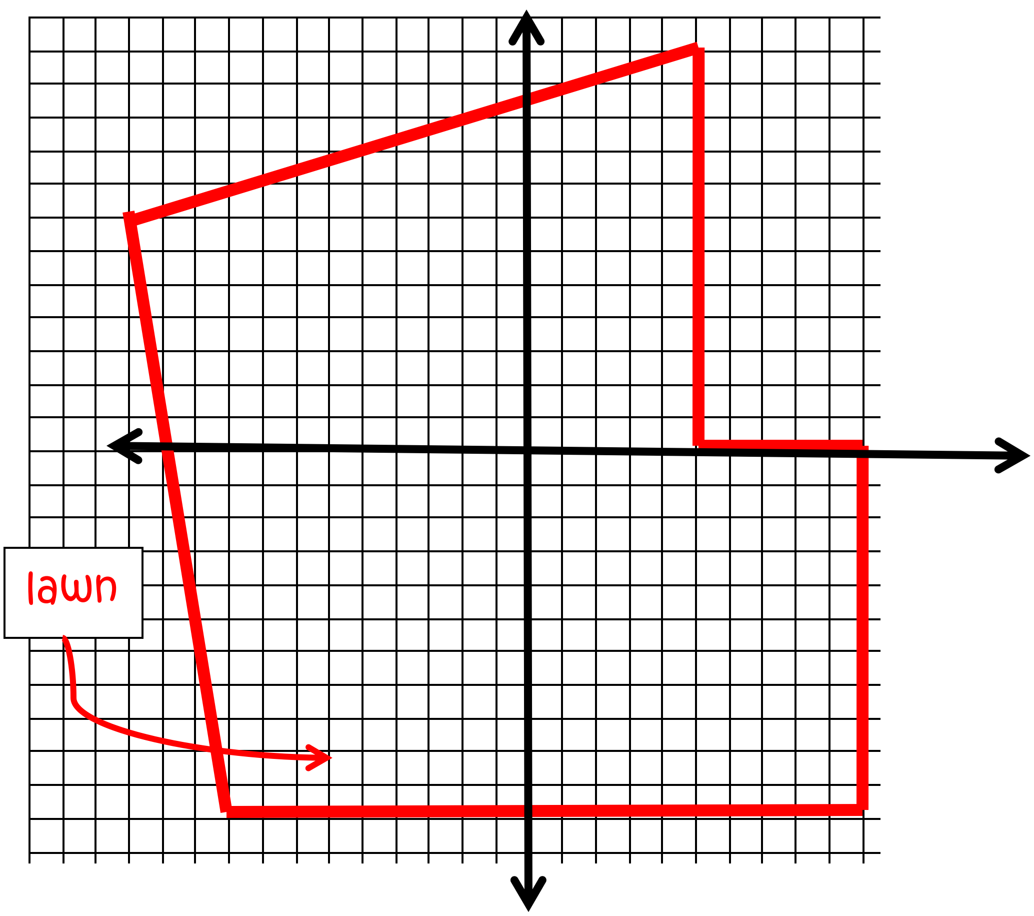
Malik's family has decided to put in a new sprinkling system in their yard. Malik has volunteered to lay the system out. Sprinklers are available at the hardware store in the following sizes:

- Full circle, maximum 15' radius
- Half circle, maximum 15' radius
- Quarter circle, maximum 15' radius

All of the sprinklers can be adjusted so that they spray a smaller radius. Malik needs to be sure that the entire yard gets watered, which he knows will require that some of the circular water patterns will overlap. He gets out a piece of graph paper and begins with a scale diagram of the yard. In this diagram, the length of the side of each square represents 5 feet.



1. As he begins to think about locating sprinklers on the lawn, his parents tell him to try to cover the whole lawn with the fewest number of sprinklers possible so that they can save some money. The equation of the first circle that Malik draws to represent the area watered by the sprinkler is:

$$(x+25)^2+(y+20)^2=225$$

   Draw this circle on the diagram using a compass.

2. Lay out a possible configuration for the sprinkling system that includes the first sprinkler pattern that you drew in #1.

3. Find the equation of each of the full circles that you have drawn.

Malik wrote the equation of one of the circles and just because he likes messing with the algebra, he did this:

Original equation: $(x-3)^2+(y+2)^2 = 225$

$$x^2-6x+9+ y^2+4y+4=225$$

$$x^2+ y^2-6x+4y-212=0$$

Malik thought, "That's pretty cool. It's like a different form of the equation. I guess that there could be different forms of the equation of a circle like there are different forms of the equation of a parabola or the equation of a line." He showed his equation to his sister, Sapana, and she thought he was nuts. Sapana, said, "That's a crazy equation. I can't even tell where the center is or the length of the radius anymore." Malik said, "Now it's like a puzzle for you. I'll give you an equation in the new form. I'll bet you can't figure out where the center is."

Sapana said, "Of course, I can. I'll just do the same thing you did, but work backwards."

4. Malik gave Sapana this equation of a circle:

$$x^2 + y^2 - 4x + 10y + 20 = 0$$

Help Sapana find the center and the length of the radius of the circle.

5. Sapana said, "Ok. I made one for you. What's the center and length of the radius for this circle?"

$$x^2 + y^2 + 6x - 14y - 42 = 0$$

6. Sapana said, "I still don't know why this form of the equation might be useful. When we had different forms for other equations like lines and parabolas, each of the various forms highlighted different features of the relationship." Why might this form of the equation of a circle be useful?

$$x^2 + y^2 + Ax + By + C = 0$$

Name: Circles and Other Conics | 8.2

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Ready, Set, Go!

Ready

Topic: Making perfect square trinomials.

Fill in the number that completes the square. Then write the trinomial in factored form.

1. x^2+6x+ __________ 2. x^2-14x+ __________

3. x^2-50x+ __________ 4. x^2-28x+ __________

On the next set of problems, leave the number that completes the square as a fraction. Then write the trinomial in factored form.

5. x^2-11x+ __________ 6. x^2+7x+ __________ 7. x^2+15x+ __________

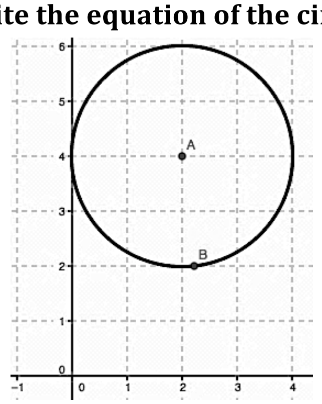
8. $x^2+\frac{2}{3}x+$ __________ 9. $x^2-\frac{1}{5}x+$ __________ 10. $x^2-\frac{3}{4}x+$ __________

Set

Topic: Writing equations of circles with center (h, k) and radius *r*.

Write the equation of the circle.

11.



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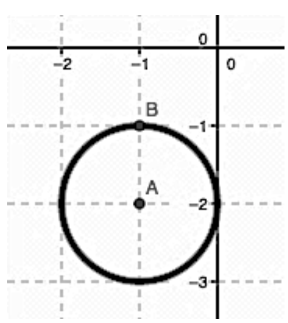

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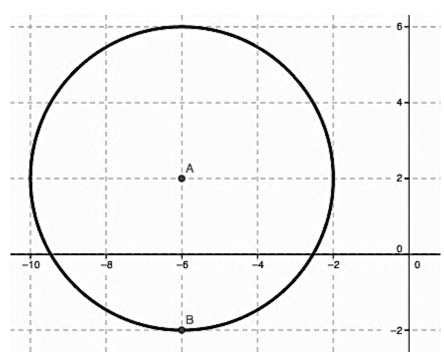


Write the equation of the circle with the given center and radius. Then write it in expanded form.

14. Center: (5, 2), Radius: 13

15. Center: (-6, -10), Radius: 9

16. Center: (0, 8), Radius: 15

17. Center: (19, -13), Radius: 1

18. Center: (-1, 2), Radius: 10

19. Center: (-3, -4), Radius: 8

Go

Topic: Verifying if a point is a solution.

Identify which point is a solution to the given equation. Show your work.

20. $y = \frac{4}{5}x - 2$ a. (-15, -14) b. (10, 10)

21. $y = 3|x|$ a. (-4, -12) b. $(-\sqrt{5}, 3\sqrt{5})$

22. $y = x^2 + 8$ a. $(\sqrt{7}, 15)$ b. $(\sqrt{7}, -1)$

23. $y = -4x^2 + 120$ a. $(5\sqrt{3}, -180)$ b. $(5\sqrt{3}, 40)$

24. $x^2 + y^2 = 9$ a. $(8, -1)$ b. $(-2, \sqrt{5})$

25. $4x^2 - y^2 = 16$ a. $(-3, \sqrt{10})$ b. $(-2\sqrt{2}, 4)$

8.3 Circle Challenges

A Practice Understanding Task

Once Malik and Sapana started challenging each other with circle equations, they got a little more creative with their ideas. See if you can work out the challenges that they gave each other to solve. Be sure to justify all of your answers.

1. Malik's challenge:
   What is the equation of the circle with center (-13,-16) and containing the point (-10,-16) on the circle?

2. Sapana's challenge:
   The points (0, 5) and (0,-5) are the endpoints of the diameter of a circle. The point (3, y) is on the circle. What is a value for y?

3. Malik's challenge:
   Find the equation of a circle with center in the first quadrant and is tangent to the lines $x = 8$, $y = 3$, and $x = 14$.

4. Sapana's challenge:
   The points (4,-1) and (-6,7) are the endpoints of the diameter of a circle. What is the equation of the circle?

5. Malik's challenge:
   Is the point (5,1) inside, outside, or on the circle $x^2 - 6x + y^2 + 8y = 24$? How do you know?

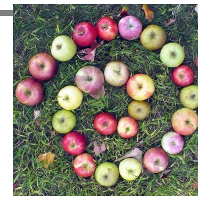
6. Sapana's challenge:
   The circle defined by $(x-1)^2 + (y+4)^2 = 16$ is translated 5 units to the left and 2 units down. Write the equation of the resulting circle.

7. Malik's challenge:
   There are two circles, the first with center (3,3) and radius r_1, and the second with center (3, 1) and radius r_2.
   a. Find values r_1 and r_2 of so that the first circle is completely enclosed by the second circle.

   b. Find one value of r_1 and one value of r_2 so that the two circles intersect at two points.

   c. Find one value of r_1 and one value of r_2 so that the two circles intersect at exactly one point.

Name: Circles and Other Conics | 8.3



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Ready, Set, Go!

Ready

Topic: Finding the distance between 2 points.

Use the distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ **to find the distance between the given points. Leave your answer in simplest radical form.**

1. $A(18,-12)\ B(10,4)$

2. $G(-11,-9)\ H(-3,7)$

3. $J(14,-20)\ K(5,5)$

4. $M(1,3)\ P(-2,7)$

5. $Q(8,2)\ R(3,7)$

6. $S(-11,2\sqrt{2})\ T(-5,-4\sqrt{2})$

7. $W(-12,-2\sqrt{2})\ Z(-7,-3\sqrt{2})$

Set

Topic: Writing equations of circles

Use the information provided to write the equation of the circle in standard form.

8. Center $(-16,-5)$ and the circumference is 22π

9. Center $(13,-27)$ and the area is 196π

10. Diameter measures 15 units and the center is at the intersection of y = x + 7 and y = 2x – 5

11. Lies in quadrant 2 Tangent to $x=-12$ *and* $x=-4$

12. Center (-14 , 9) Point on circle (1 , 11)

13. Center lies on the y axis Tangent to y = -2 and y = -17

14. Three points on the circle are $(-8,5),(3,-6),(14,5)$

15. I know three points on the circle are (-7,6), (9,6), and (-4,13). I think that the equation of the circle is $(x-1)^2+(y-6)^2=64$. Is this the correct equation for the circle?
Convince me!

Go

Topic: Finding the value of "B" in a quadratic of the form Ax^2+Bx+C in order to create a perfect square trinomial.

Find the value of "B," that will make a perfect square trinomial. Then write the trinomial in factored form.

16. $x^2+\_\_\_\_\_x+36$

17. $x^2+\_\_\_\_\_x+100$

18. $x^2+\_\_\_\_\_x+225$

19. $9x^2+\_\_\_\_\_x+225$

20. $16x^2+\_\_\_\_\_x+169$

21. $x^2+\_\_\_\_\_x+5$

22. $x^2+\_\_\_\_\_x+\frac{25}{4}$

23. $x^2+\_\_\_\_\_x+\frac{9}{4}$

24. $x^2+\_\_\_\_\_x+\frac{49}{4}$

8.4 Directing Our Focus

A Develop Understanding Task

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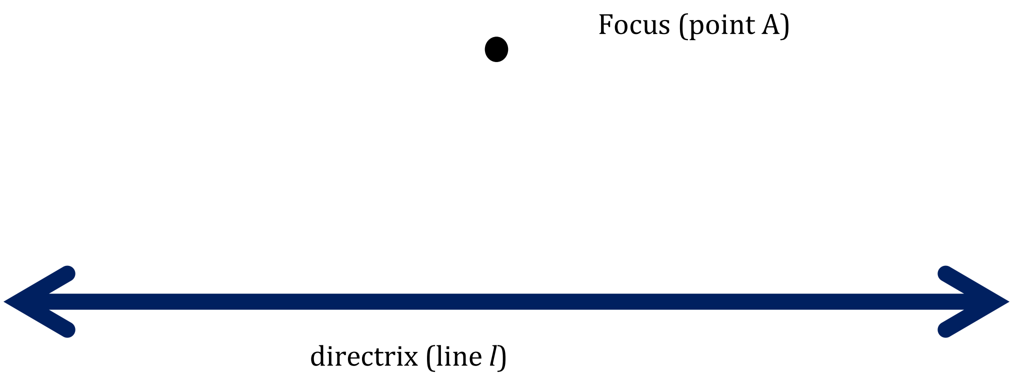
On a board in your classroom, your teacher has set up a point and a line like this:

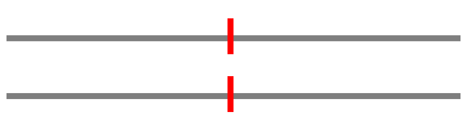


We're going to call the line a directrix and the point a focus. They've been labeled on the drawing.

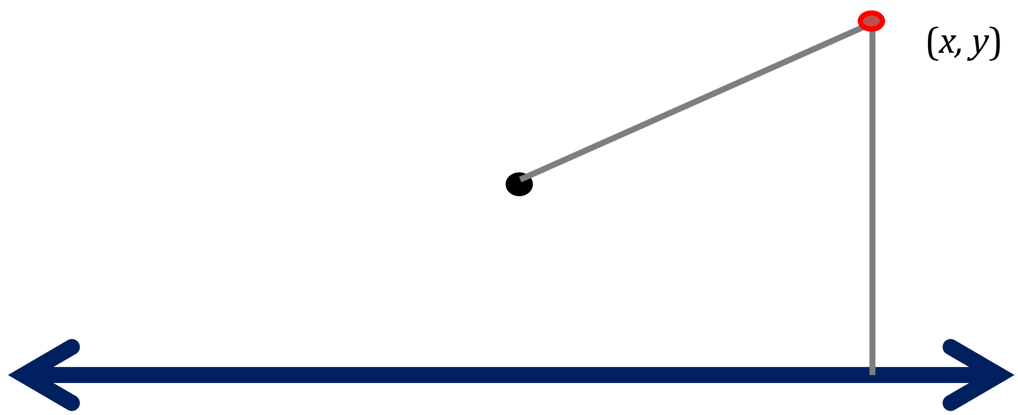
Similar to the circles task, the class is going to construct a geometric figure using the focus (point A) and directrix (line *l*).

1. Cut two pieces of string with the same length.

2. Mark the midpoint of each piece of string with a marker.



3. Position the string on the board so that the midpoint is equidistant from the focus (point A) and the directrix (line *l*), which means that it must be perpendicular to the directrix. While holding the string in this position, put a pin through the midpoint. Depending on the size of your string, it will look something like this:



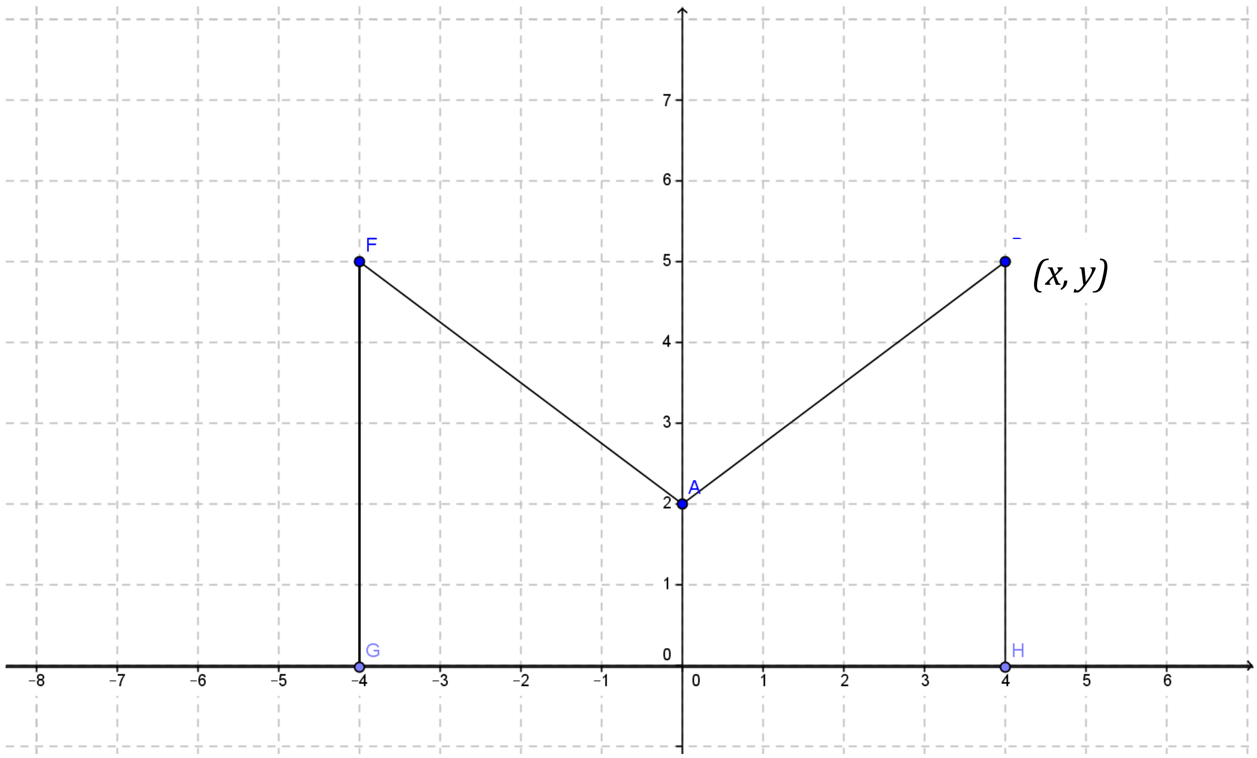
4. Using your second string, use the same procedure to post a pin on the other side of the focus.

5. As your classmates post their strings, what geometric figure do you predict will be made by the tacks (the collection of all points like (x, y) show in the figure above)? Why?

6. Where is the vertex of the figure located? How do you know?

7. Where is the line of symmetry located? How do you know?

8. Consider the following construction with focus point A and the x axis as the directrix. Use a ruler to complete the construction of the parabola in the same way that the class constructed the parabola with string.



9. You have just constructed a parabola based upon the definition: A parabola is the set of all points *(x, y)* equidistant from a line *l* (the directrix) and a point not on the line (the focus). Use this definition to write the equation of the parabola above, using the point (*x, y*) to represent any point on the parabola.

10. How would the parabola change if the focus was moved up, away from the directrix?

11. How would the parabola change if the focus was moved down, toward the directrix?

12. How would the parabola change if the focus was moved down, below the directrix?

Name:

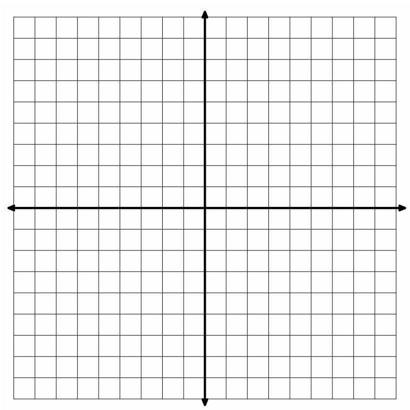
Ready, Set, Go!

Ready

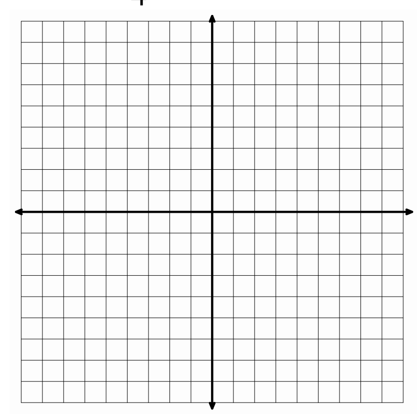
Topic: Graphing quadratics

Graph each set of functions on the same coordinate axes. Describe in what way the graphs are the same and in what way they are different.

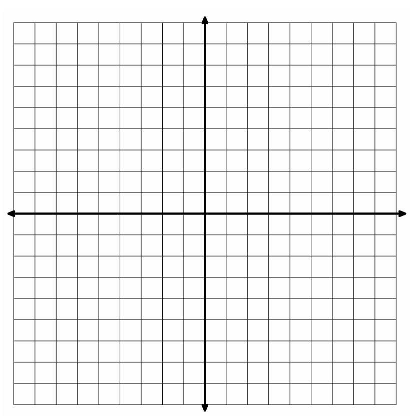
1. $y = x^2,\ y = 2x^2,\ y = 4x^2$



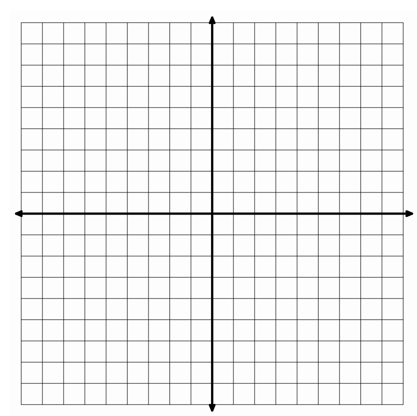
2. $y = \frac{1}{4}x^2,\ y = -x^2,\ y = -4x^2$



3. $y = \frac{1}{4}x^2, y = x^2 - 2,\ y = \frac{1}{4}x^2 - 2,\ y = 4x^2 - 2$



4. $y = x^2,\ y = -x^2,\ y = x^2 + 2,\ y = -x^2 + 2$



Set
Topic: Sketching a parabola using the conic definition.

Use the conic definition of a parabola to sketch a parabola defined by the given focus *F* and the equation of the directrix.

Begin by graphing the focus, the directrix, and point P_1. Use the distance formula to find FP_1 and find the vertical distance between P_1 and the directrix by identifying point H on the directrix and counting the distance. Locate the point P_2, (the point on the parabola that is a reflection of P_1 across the axis of symmetry.) Locate the vertex V. Since the vertex is a point on the parabola, it must lie equidistant between the focus and the directrix. Sketch the parabola. Hint: the parabola always "hugs" the focus.

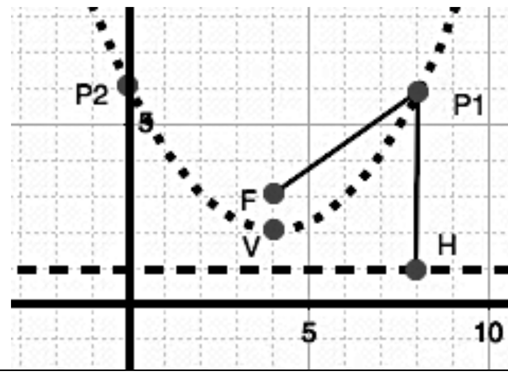
Example: ***F*** (4,3), P_1 (8, 6), y = 1

$FP_1 = \sqrt{(4-8)^2+(3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5$

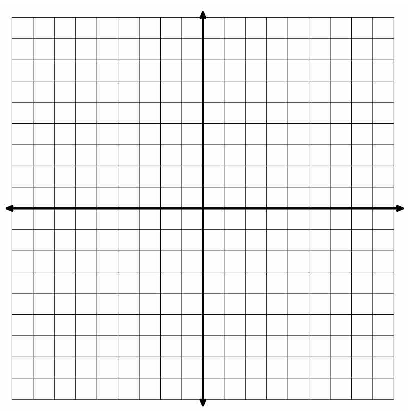
$P_1H = 5$

P_2 is located at (0, 6)

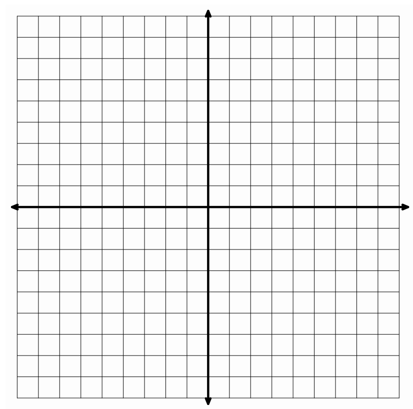
V is located at (4, 2)



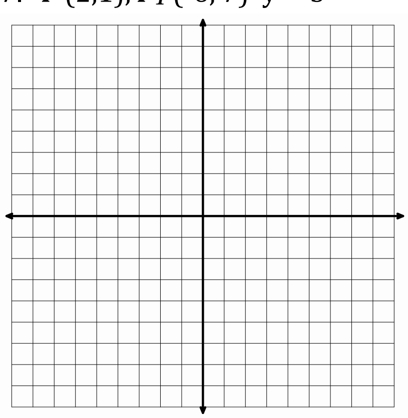
5. ***F*** (1,-1), P_1 (3, -1) y = -3



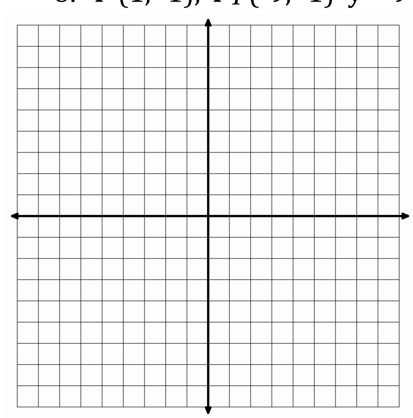
6. ***F*** (-5,3), P_1 (-1, 3) y = 7



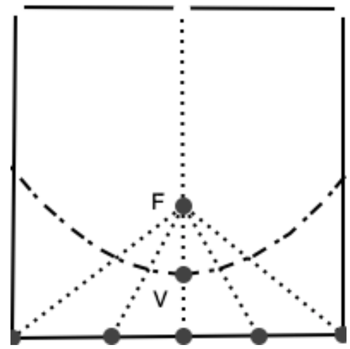
7. ***F*** (2,1), P_1 (-6, 7) y =-3



8. ***F*** (1, -1), P_1 (-9, -1) y = 9



9. Find a square piece of paper (a post-it note will work). Fold the square in half vertically and put a dot anywhere on the fold. Let the edge of the paper be the directrix and the dot be the focus. Fold the edge of the paper (the directrix) up to the dot repeatedly from different points along the edge. The fold lines between the focus and the edge should make a parabola.



Experiment with a new paper and move the focus. Use your experiments to answer the following questions.

10. How would the parabola change if the focus were moved up, away from the directrix?

11. How would the parabola change if the focus were moved down, toward the directrix?

12. How would the parabola change if the focus were moved down, below the directrix?

Go

Topic: Finding the center and radius of a circle.

Write each equation in standard form. Find the center (*h, k*) and radius *r* of the circle.

13. $x^2 + y^2 + 4y - 12 = 0$

14. $x^2 + y^2 - 6x - 3 = 0$

15. $x^2 + y^2 + 8x + 4y - 5 = 0$

16. $x^2 + y^2 - 6x - 10y - 2 = 0$

17. $x^2 + y^2 - 6y - 7 = 0$

18. $x^2 + y^2 - 4x + 8y + 6 = 0$

19. $x^2 + y^2 - 4x + 6y - 72 = 0$

20. $x^2 + y^2 + 12x + 6y - 59 = 0$

21. $x^2 + y^2 - 2x + 10y + 21 = 0$

22. $4x^2 + 4y^2 + 4x - 4y - 1 = 0$

8.5 Functioning With Parabolas

A Solidify Understanding Task

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Sketch the graph (accurately), find the vertex and use the geometric definition of a parabola to find the equation of these parabolas.

1. Directrix y = -4, Focus A(2, -2)

Vertex ________________

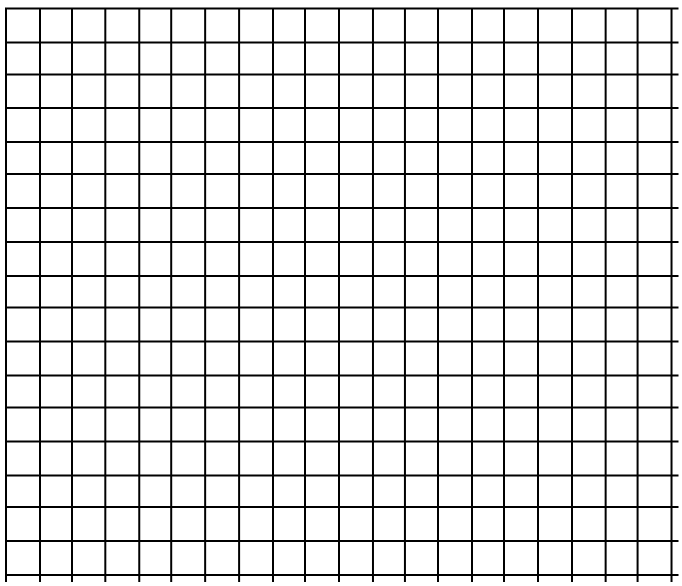
Equation:

2. Directrix y = 2, Focus A(-1, 0)

Vertex ________________

Equation:

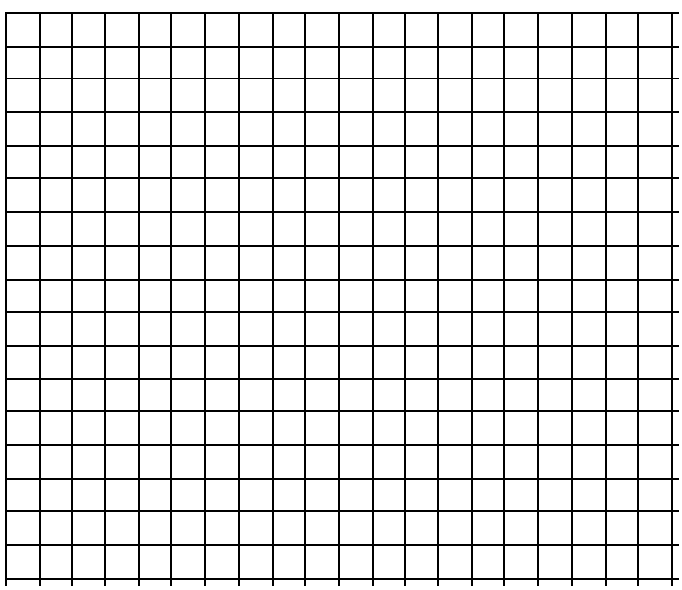
3. Directrix y = 3, Focus A(1, 7)



Vertex ________________

Equation:

4. Directrix y = 3, Focus A(2, -1)



Vertex ________________

Equation:

5. Given the focus and directrix, how can you find the vertex of the parabola?

6. Given the focus and directrix, how can you tell if the parabola opens up or down?

7. How do you see the distance between the focus and the vertex (or the vertex and the directrix) showing up in the equations that you have written?

8. Describe a pattern for writing the equation of a parabola given the focus and directrix.

9. Annika wonders why we are suddenly thinking about parabolas in a completely different way than when we did quadratic functions. She wonders how these different ways of thinking match up. For instance, when we talked about quadratic functions earlier we started with $y = x^2$. "Hmmmm. .... I wonder where the focus and directrix would be on this function," she thought. Help Annika find the focus and directrix for $y = x^2$.

10. Annika thinks, "Ok, I can see that you can find the focus and directrix for a quadratic function, but what about these new parabolas. Are they quadratic functions? When we work with families of functions, they are defined by their rates of change. For instance, we can tell a linear function because it has a constant rate of change." How would you answer Annika? Are these new parabolas quadratic functions? Justify your answer using several representations and the parabolas in problems 1-4 as examples.

Name: Circles and Other Conics | 8.5

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Ready, Set, Go!

Ready

Topic: Standard form of a quadratic.

Verify that the given point lies on the graph of the parabola described by the equation. (Show your work.)

1. $(6,0)$ $y=2x^2-9x-18$

2. $(-2,49)$ $y=25x^2+30x+9$

3. $(5,53)$ $y=3x^2-4x-2$

4. $(8,2)$ $y=\frac{1}{4}x^2-x-6$

Set

Topic: the equation of a parabola based on the geometric definition

5. Verify that $(y-1)=\frac{1}{4}x^2$ is the equation of the parabola in *figure 1* by plugging in the 3 points V (0,1), C (4,5) and E (2,2). Show your work for each point!

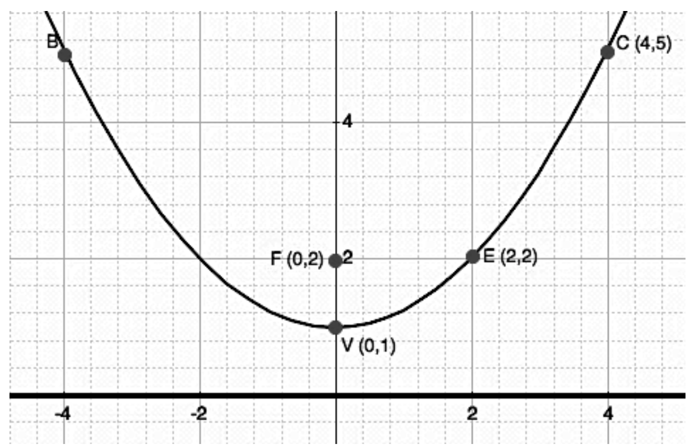
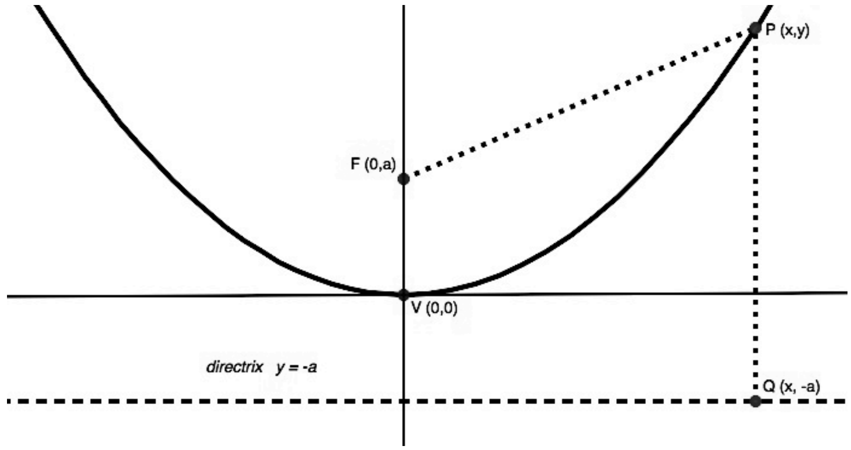


Figure 1

6. If you didn't know that (0,1) was the vertex of the parabola, could you have found it by just looking at the equation? Explain.

7. Use the diagram in *figure 2* to derive the general equation of a parabola based on the **geometric definition** of a parabola. Remember that the definition states that PF = PQ.

figure 2

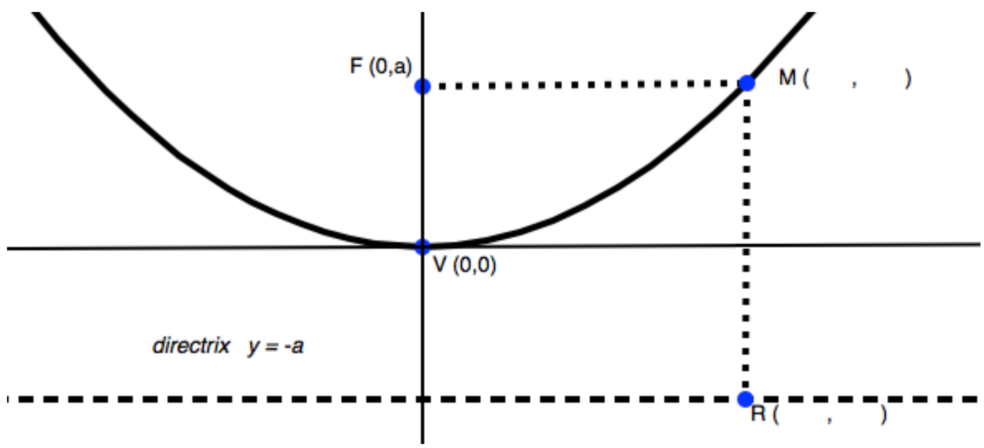


8. Recall the equation in #5, $(y-1)=\frac{1}{4}x^2$, what is the value of **a**?

9. In general, what is the value of **a** in any parabola?

10. In *figure 3*, the point M is the same height as the focus and $\overline{FM} \cong \overline{MR}$. How do the coordinates of this point compare with the coordinates of the focus?
Fill in the missing coordinates for M and R in the diagram.

figure 3

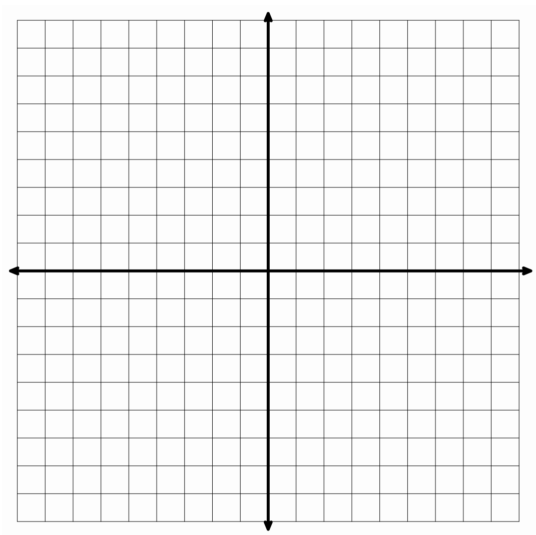


Sketch the graph by finding the vertex and the point M and M' (the reflection of M) as defined in the diagram above. Use the geometric definition of a parabola to find the equation of these parabolas.

11. Directrix y = 9, Focus A(-3, 7)

Vertex ___________

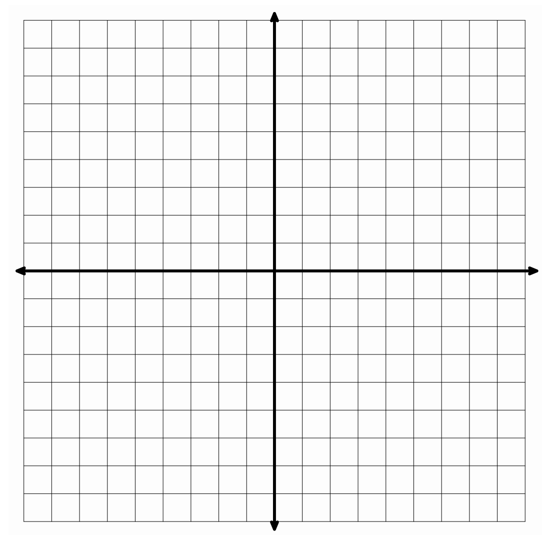
Equation ________________________________



12. Directrix y = -6, Focus A(2, -2)

Vertex ___________

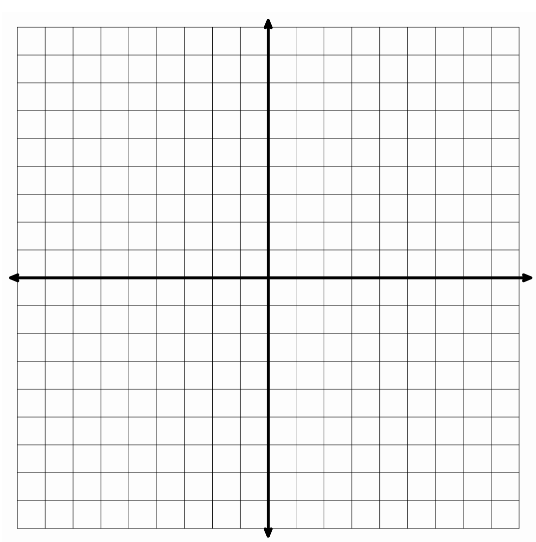
Equation ________________________________



13. Directrix y = 5, Focus A(-4, -1)

Vertex ___________

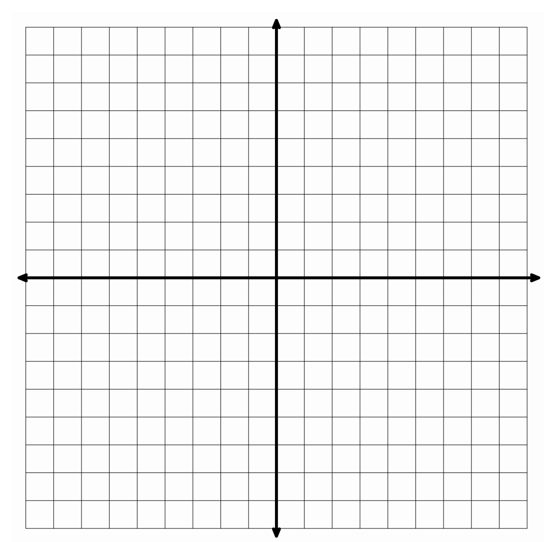
Equation ________________________________



14. Directrix y = -1, Focus A(4, -3)

Vertex ___________

Equation ________________________________



Go

Find the maximum or minimum value of the quadratic. Indicate which it is.

15. $y = x^2 + 6x - 5$

16. $y = 3x^2 - 12x + 8$

17. $y = -\frac{1}{2}x^2 + 10x + 13$

18. $y = -5x^2 + 20x - 11$

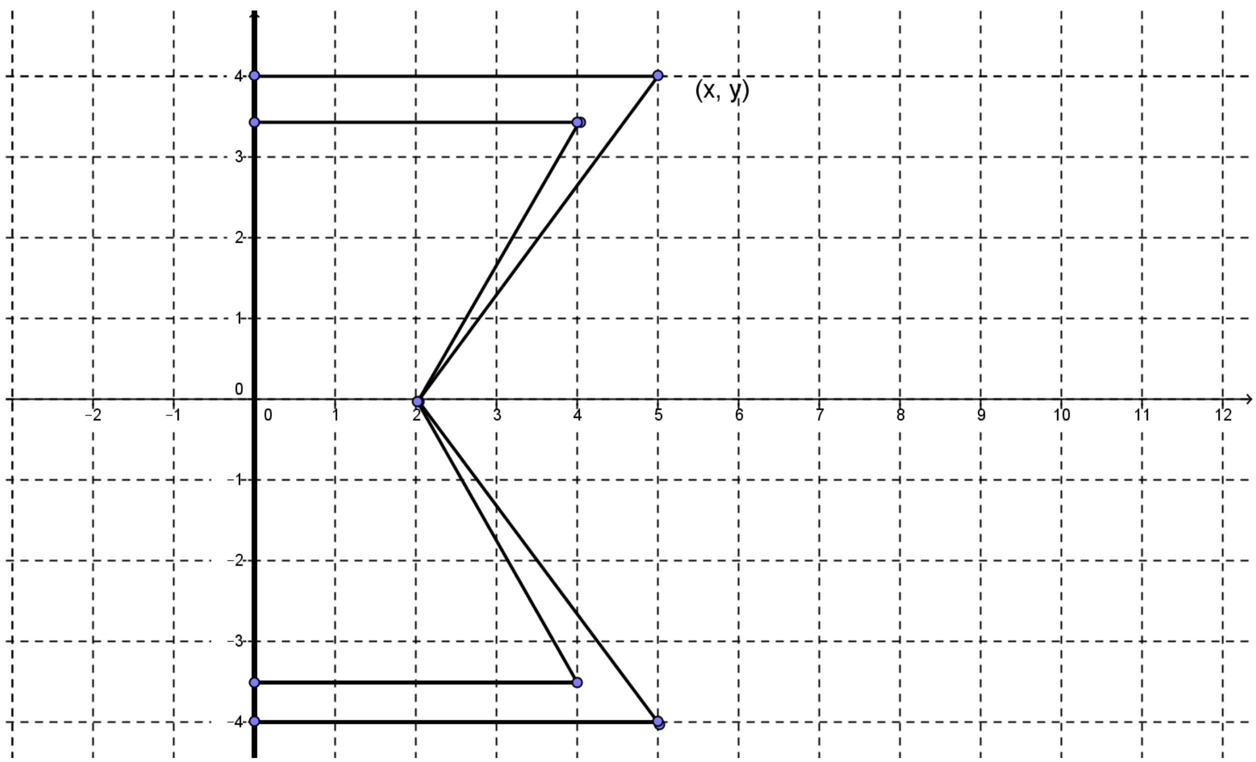
19. $y = \frac{7}{2}x^2 - 21x - 3$

20. $y = -\frac{3}{2}x^2 + 9x + 25$

8.6 Turn It Around

A Solidify Understanding Task

Annika is thinking more about the geometric view of parabolas that she has been working on in math class. She thinks, "Now I see how all the parabolas that come from graphing quadratic functions could also come from a given focus and directrix. I notice that all the parabolas have opened up or down when the directrix is horizontal. I wonder what would happen if I rotated the focus and directrix 90 degrees so that the directrix is vertical. How would that look? What would the equation be? Hmmm…." So Annika starts trying to construct a parabola with a vertical directrix. Here's the beginning of her drawing. Use a ruler to complete Annika's drawing.



1. Use the definition of a parabola to write the equation of Annika's parabola.

2. What similarities do you see to the equations of parabolas that open up or down? What differences do you see?

3. Try another one: Write the equation of the parabola with directrix $x = 4$ and focus (0, 3).

4. One more for good measure: Write the equation of the parabola with directrix $x = -3$ and focus (-2, -5).

5. How can you predict if a parabola will open left, right, up, or down?

6. How can you tell how wide or narrow a parabola is?

7. Annika has two big questions left. Write and explain your answers to these questions.
   a. Are parabolas functions?

   b. Are all parabolas similar?

Name: Circles and Other Conics | 8.6

Ready, Set, Go!

Ready

Topic: Are you ready for a test on module 8? Review of circles

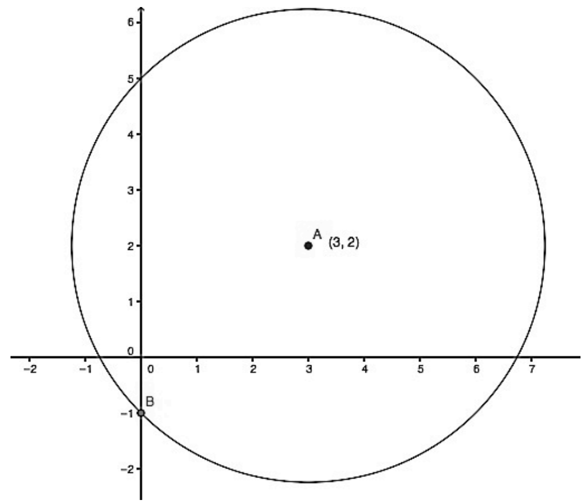
Use the given information to write the equation of the circle in standard form.

1. Center: (-5, -8) , Radius: 11

2. Endpoints of the diameter: (6, 0) and (2, -4)

3. Center (-5, 4) : Point on the circle (-9, 1)

4. Equation of the circle in the diagram.

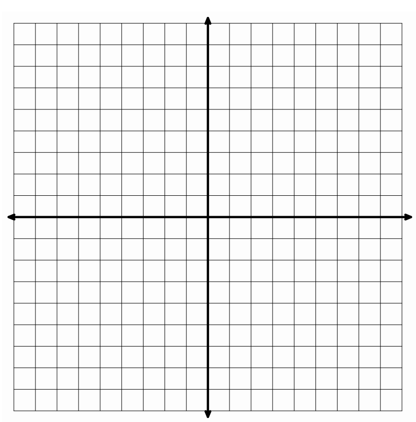


Set

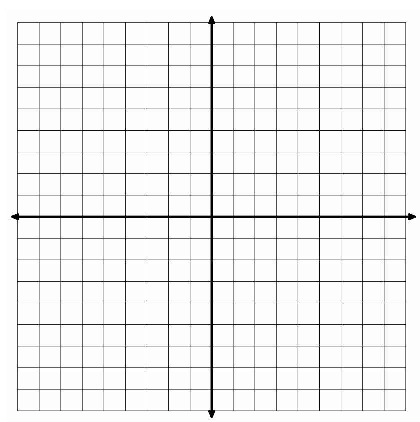
Topic: Writing equations of horizontal parabolas

Use the focus F, point P, a point on the parabola, and the equation of the directrix to sketch the parabola (label your points) and write the equation. Put your equation in the form $(x-h)=\frac{1}{4a}(y-k)^2$ where "a" is the distance from the focus to the vertex.

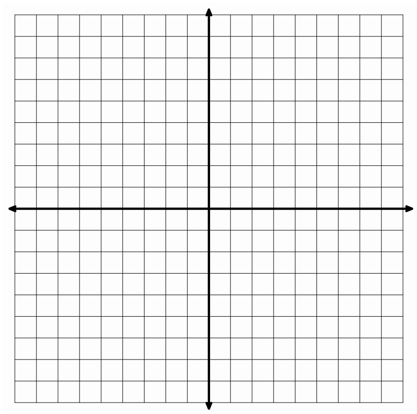
5. ***F*** (1,0), ***P*** (1,4) x = -3



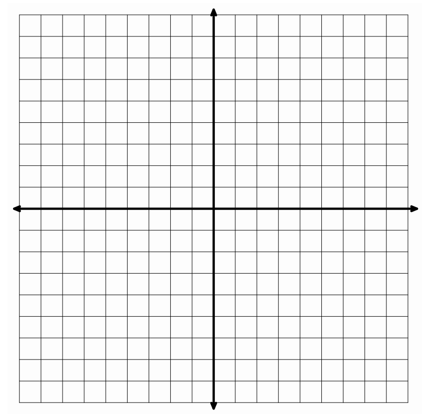
6. ***F*** (3,1), ***P*** (2,-5) x = 9



7. ***F*** (7,-5), ***P*** (4,-1) x = 9



8. ***F*** (-1,2), ***P*** (6,-9) x = -7



Go

Topic: Identifying key features of a quadratic written in vertex form

State (a) the coordinates of the vertex, (b) the equation of the axis of symmetry, (c) the domain, and (d) the range for each of the following functions.

9. $f(x) = (x-3)^2 + 5$

10. $f(x) = (x+1)^2 - 2$

11. $f(x) = -(x-3)^2 - 7$

12. $f(x) = -3\left(x - \frac{3}{4}\right)^2 + \frac{4}{5}$

13. $f(x) = \frac{1}{2}(x-4)^2 + 1$

14. $f(x) = \frac{1}{4}(x+2)^2 - 4$

15. Compare the vertex form of a quadratic to the geometric equation of a parabola. Describe how they are similar and how they are different.

8.7H Operating on a Shoestring

A Solidify Understanding Task

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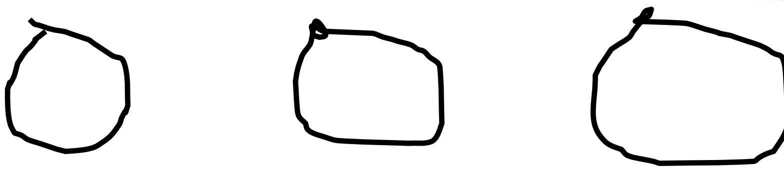
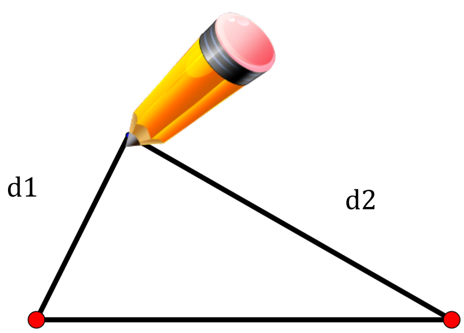
You will need 3 pieces of paper, a piece of cardboard that is at least 8" x 8", 2 tacks, 36 inches of string, and a pencil

1. Cut three pieces of string: a 10 inch piece, a 12 inch piece, and a 14 inch piece. Tie the ends of each piece of string together, making 3 loops.



2. Place a piece of paper on top of the cardboard.
3. Place the two tacks 4 inches apart, wrap the string around the tacks and then press the tacks down.
4. Pull the string tight between the two tacks and hold them down between your finger and thumb. Pull the string tight so that it forms a triangle, as shown below. What is the length of the part of the string that is not on the segment between the two tacks, the sum of the lengths of the segments marked d1 and d2 in the diagram?
5. With your pencil in the loop and the string pulled tight, move your pencil around the path that keeps the string tight.



6. What shape is formed? What geometric features of the figure do you notice?

7. Repeat the process again using the other strings. What is the effect of the length of the string?

8. What is the effect of changing the distance between the two tacks? (You may have to experiment to find this answer.)

The geometric figure that you have created is called an ellipse. The two tacks each represent a focus point for the ellipse. (The plural of the word "focus" is "foci", but "focuses" is also correct.)

To focus our observations about the ellipse, we're going to slow the process down and look at points on the ellipse in particular positions. To help make the labeling easier, we will place the ellipse on the coordinate plane.

9. The distances from the point on the ellipse to each of the two foci is labeled d1 and d2.

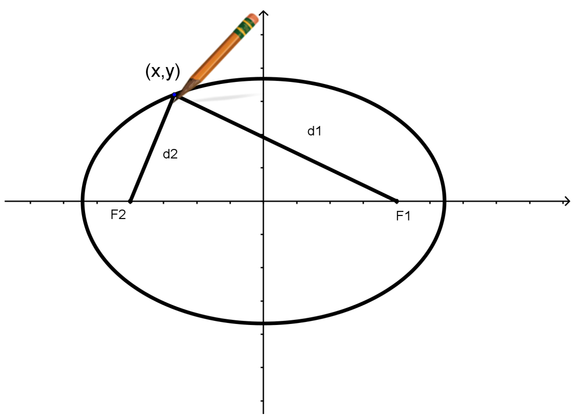


Figure 1

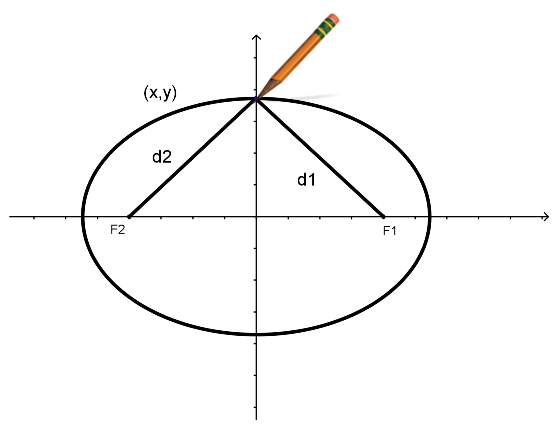
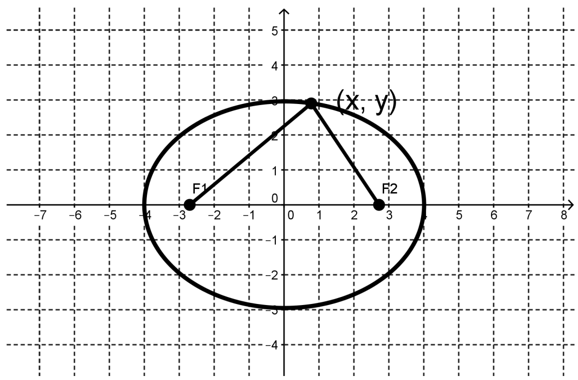


Figure 2

How does d1 + d2 in Figure 1 compare to d1 + d2 in Figure 2? (Figure 1 and Figure 2 are the same ellipse.)

10. How does d1 + d2 compare to the length of the ellipse, measured from one end to the other along the x-axis? Explain your answer with a diagram.

You have just constructed an ellipse based upon the definition: An ellipse is the set of all points *(x, y)* in a plane which have the same total distance from two fixed points called the foci. Like circles and parabolas, ellipses also have equations. The basic equation of the ellipse is derived in a way that is similar to the equation of a parabola or a circle. Since it's usually easier to start with a specific case and then generalize, we'll start with this ellipse:



11. Now, use the conclusions that you drew earlier to help you to write an equation. (We'll help with a few prompts.)
    a. What is the sum of the distances from a point (x, y) on this ellipse to F1 and F2?
    b. Write an expression for the distance between the point (x, y) on the ellipse and F1(-3,0).
    c. Write an expression for the distance between (x,y) on the ellipse and F2 (3,0).
    d. Use your answers to a, b, and c to write an equation.

12. The equation of this ellipse in standard form is:

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

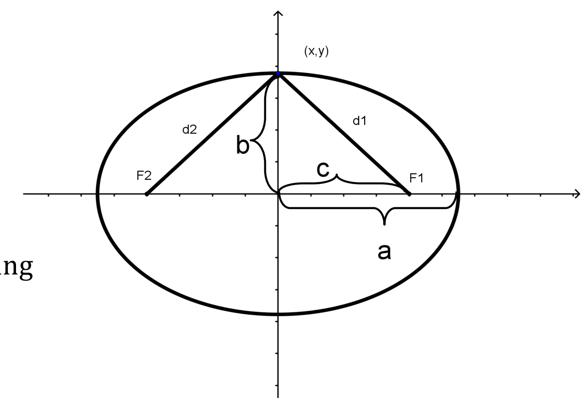
It might be much trickier than you would imagine to re-arrange your equation to check it, so we'll try a different strategy. This equation would say that the ellipse contains the points (4,0) and (0,-3). Do both of these points make your equation true? Show how you checked them here.

13. Using the standard form of the equation is actually pretty easy, but you have to notice a few more relationships. Here's another picture with some different parts labeled.

a = horizontal distance from the center to the ellipse

b = vertical distance from the center to the ellipse

c = distance from the center to a focus



Based on the diagram, describe in words the following expressions:

2a

2b

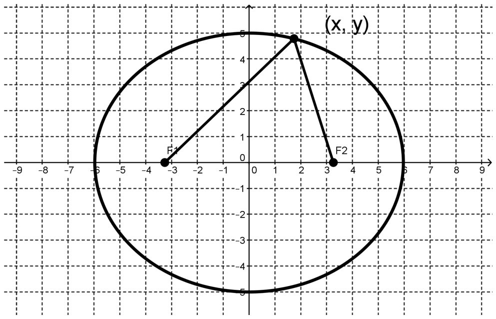
2c

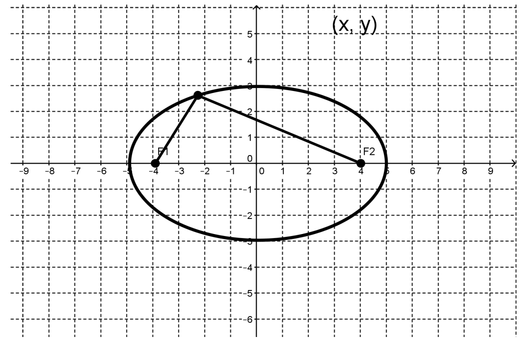
14. What is the mathematical relationship between a, b, and c?

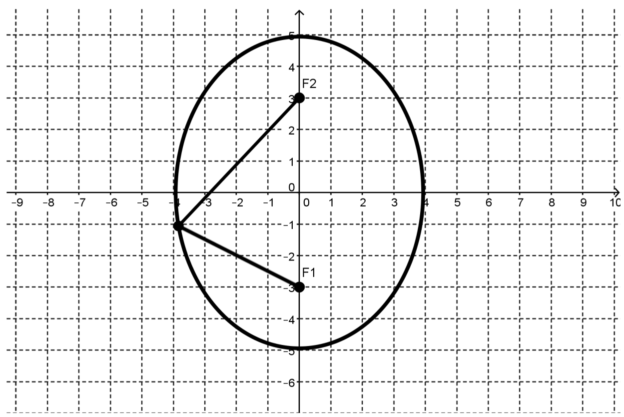
15. Now you can use the standard form of the equation of an ellipse centered at (0,0) which is:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

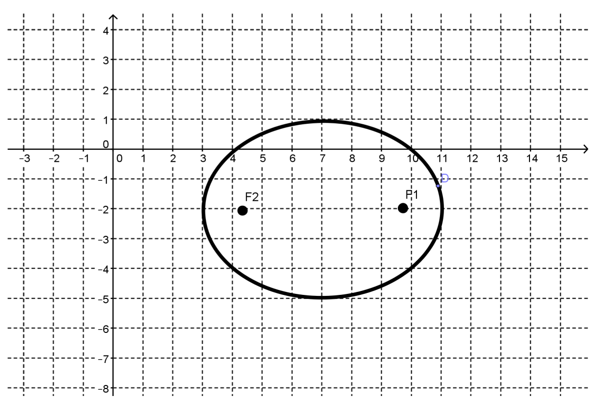
Write the equation of each of the ellipses pictured below:







16. Based on your experience with shifting circles and parabolas away from the origin, write an equation for of this ellipse. Test your equation with some points on the ellipse that you can identify.



Ready, Set, Go!

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Ready

Topic: Solving radical equations.

Solve for x. Beware of *extraneous solutions.*

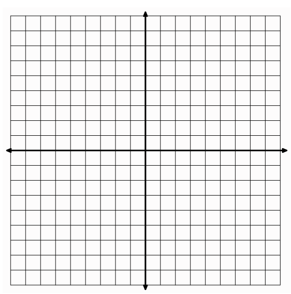
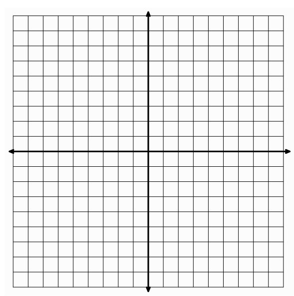
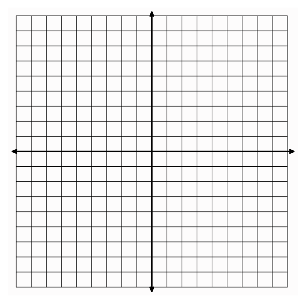
1. $\sqrt{2x-5}=3$
2. $\sqrt{10x+9}=13$
3. $\sqrt{2x}=x-4$
4. $3\sqrt{2x+2}=2\sqrt{5x-1}$
5. $x-3=\sqrt{3x+1}$
6. $4-\sqrt{10-3x}=x$

Set

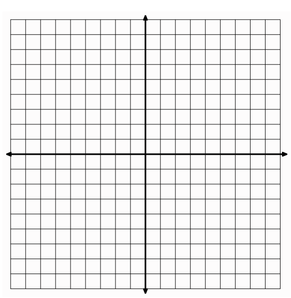
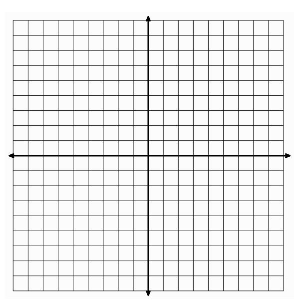
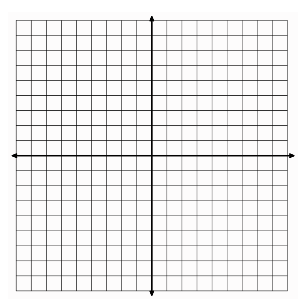
Topic: Graphing Ellipses

Find the x- and y- intercepts of the ellipse whose equation is given. Then draw the graph.

7. $\frac{x^2}{36}+\frac{y^2}{16}=1$
8. $\frac{x^2}{9}+\frac{y^2}{64}=1$
9. $\frac{x^2}{25}+\frac{y^2}{4}=1$

10. $x^2+4y^2=64$
11. $9x^2+y^2=36$
12. $x^2+3y^2=75$

Write an equation, in standard form, for each ellipse, given its center C and its x- and y- intercepts.

12. C (-2,3), a = ±4, b = ± 2

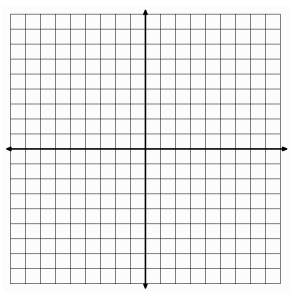
13. C (5,2), a = ±3, b = ± 5

14. C (-4,-7), a = ±10, b = ± 8

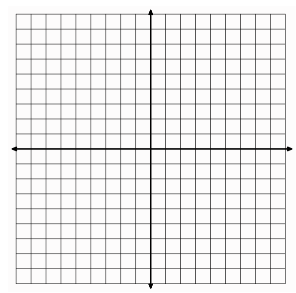
15. C (6,-5), a = ±7, b = ± $\sqrt{11}$

Write the equation of each ellipse in standard form. Identify the center. Then graph the ellipse.

16. $4x^2 + y^2 - 32x - 4y + 52 = 0$



17. $16x^2 + 9y^2 - 96x + 72y + 144 = 0$



Go

Topic: Point-Slope form of a line.

The rectangle in figure B is a translation of the rectangle in figure A. Write the equations of the 2 diagonals of rectangle ABCD in <u>point-slope form</u>. Then write the equations of the 2 diagonals of A'B'C'D'.

18. *figure* A

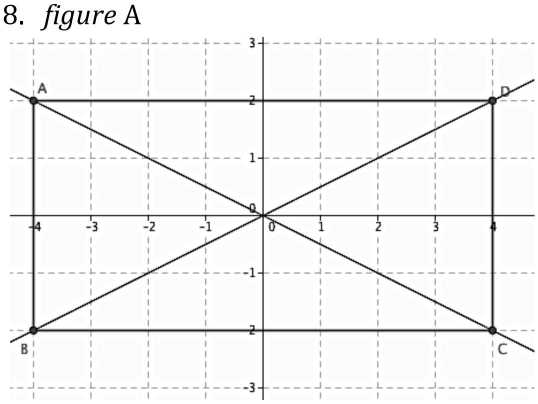
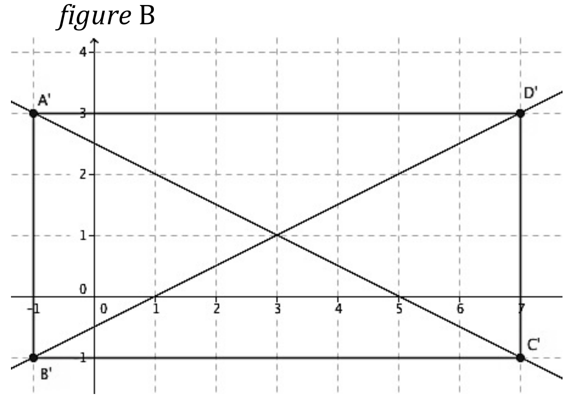


figure B



19. *figure* A

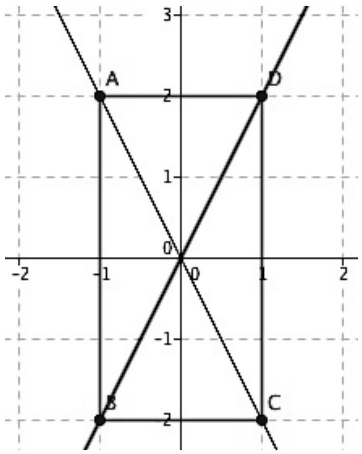
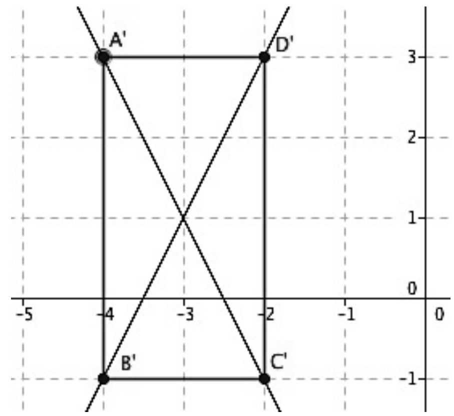


figure B



20. *figure* A

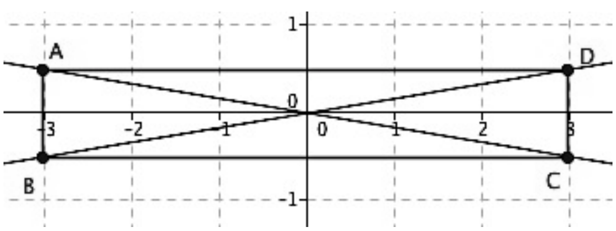
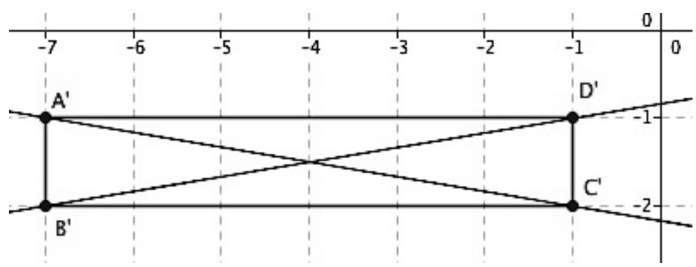


figure B



21. The equations of the diagonals of rectangle JKLM are $y_1 = 5/8x$ and $y_2 = -5/8x$. Rectangle JKLM is then translated so that its diagonals intersect at the point (12, -9). Write the equation of the diagonals of the translated rectangle.

8.8H What Happens If . . .?

A Solidify Understanding Task

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After spending some time working with circles and ellipses, Maya notices that the equations are a lot alike. For example, here's an equation of an ellipse and a circle:

$$\frac{x^2}{16}+\frac{y^2}{9}=1 \qquad x^2+y^2=16$$

1. What are some of the similarities between the circle and the ellipse given in the equations above? What are some of the differences?

2. Maya wonders what would happen if she took the equation of the circle and rearranged it so the right hand side was 1, like the standard form of an ellipse. What does the equation of the circle become?

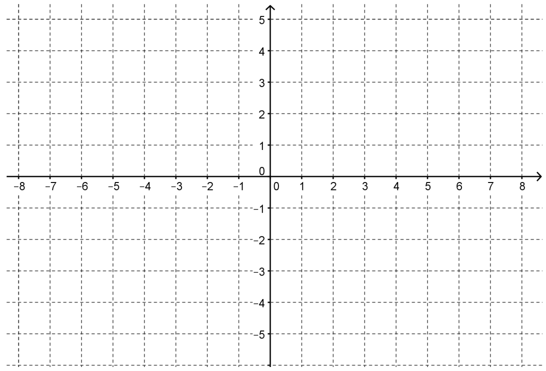
3. After seeing this equation Maya wonders if a circle is really an ellipse, or if an ellipse is really a circle. How would you answer this question?

4. Maya looks at the equation of the ellipse and wonders what would happen if the "+" in the equation was replaced with a "—", making the equation:

   $$\frac{x^2}{16}-\frac{y^2}{9}=1$$

   Without making any further calculations or graphing any points, predict whether or not the graph of this equation will be an ellipse? Using what you know about ellipses, explain your answer.

5. Graph the equation to determine whether or not your prediction was correct. Be sure to use enough points to get a full picture of the figure.

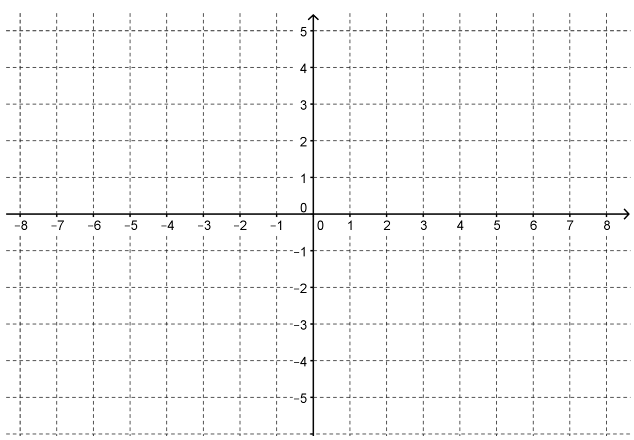


6. What are some of the features of the figure that you have graphed?

7. Maya's teacher tells her that the name of the figure represented in each of the two equations is a hyperbola. Maya wonders what would happen if the x^2 term in the equation was switched with the y^2 term, making the equation:

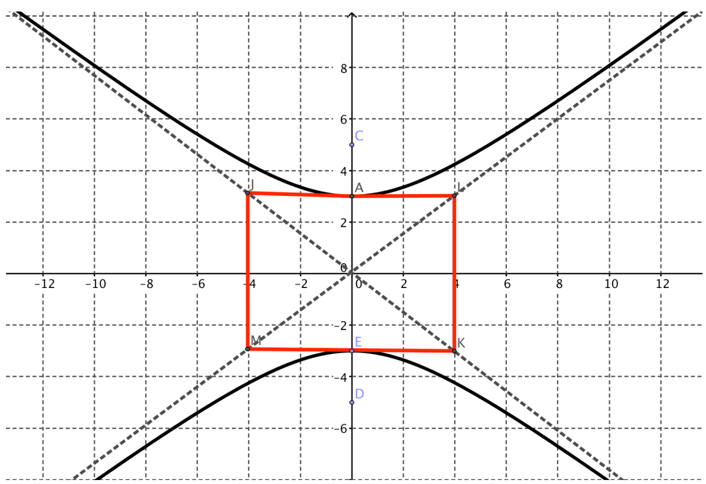
$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

Graph this equation and compare it to the hyperbola that you graphed previously.



8. What similarities and differences do you see between this hyperbola and the one that you graphed in #5?

One strategy that makes it easier to graph the hyperbola from an equation is to notice that the square root of the numbers under the x^2 and y^2 terms can be used to make a rectangle and then to draw dotted lines through the diagonals that form the boundaries of the hyperbola. Using this strategy to graph the equation: $\frac{y^2}{9} - \frac{x^2}{16} = 1$, you would start by taking the square root of 9 = 3 and going up and down 3 units from the origin. Then you take the square root of 16 = 4 and go left and right 4 units from the origin. Make a rectangle with these points on the sides and draw the diagonals. You will get this:



9. So, Maya, the bold math adventurer, decides to try it with a new equation of a hyperbola. The standard form of the equation of an hyperbola centered at (0,0) is:

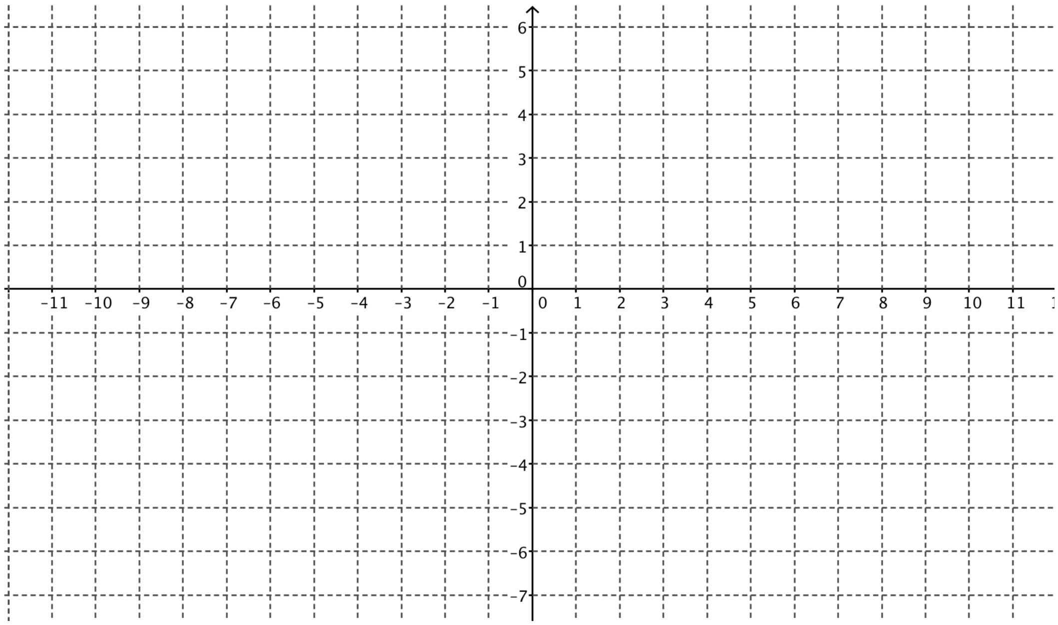
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (opens left and right)

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (opens up and down)

Maya goes to work graphing the equation:

$$\frac{x^2}{36} - \frac{y^2}{25} = 1$$

Try it yourself and see what you can come up with.

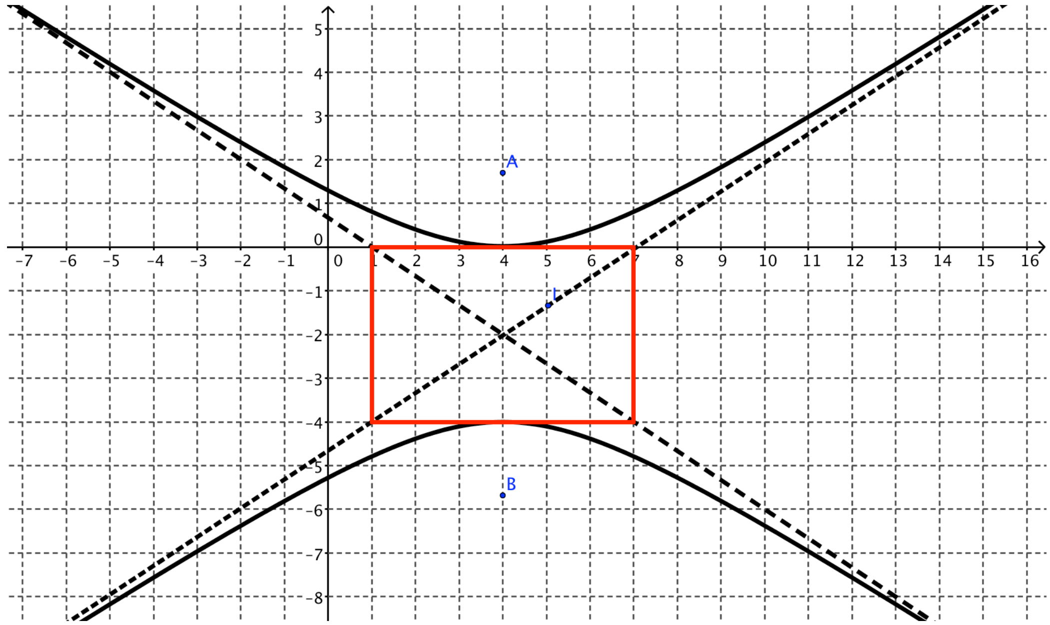


10. Maya wonders what happens if the equation becomes:

$$\frac{(x-1)^2}{36} - \frac{(y+2)^2}{25} = 1$$

What is your prediction? Why?

11. Write the equation of the hyperbola shown below:



11. What similarities and differences do you see between a hyperbola and an ellipse?

Name: Circles and Other Conics | 8.8H

Ready, Set, Go!

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Ready

Topic: Identifying conic sections by their equations.

Identify each conic section by the given equation.

1. $\frac{x^2}{25}+\frac{y^2}{12}=1$

2. $\frac{x^2}{4}-\frac{y^2}{16}=1$

3. $\frac{x^2}{49}+\frac{y^2}{49}=1$

4. $x^2=16+y$

5. $9x^2=36+4y^2$

6. $9x^2=36-9y^2$

7. $y=\frac{x+4}{y}$

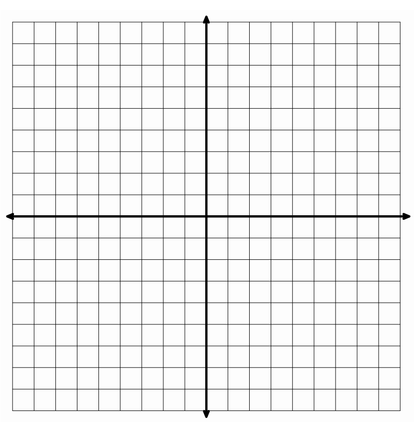
8. $7x^2-8y^2=35$

9. $5x^2-2y^2-15=-6y^2+5$

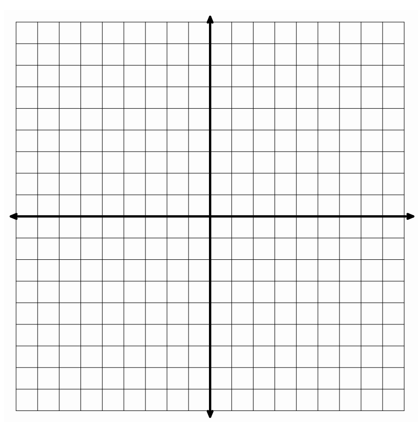
Set Topic: Graphing hyperbolas

Write the equation of the asymptotes. Then sketch the graph of the given equation.

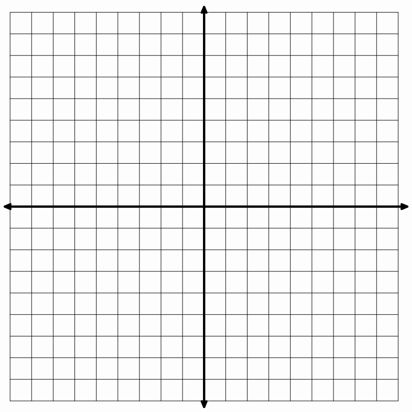
10. $\frac{x^2}{16}-\frac{y^2}{25}=1$



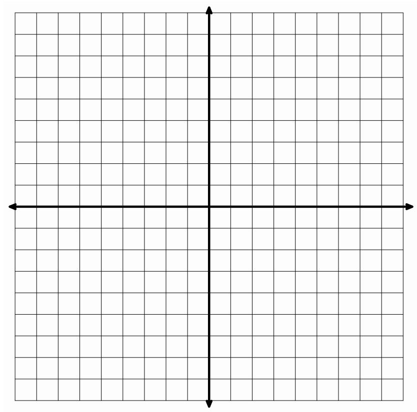
11. $\frac{y^2}{16}-\frac{x^2}{25}=1$



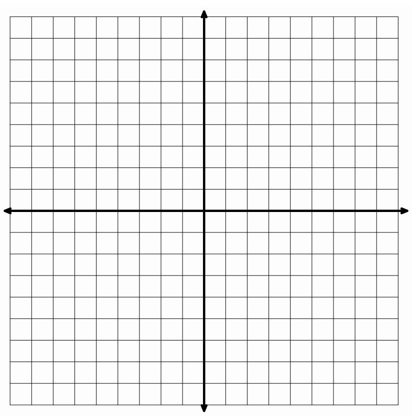
12. $\frac{y^2}{9} - \frac{x^2}{4} = 1$



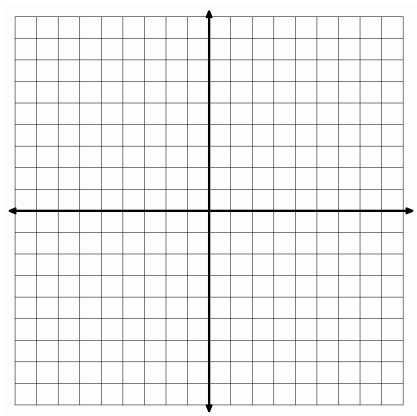
13. $\frac{x^2}{49} - \frac{y^2}{36} = 1$



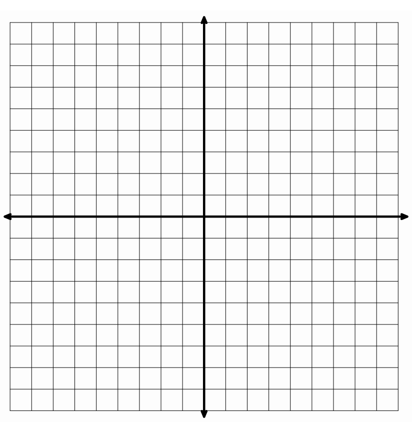
14. $4x^2 - 16y^2 = 64$



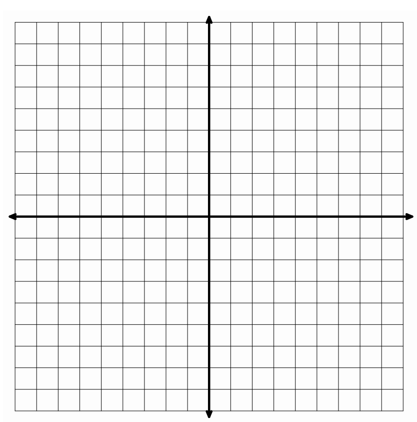
15. $12x^2 - 3y^2 = 48$



16. $\frac{(x-3)^2}{16} - \frac{(y+2)}{9} = 1$



17. $\frac{(y-5)^2}{4} - \frac{(x-3)}{9} = 1$



Go

Topic: Writing the equations of conic sections in standard form

Write the equation in standard form by completing the square. Then identify the conic section.
If the conic is:

- **a parabola, identify the vertex and the write the equation of the directrix.**
- **a circle, identify the center and the radius.**
- **an ellipse, identify the center and the lengths of the minor axis and major axis.**
- **a hyperbola, write the equations of the asymptotes.**

18. $x^2 - 4x + y^2 + 6y = 1$

19. $16x^2 - 9y^2 - 72y - 288 = 0$

20. $2y^2 - 32x + 20y + 50 = 0$

21. $2x^2 + 3y^2 + 6x - 9y - 15 = 0$