Secondary One Mathematics: An Integrated Approach
Module 6
Congruence, Construction and Proof

By

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www.mathematicsvisionproject.org

In partnership with the Utah State Office of Education

Module 6 – Congruence, Construction and Proof

Classroom Task: Leaping Lizards! - A Develop Understanding Task
Developing the definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.1

Classroom Task: Is It Right? - A Solidify Understanding Task
Examining the slope of perpendicular lines (**G.CO.1, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.2

Classroom Task: Leap Frog– A Solidify Understanding Task
Determining which rigid-motion transformations carry one image onto another congruent image (**G.CO.4, G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.3

Classroom Task: Leap Year – A Practice Understanding Task
Writing and applying formal definitions of the rigid-motion transformations: translations, reflections and rotations (**G.CO.1, G.CO.2, G.CO.4, G.GPE.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.4

Classroom Task: Symmetries of Quadrilaterals – A Develop Understanding Task
Finding rotational symmetry and lines of symmetry in special types of quadrilaterals (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.5

Classroom Task: Symmetries of Regular Polygons – A Solidify Understanding Task
Examining characteristics of regular polygons that emerge from rotational symmetry and lines of symmetry (**G.CO.3, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.6

Classroom Task: Quadrilaterals-Beyond Definition – A Practice Understanding Task
Making and justifying properties of quadrilaterals using symmetry transformations (**G.CO.3, G.CO.4, G.CO.6**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.7

Classroom Task: Can You Get There From Here? – A Develop Understanding Task
Describing a sequence of transformations that will carry congruent images onto each other (**G.CO.5**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.8

Classroom Task: Congruent Triangles – A Solidify Understanding Task
Establishing the ASA, SAS and SSS criteria for congruent triangles (**G.CO.6, G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.9

Classroom Task: Congruent Triangles to the Rescue – A Practice Understanding Task
Working with systems of linear equations, including inconsistent and dependent systems
(**G.CO.7, G.CO.8**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.10

Classroom Task: Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct rhombuses and squares
(**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.11

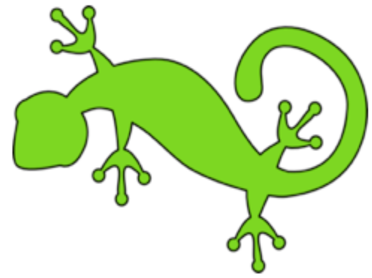
Classroom Task: More Things Under Construction – A Develop Understanding Task
Exploring compass and straightedge constructions to construct parallelograms, equilateral triangles and inscribed hexagons (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.12

Classroom Task: Justifying Constructions – A Solidify Understanding Task
Examining why compass and straightedge constructions produce the desired objects (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.13

Classroom Task: Construction Blueprints – A Practice Understanding Task
Writing procedures for compass and straightedge constructions (**G.CO.12, G.CO.13**)
Ready, Set, Go Homework: Congruence, Construction and Proof 6.14

6.1 Leaping Lizards!

A Develop Understanding Task



Animated films and cartoons are now usually produced using computer technology, rather than the hand-drawn images of the past. Computer animation requires both artistic talent and mathematical knowledge.

Sometimes animators want to move an image around the computer screen without distorting the size and shape of the image in any way. This is done using geometric transformations such as translations (slides), reflections (flips), and rotations (turns) or perhaps some combination of these. These transformations need to be precisely defined, so there is no doubt about where the final image will end up on the screen.

So where do you think the lizard shown on the grid on the following page will end up using the following transformations? (The original lizard was created by plotting the following anchor points on the coordinate grid and then letting a computer program draw the lizard. The anchor points are always listed in this order: tip of nose, center of left front foot, belly, center of left rear foot, point of tail, center of rear right foot, back, center of front right foot.)

Original lizard anchor points:

{(12,12), (15,12), (17,12), (19,10), (19,14), (20,13), (17,15), (14,16)}

Each statement below describes a transformation of the original lizard. Do the following for each of the statements:

- plot the anchor points for the lizard in its new location
- connect the **pre-image** and **image** anchor points with line segments, or circular arcs, whichever best illustrates the relationship between them

Lazy Lizard

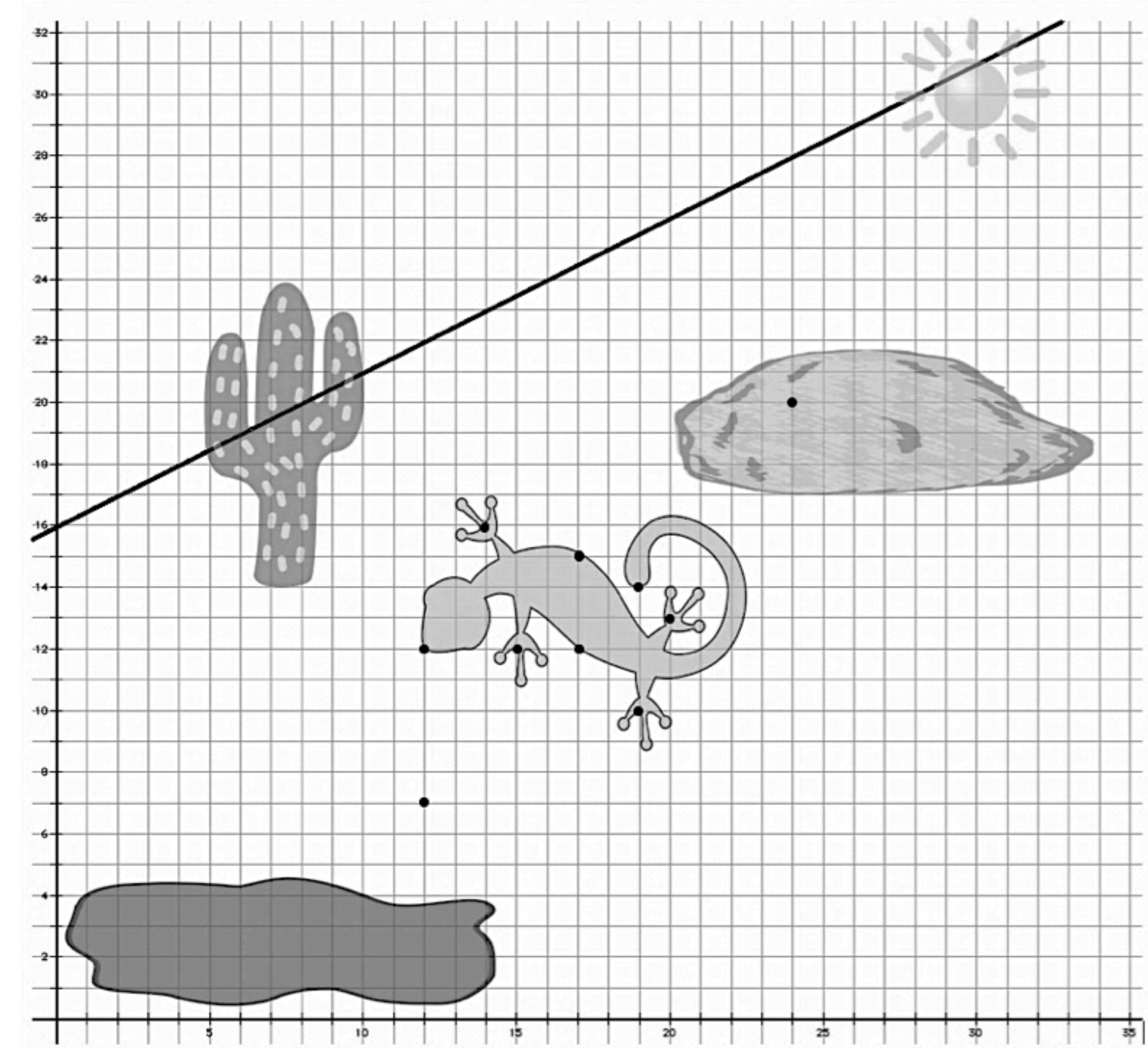
Translate the original lizard so the point at the tip of its nose is located at (24, 20), making the lizard appears to be sunbathing on the rock.

Lunging Lizard

Rotate the lizard 90° about point A (12,7) so it looks like the lizard is diving into the puddle of mud.

Leaping Lizard

Reflect the lizard about given line $y=\frac{1}{2}x+16$ so it looks like the lizard is doing a back flip over the cactus.



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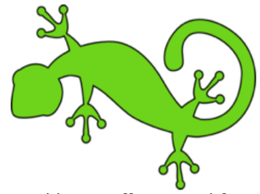
Ready, Set, Go!

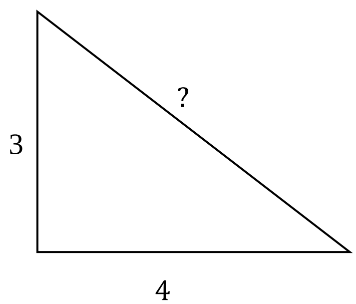


Ready

Topic: Pythagorean Theorem

For each of the following right triangles determine the number units measure for the missing side.

1.



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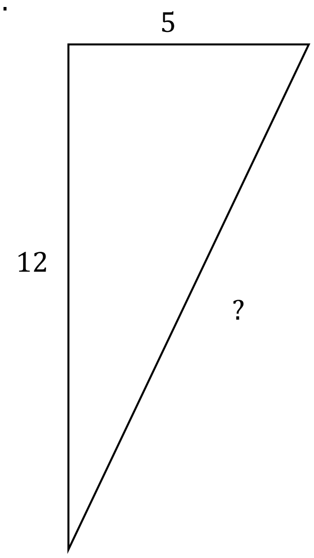

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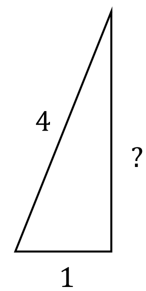

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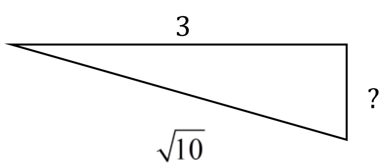

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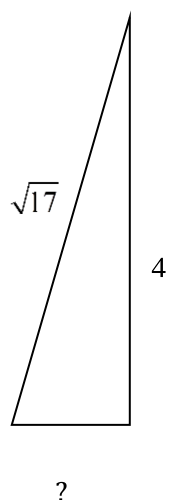

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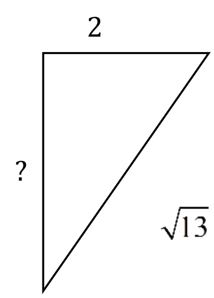


Set

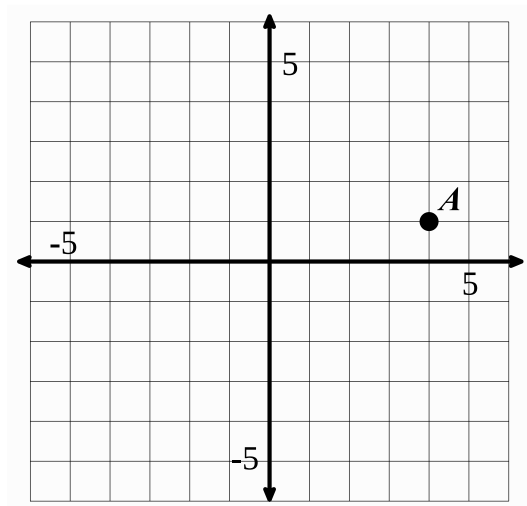
Topic: Transformations

Transform points as indicated in each exercise below.

7a. Rotate point A around the origin 90° clockwise, label as A'

b. Reflect point A over x-axis, label as A''

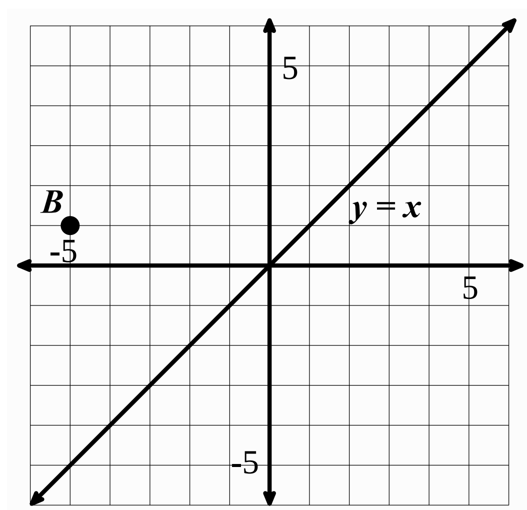
c. Apply the rule $(x-2, y-5)$, to point A and label A'''



8a. Reflect point B over the line $y = x$, label as B'

b. Rotate point B 180° about the origin, label as B''

c. Translate point B the point up 3 and right 7 units, label as B'''

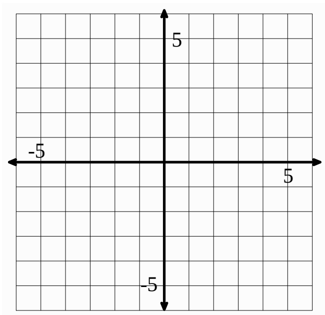


Go

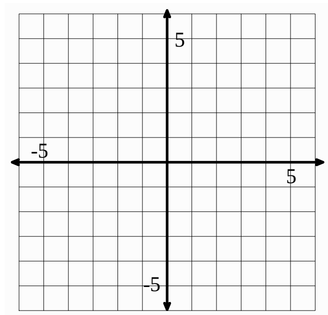
Topic: Graphing linear equations

Graph each equation on the coordinate grid provided. Extend the line as far as the grid will allow.

9. $y = 2x - 3$

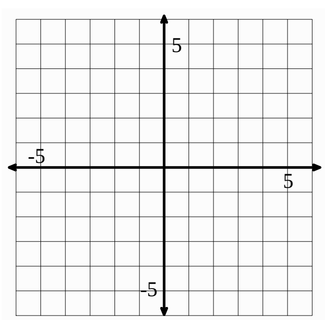


10. $y = -2x - 3$

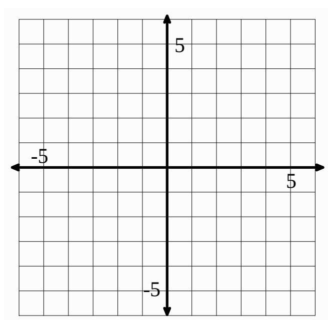


11. What similarities and differences are there between the equations in number 13 and 14?

12. $y = \frac{2}{3}x + 1$

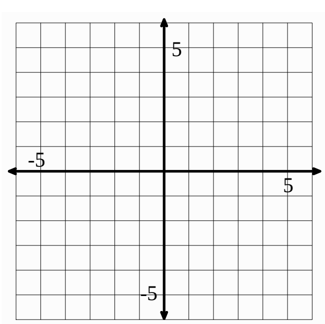


13. $y = -\frac{3}{2}x + 1$

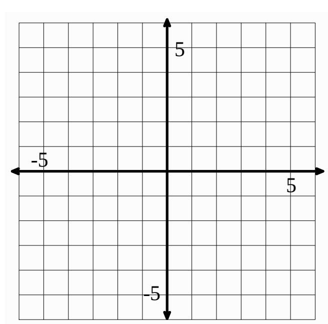


14. What similarities and differences are there between the equations in number 15 and 16?

15. $y = x + 1$



16. $y = x - 3$



17. What similarities and differences are there between the equations in number 15 and 16?

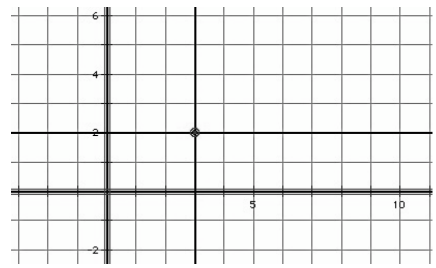
6.2 Is It Right?

A Solidify Understanding Task

In *Leaping Lizards* you probably thought a lot about perpendicular lines, particularly when rotating the lizard about a 90° angle or reflecting the lizard across a line.

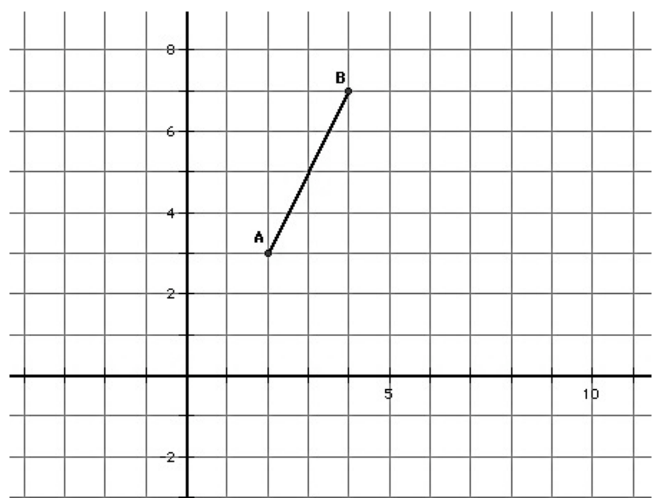
In previous tasks, we have made the observation that *parallel lines have the same slope*. In this task we will make observations about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: *Horizontal and vertical lines are perpendicular.* For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.



But what if a line or line segment is not horizontal or vertical?
How do we determine the slope of a line or line segment that will be perpendicular to it?

<u>Experiment 1</u>



1. Consider the points A (2, 3) and B (4, 7) and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

2. Locate a third point C (x, y) on the coordinate grid, so the points A (2, 3), B (4, 7) and C (x, y) form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

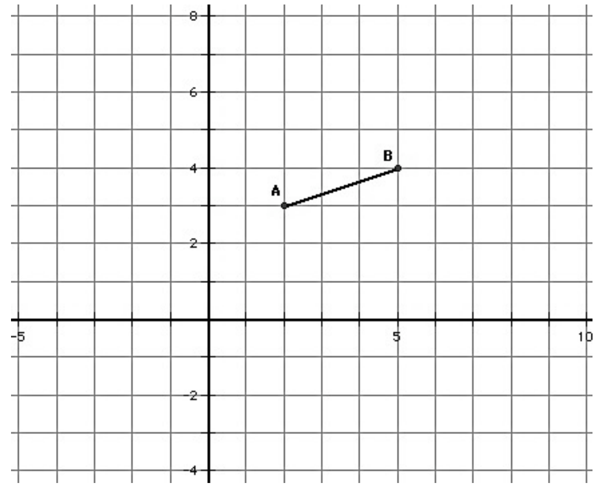
3. Explain how you know that the triangle you formed contains a right angle?

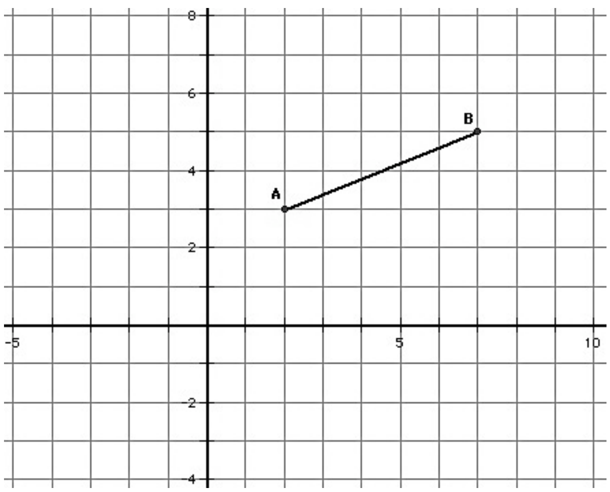
4. Now rotate this right triangle 90° about the vertex point (2, 3). Explain how you know that you have rotated the triangle 90°.

5. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

Experiment 2

Repeat steps 1-5 above for the points A (2, 3) and B (5, 4).



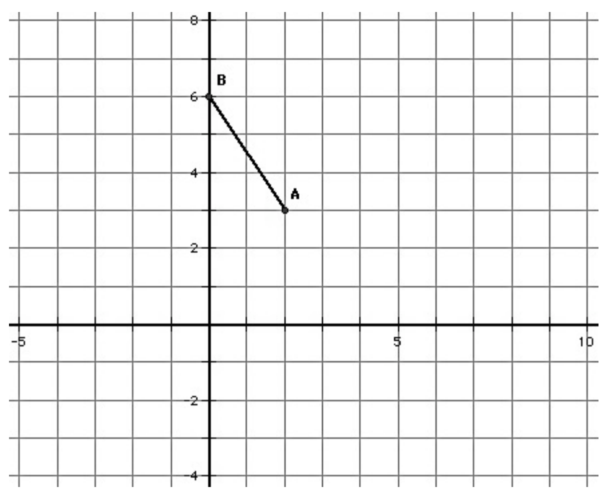


Experiment 3

Repeat steps 1-5 above for the points A (2, 3) and B (7, 5).

Experiment 4

Repeat steps 1-5 above for the points A (2, 3) and B (0, 6).



Based on experiments 1-4, state an observation about the slopes of perpendicular lines.

While this observation is based on a few specific examples, can you create an argument or justification for why this is always true? (Note: You will examine a formal proof of this observation in the next module.)

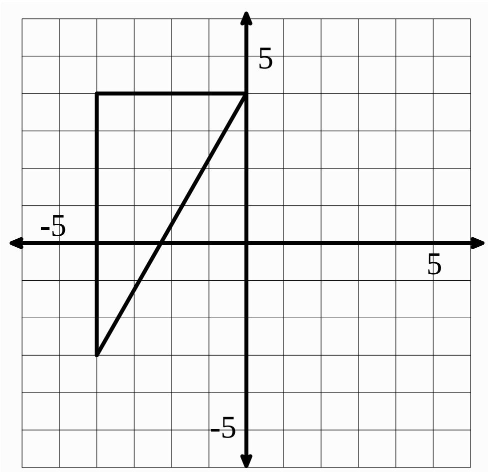
Ready, Set, Go!

Ready

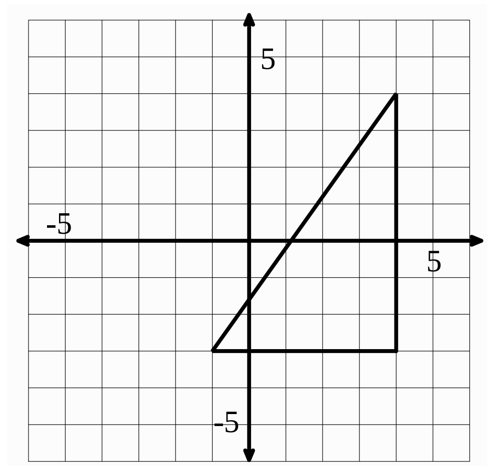
Topic: Finding Distance using Pythagorean Theorem

Use the coordinate grid to find the length of each side of the triangles provided.

1.



2.



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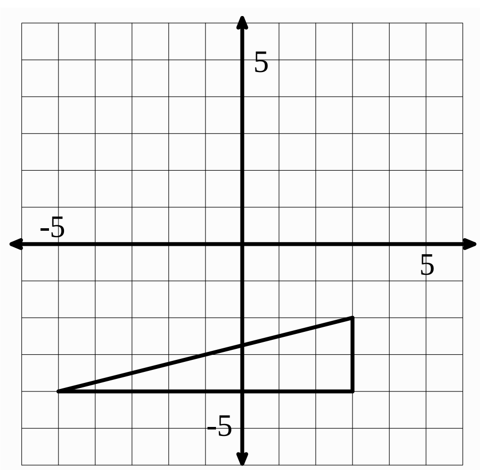

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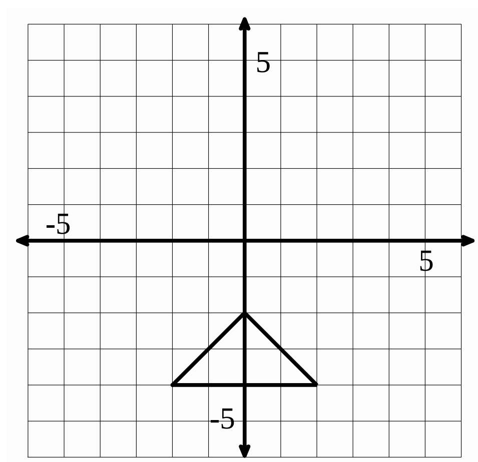
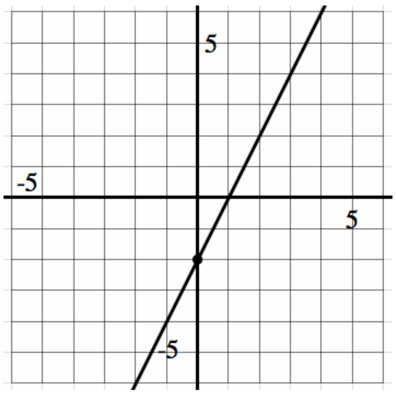


Set

Topic: Slopes of parallel and perpendicular lines.

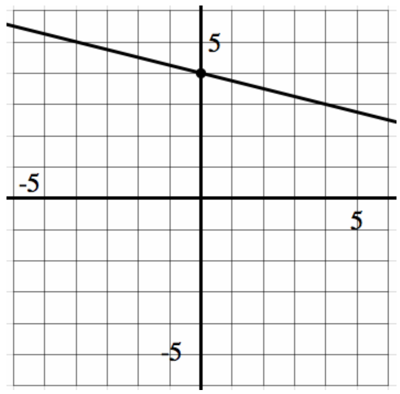
5. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

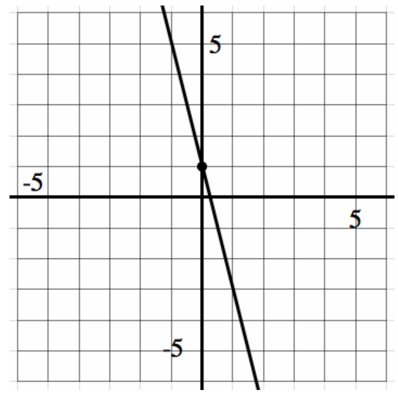
6. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

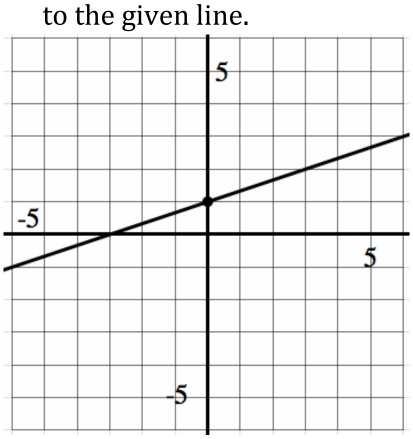
7. Graph a line *parallel* to the given line.



Equation for given line:

Equation for new line:

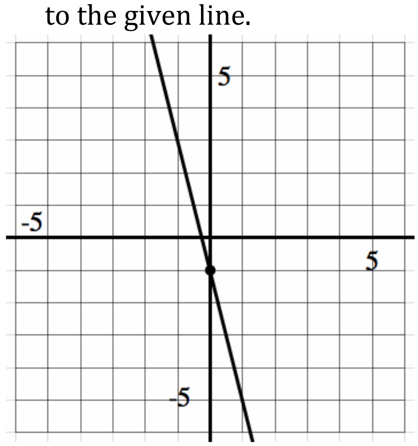
8. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

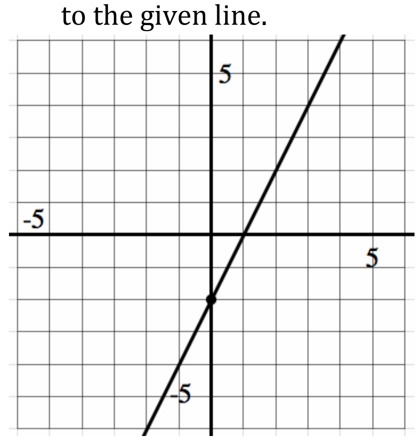
9. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

10. Graph a line *perpendicular* to the given line.



Equation for given line:

Equation for new line:

Go

Topic: Solve the following equations.

Solve each equation for the indicated variable.

11. $3(x-2)=5x+8$; Solve for *x*.

12. $-3+n=6n+22$; Solve for *n*.

13. $y-5=m(x-2)$; Solve for *x*.

14. $Ax+By=C$; Solve for *y*.

6.3 Leap Frog

A Solidify Understanding Task

Josh is animating a scene where a troupe of frogs is auditioning for the Animal Channel reality show, "The Bayou's Got Talent". In this scene the frogs are demonstrating their "leap frog" acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a rotation, you will need to give the center of rotation, the direction of the rotation (clockwise or counterclockwise), and the measure of the angle of rotation.

- If you decide the transformation is a reflection, you will need to give the equation of the line of reflection.

- If you decide the transformation is a translation you will need to describe the "rise" and "run" between pre-image points and their corresponding image points.

- If you decide it takes a combination of transformations to get from the pre-image to the final image, describe each transformation in the order they would be completed.

Pre-image	Final Image	Description
image 1	image 2	
image 2	image 3	
image 3	image 4	
image 1	image 5	
image 2	image 4	

images this page:
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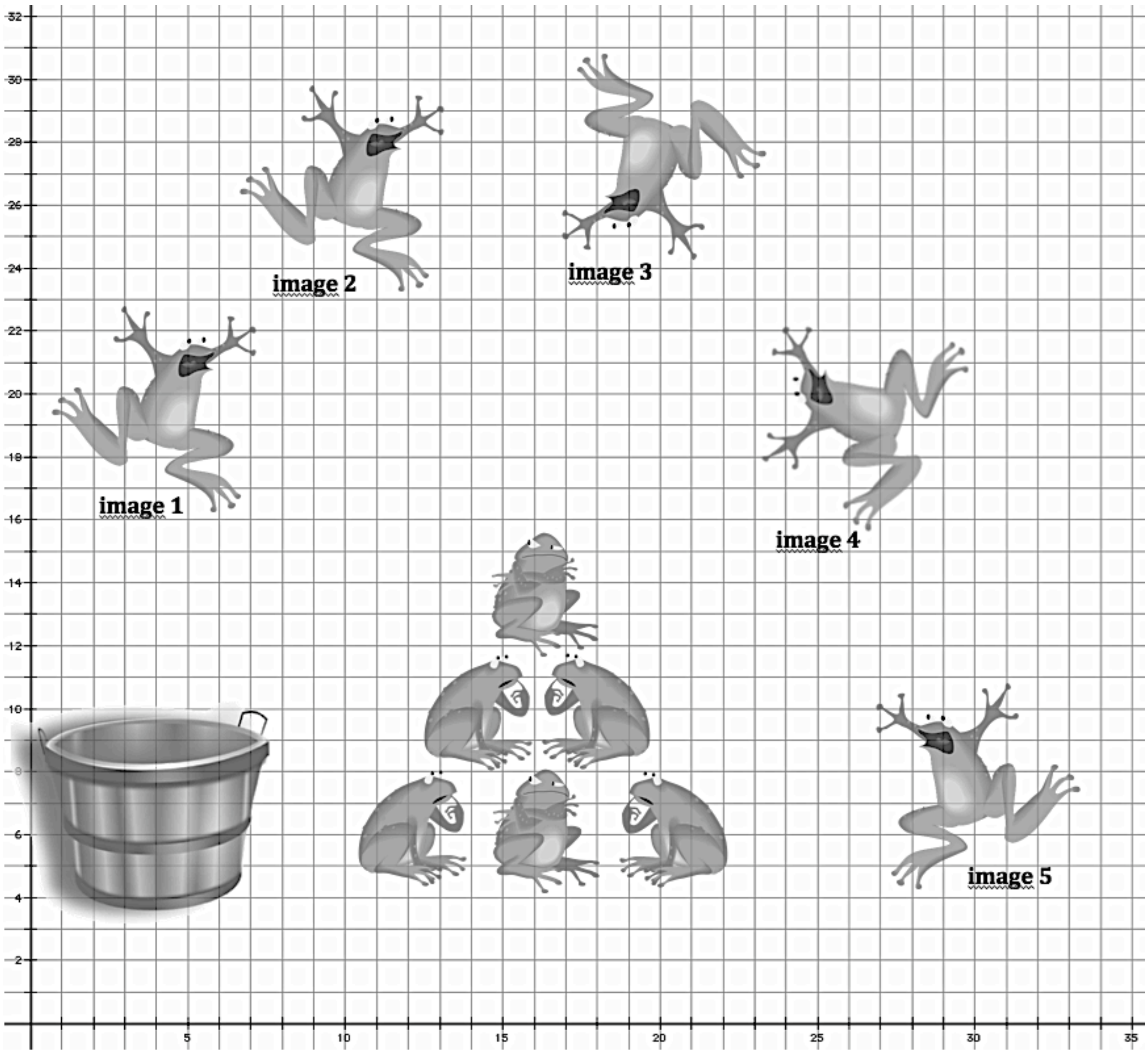

Ready, Set, Go!

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Ready

Topic: Basic Rotations and Reflections of objects

In each problem there will be a preimage and several images based on the give preimage. Determine which of the images are rotations of the given preimage and which of them are reflections of the preimage. If an image appears to be created as the result of a rotation and a reflection then state both.

1.

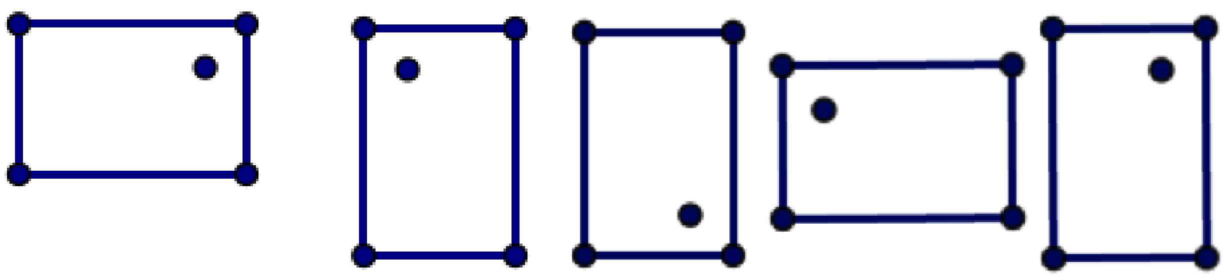
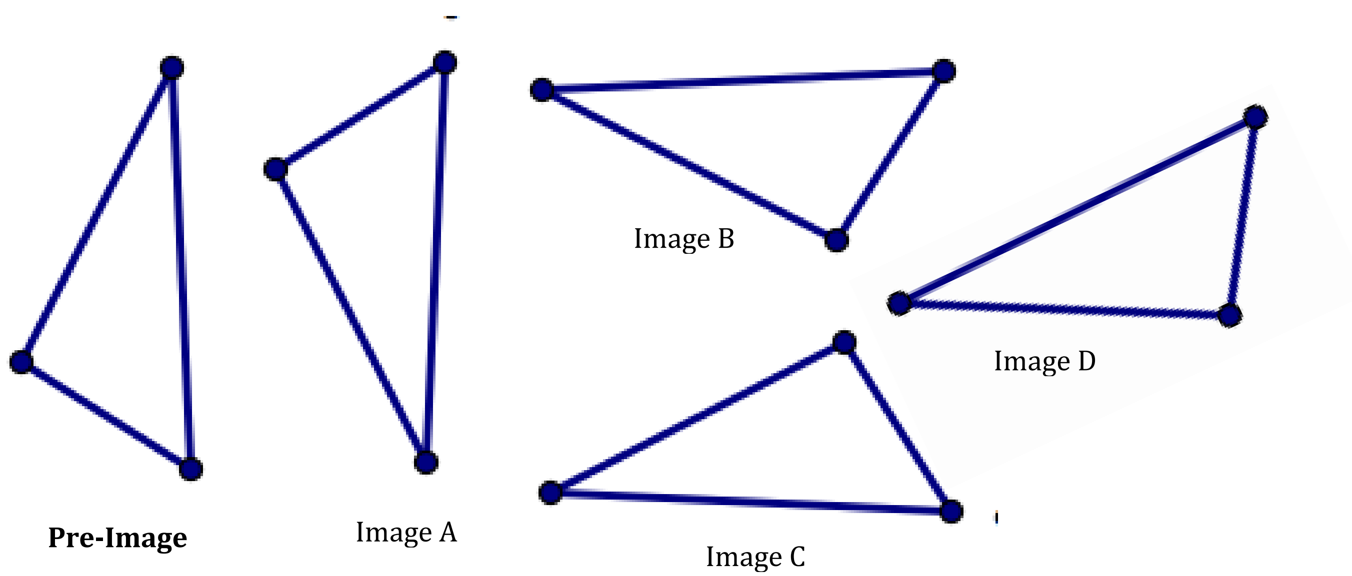


Image A Image B Image C Image D

Pre-Image

2.

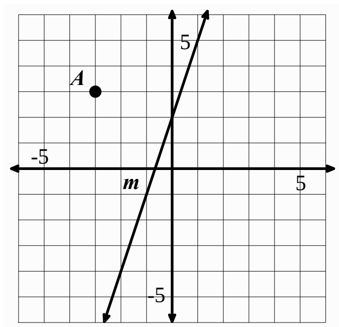


Set

Topic: Reflecting and Rotating points

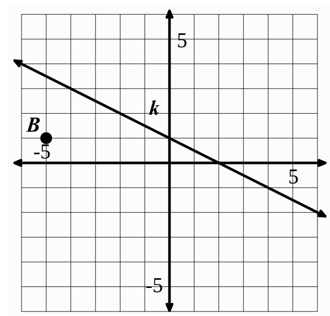
On each of the coordinate grids there is a labeled point and line. Use the line as a line of reflection to reflect the given point and create its reflected image over the line of reflection.

3.



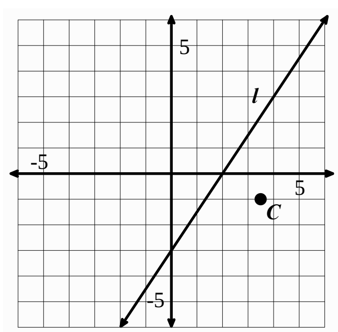
Reflect point ***A*** over line ***m*** and label the image ***A'***

4.



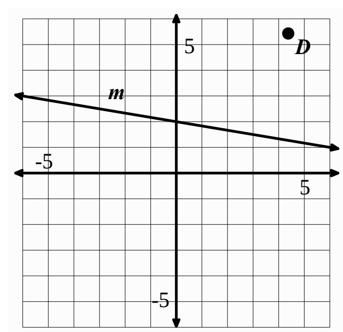
Reflect point ***B*** over line ***k*** and label the image ***B'***

5.



Reflect point ***C*** over line ***l*** and label the image ***C'***

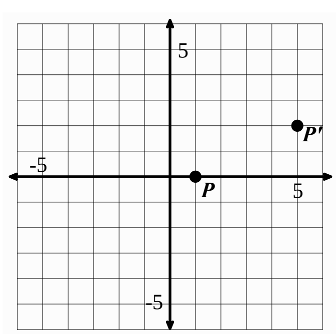
6.



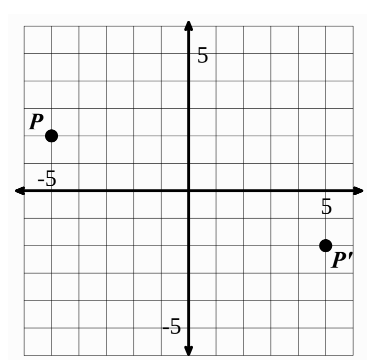
Reflect point ***D*** over line ***m*** and label the image ***D'***

For each pair of point, *P* and *P'* draw in the line of reflection that would need to be used to reflect *P* onto *P'*. Then find the equation of the line of reflection.

7.

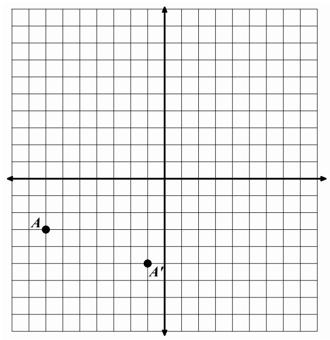


8.

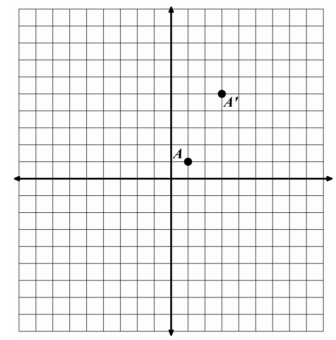


For each pair of point, *A* and *A'* draw in the line of reflection that would need to be used to reflect *A* onto *A'*. Then find the equation of the line of reflection. Also, draw a line connecting *A* to *A'* and find the equation of this line. Compare the slopes of the lines of reflection containing *A* and *A'*.

9.



10.



Go

Topic: Slopes of parallel and perpendicular lines and finding both distance and slope between two points.

For each linear equation write the slope of a line parallel to the given line.

11. $y = -3x + 5$

12. $y = 7x - 3$

13. $3x - 2y = 8$

For each linear equation write the slope of a line perpendicular to the given line.

14. $y = -\frac{2}{7}x + 5$

15. $y = \frac{1}{5}x - 4$

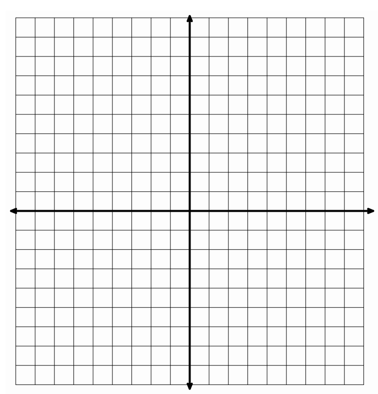
16. $3x + 5y = -15$

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points. You may use the graph to help you as needed.

17. (-2, -3) (1, 1)
a. Slope: b. Distance:

18. (-7, 5) (-2, -7)
a. Slope: b. Distance:

19. (2, -4) (3, 0)
a. Slope: b. Distance:

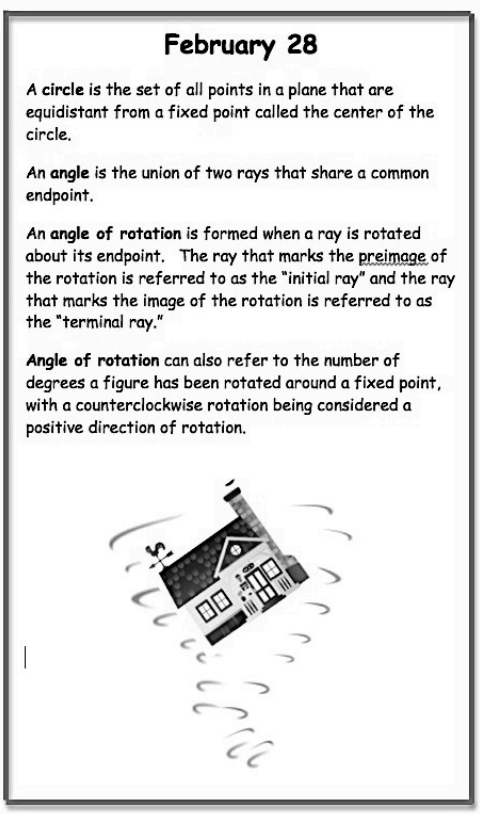


6.4 Leap Year
A Practice Understanding Task

Carlos and Clarita are discussing their latest business venture with their friend Juanita. They have created a daily planner that is both educational and entertaining. The planner consists of a pad of 365 pages bound together, one page for each day of the year. The planner is entertaining since images along the bottom of the pages form a flip-book animation when thumbed through rapidly. The planner is educational since each page contains some interesting facts. Each month has a different theme, and the facts for the month have been written to fit the theme. For example, the theme for January is astronomy, the theme for February is mathematics, and the theme for March is ancient civilizations. Carlos and Clarita have learned a lot from researching the facts they have included, and they have enjoyed creating the flip-book animation.

The twins are excited to share the prototype of their planner with Juanita before sending it to printing. Juanita, however, has a major concern. "Next year is leap year," she explains, "you need 366 pages."

So now Carlos and Clarita have the dilemma of having to create an extra page to insert between February 28 and March 1. Here are the planner pages they have already designed.



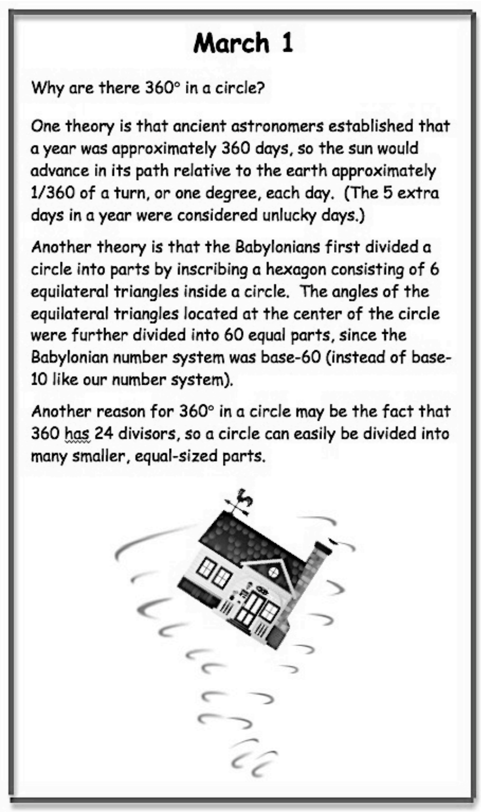
February 28

A **circle** is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

An **angle** is the union of two rays that share a common endpoint.

An **angle of rotation** is formed when a ray is rotated about its endpoint. The ray that marks the preimage of the rotation is referred to as the "initial ray" and the ray that marks the image of the rotation is referred to as the "terminal ray."

Angle of rotation can also refer to the number of degrees a figure has been rotated around a fixed point, with a counterclockwise rotation being considered a positive direction of rotation.



March 1

Why are there 360° in a circle?

One theory is that ancient astronomers established that a year was approximately 360 days, so the sun would advance in its path relative to the earth approximately 1/360 of a turn, or one degree, each day. (The 5 extra days in a year were considered unlucky days.)

Another theory is that the Babylonians first divided a circle into parts by inscribing a hexagon consisting of 6 equilateral triangles inside a circle. The angles of the equilateral triangles located at the center of the circle were further divided into 60 equal parts, since the Babylonian number system was base-60 (instead of base-10 like our number system).

Another reason for 360° in a circle may be the fact that 360 has 24 divisors, so a circle can easily be divided into many smaller, equal-sized parts.

Part 1

Since the theme for the facts for February is mathematics, Clarita suggests that they write formal definitions of the three rigid-motion transformations they have been using to create the images for the flip-book animation.

How would you complete each of the following definitions?

1. A translation of a set of points in a plane . . .

2. A rotation of a set of points in a plane . . .

3. A reflection of a set of points in a plane . . .

4. Translations, rotations and reflections are rigid motion transformations because . . .

Carlos and Clarita used these words and phrases in their definitions: perpendicular bisector, center of rotation, equidistant, angle of rotation, concentric circles, parallel, image, pre-image, preserves distance and angle measures.

Revise your definitions so they also use these words or phrases.

Part 2

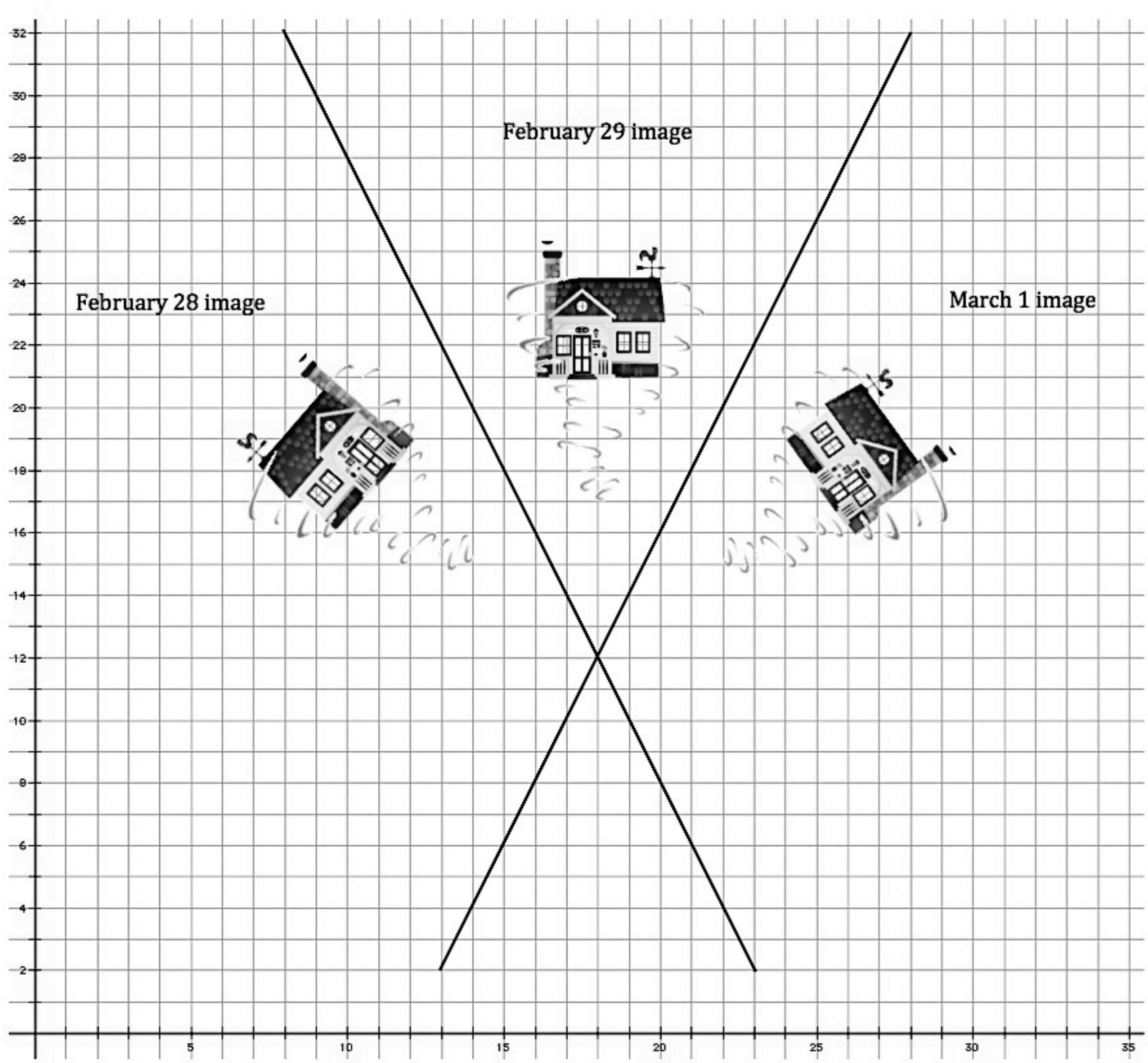
In addition to writing new facts for February 29, the twins also need to add another image in the middle of their flip-book animation. The animation sequence is of Dorothy's house from the Wizard of Oz as it is being carried over the rainbow by a tornado. The house in the February 28 drawing has been rotated to create the house in the March 1 drawing. Carlos believes that he can get from the February 28 drawing to the March 1 drawing by reflecting the February 28 drawing, and then reflecting it again.

Verify that the image Carlos inserted between the two images that appeared on February 28 and March 1 works as he intended. For example,

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the February 29 image is a reflection of the February 28 image about the given line of reflection?

- What convinces you that the two reflections together complete a rotation between the February 28 and March 1 images?



Images this page:
http://openclipart.org/detail/168722/simple-farm-pack-by-viscious-speed
www.clker.com/clipart.tornado-gray

Ready, Set, Go!

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Ready

Topic: Defining geometric shapes and components

For each of the geometric words below write a definition of the object that addresses the essential elements. Also, list necessary attributes and characteristics.

1. Quadrilateral:

2. Parallelogram:

3. Rectangle:

4. Square:

5. Rhombus:

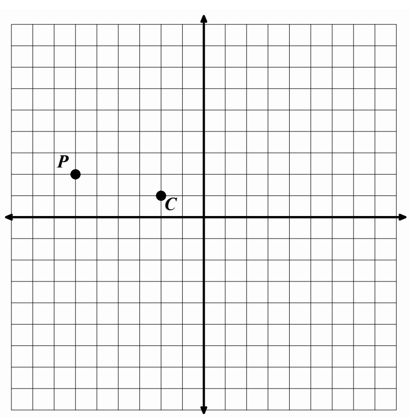
6. Trapezoid:

Set

Topic: Reflections and Rotations, composing reflections to create a rotation

Perform the indicated rotations.

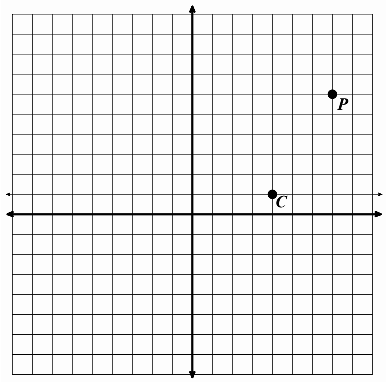
7.



Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P***'.

With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P***''.

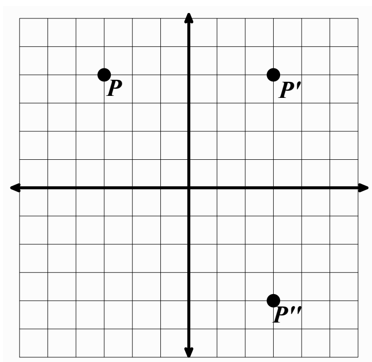
8.



Use the center of rotation point ***C*** and rotate point ***P*** clockwise around it 90^0. Label the image ***P'***.

With point ***C*** as a center of rotation also rotate point ***P*** 180^0. Label this image ***P''***.

9.

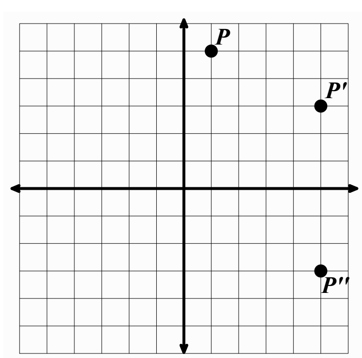


a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

10.

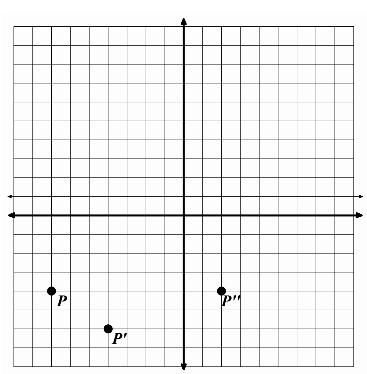


a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

11.



a. What is the equation for the line for reflection that reflects point ***P*** onto ***P'***?

b. What is the equation for the line of reflections that reflects point ***P'*** onto ***P''*** ?

c. Could ***P''*** also be considered a rotation of point ***P***? If so what is the center of rotation and how many degrees was point ***P*** rotated?

Go

Topic: Rotations about the origin

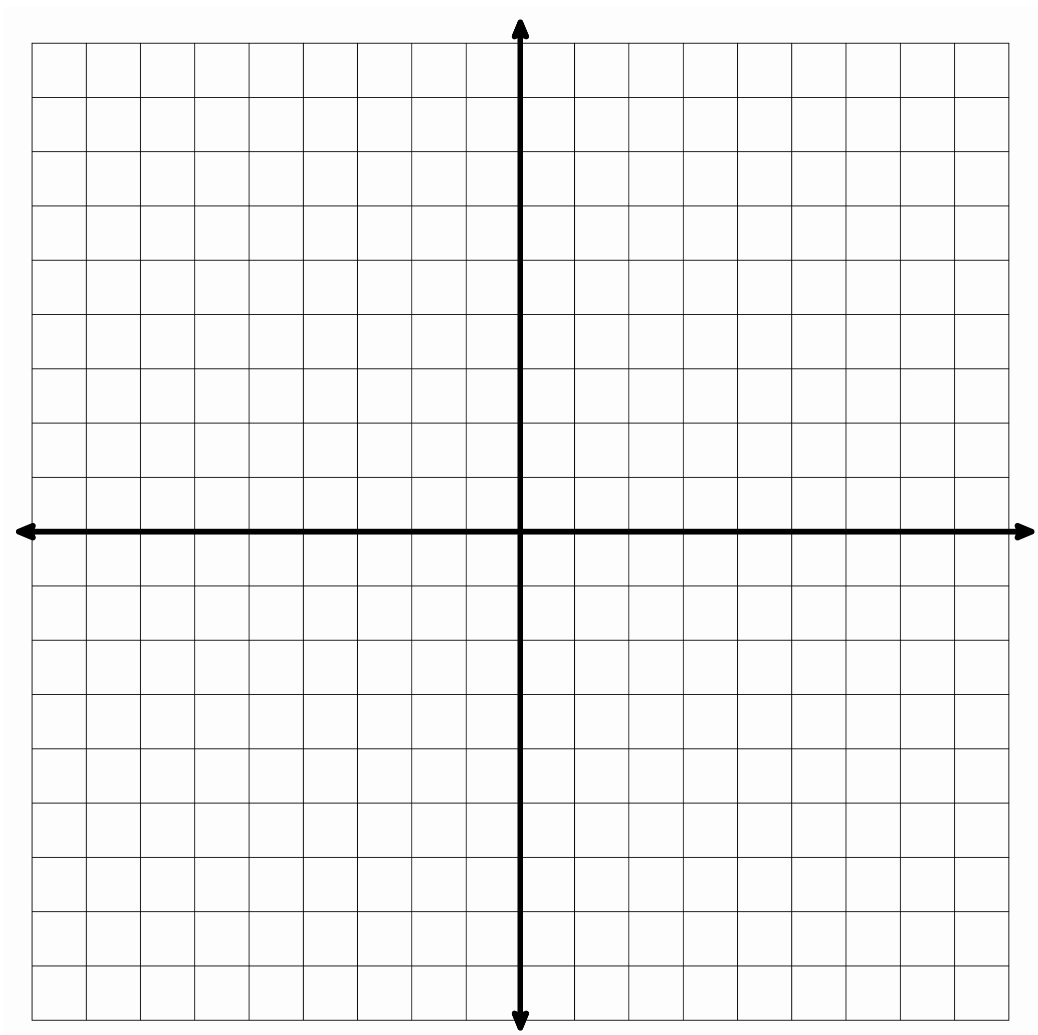
Plot the given coordinate and then perform the indicated rotation in a clockwise direction around the origin, the point (0 , 0), and plot the image created. State the coordinates of the image.

12. Point ***A*** (4, 2) rotate 180^0
    Coordinates for Point ***A'*** (___ , ___)

13. Point ***B*** (-5, -3) rotate 90^0 clockwise
    Coordinates for Point ***B'*** (___ , ___)

14. Point ***C*** (-7, 3) rotate 180^0
    Coordinates for Point ***C'*** (___ , ___)

15. Point ***D*** (1, -6) rotate 90^0 clockwise
    Coordinates for Point ***D'*** (___ , ___)



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6.5 Symmetries of Quadrilaterals
A Develop Understanding Task

A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

1. A **rectangle** is a quadrilateral that contains four right angles. Is it possible to reflect or rotate a rectangle onto itself?

For the rectangle shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rectangle onto itself.

Describe the rotations and/or reflections that carry a rectangle onto itself. (Be as specific as possible in your descriptions.)

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel. Is it possible to reflect or rotate a parallelogram onto itself?

For the parallelogram shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the parallelogram onto itself.

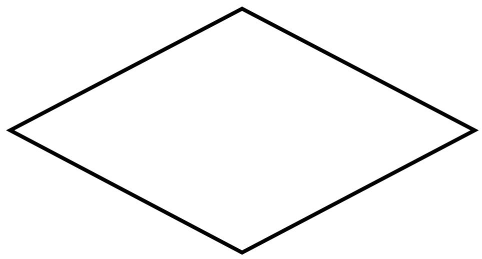
Describe the rotations and/or reflections that carry a parallelogram onto itself. (Be as specific as possible in your descriptions.)

3. A **rhombus** is a quadrilateral in which all sides are congruent. Is it possible to reflect or rotate a rhombus onto itself?

For the rhombus shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rhombus onto itself.



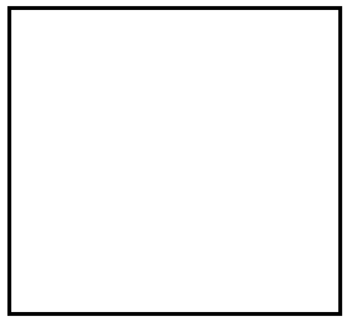
Describe the rotations and/or reflections that carry a rhombus onto itself. (Be as specific as possible in your descriptions.)

4. A **square** is both a rectangle and a rhombus. Is it possible to reflect or rotate a square onto itself?

For the square shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the square onto itself.



Describe the rotations and/or reflections that carry a square onto itself. (Be as specific as possible in your descriptions.)

5. A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find

- any lines of symmetry, or
- any centers of rotational symmetry

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

Ready, Set, Go!

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Ready

Topic: Polygons, definition and names

1. What is a polygon? Describe in your own words what a polygon is.

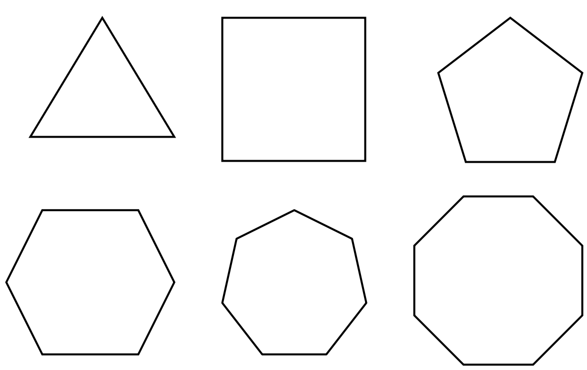
2. Fill in the names of each polygon based on the number of sides the polygon has.

Number of Sides	Name of Polygon
3	
4	
5	
6	
7	
8	
9	
10	

Set

Topic: Lines of symmetry and diagonals

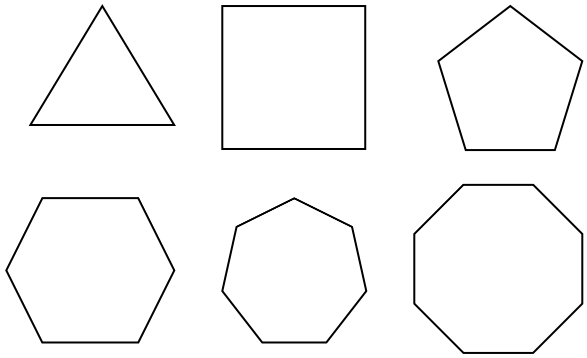
3. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a ***n***-sided polygon.



4. Find

Number of Sides	Number of lines of symmetry
3	
4	
5	
6	
7	
8	
n	

all of the diagonals in each regular polygon. Fill in the table including an expression for the number of diagonals in a ***n***-sided polygon.



Number of Sides	Number of diagonals
3	
4	
5	
6	
7	
8	
n	

5. Are all lines of symmetry also diagonals? Explain.

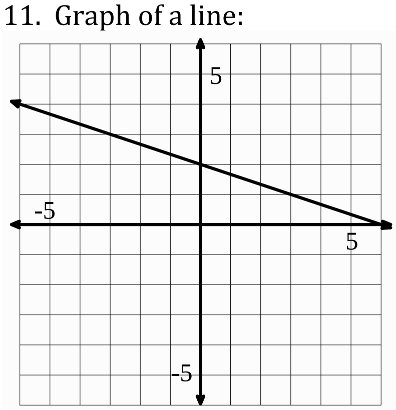
6. Are all diagonals also lines of symmetry? Explain.

7. What shapes will have diagonals that are not lines of symmetry? Name some and draw them.

8. Will all parallelograms have diagonals that are lines of symmetry? If so, draw and explain. If not draw and explain.

Go

Topic: Equations for parallel and perpendicular lines.

<table>
<tr><th></th><th>Find the equation of a line PARALLEL to the given info and through the indicated point.</th><th>Find the equation of a line PERPENDICULAR to the given line and through the indicated point.</th></tr>
<tr><td>9. Equation of a line:
$y = 4x + 1$.</td><td>a. Parallel line through point (-1 , -7):</td><td>b. Perpendicular to the line line through point (-1 , -7):</td></tr>
<tr><td>10. Table of a line:
<table><tr><th>x</th><th>y</th></tr><tr><td>3</td><td>-8</td></tr><tr><td>4</td><td>-10</td></tr><tr><td>5</td><td>-12</td></tr><tr><td>6</td><td>-14</td></tr></table></td><td>a. Parallel line through point (3 , 8):</td><td>b. Perpendicular to the line through point (3 , 8):</td></tr>
<tr><td>11. Graph of a line:
</td><td>a. Parallel line through point (2 , -9):</td><td>b. Perpendicular to the line through point (2 , -9):</td></tr>
</table>

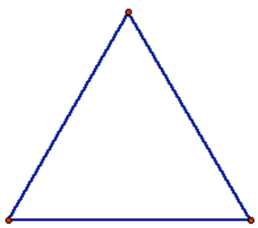
6.6 Symmetries of Regular Polygons

A Solidify Understanding Task

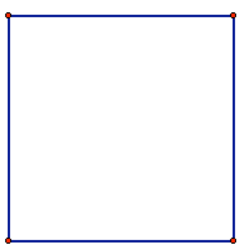
A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

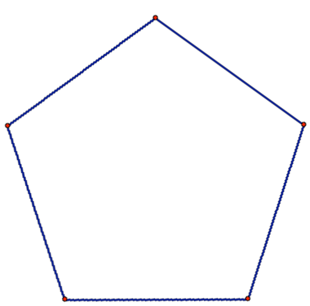
1. An equilateral triangle



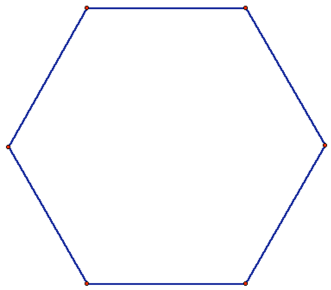
2. A square



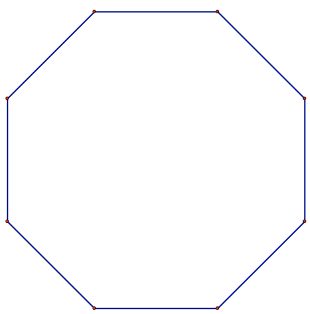
3. A regular pentagon



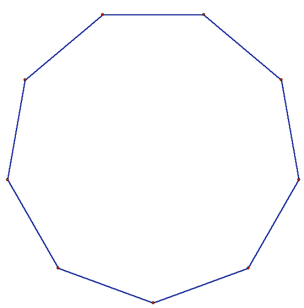
4. A regular hexagon



5. A regular octagon



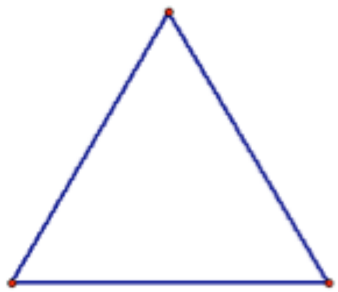
6. A regular nonagon



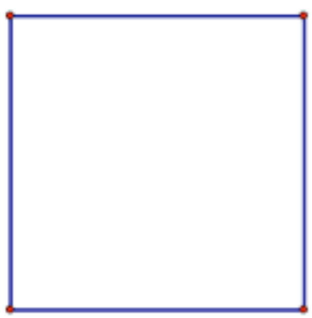
What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

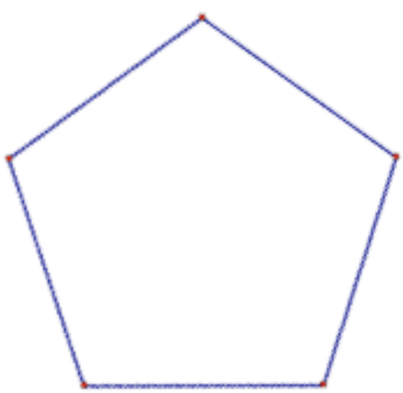
1. An equilateral triangle



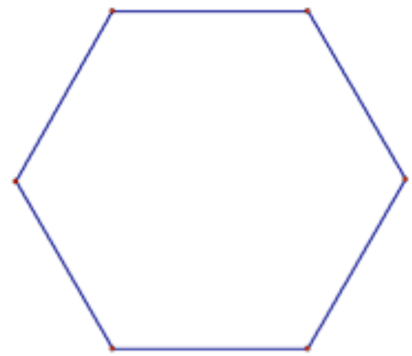
2. A square



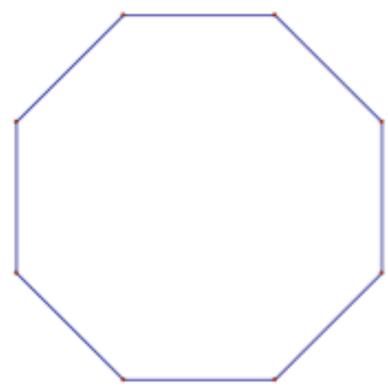
3. A regular pentagon



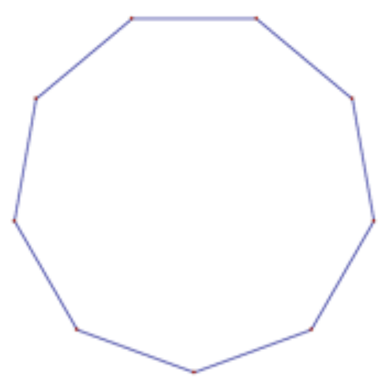
4. A regular hexagon



5. A regular octagon



6. A regular nonagon



Ready, Set, Go!

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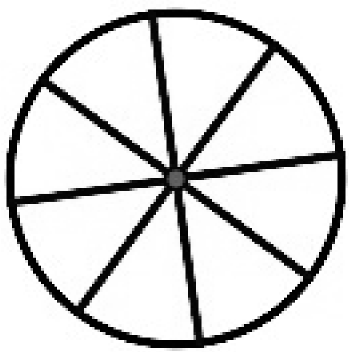
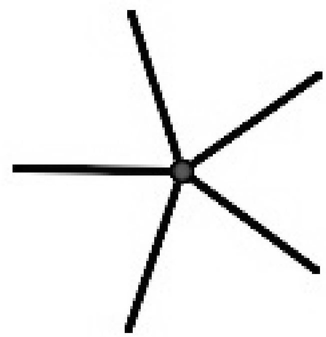
Ready

Topic: Rotation as a transformation, what does it mean?

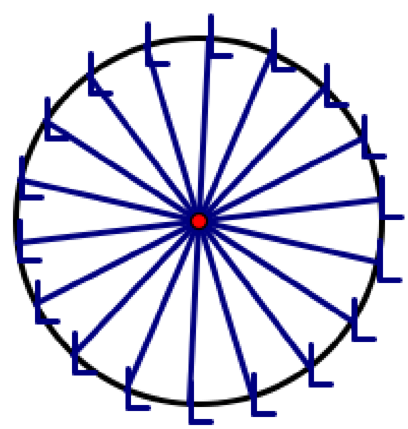
1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



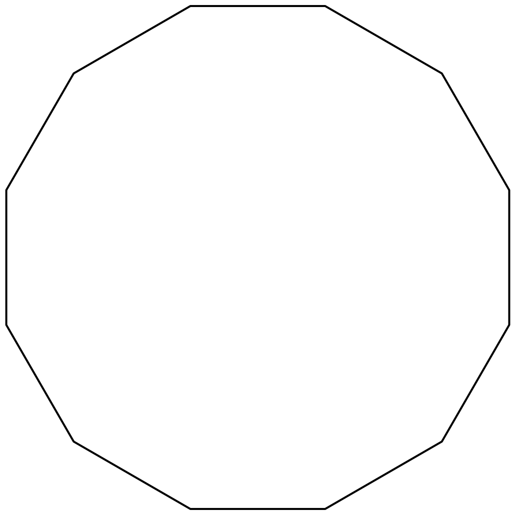
3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



Set

Topic: Finding angels of rotation for regular polygons.

4. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



5. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

6. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

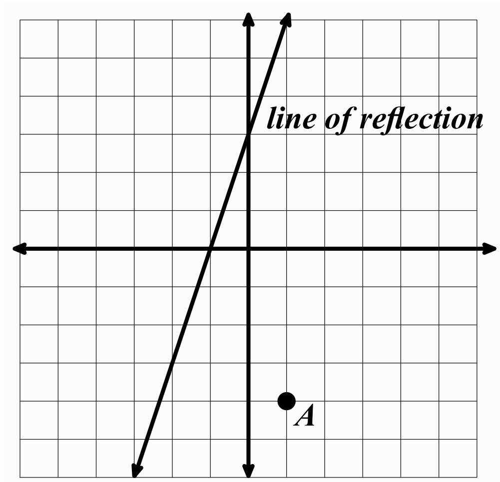
7. How many sides does a regular polygon have that has an angle of rotation equal to 18^0? Explain.

8. How many sides does a regular polygon have that has an angle of rotation equal to 20^0? How many lines of symmetry will it have?

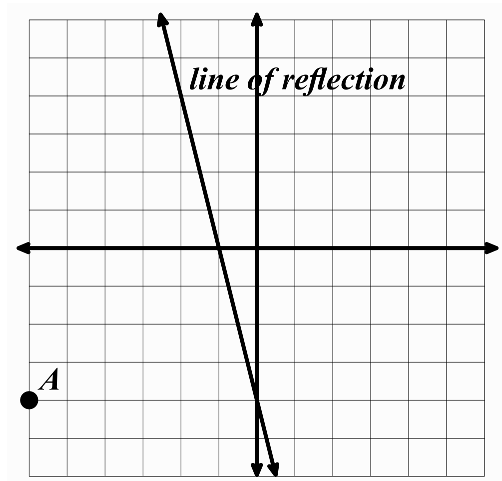
Go

Topic: Reflecting and Rotating points on the coordinate plane.

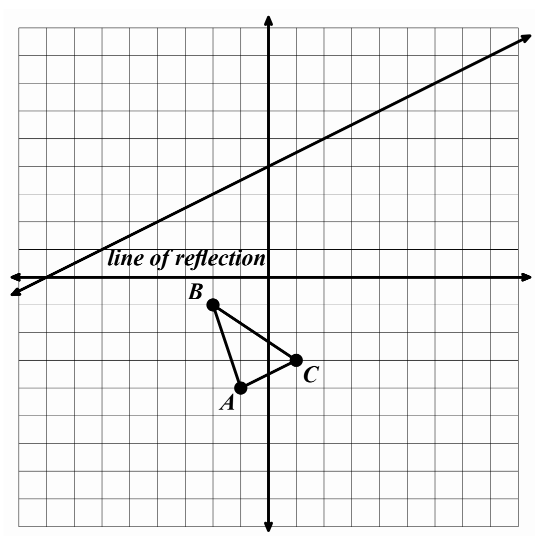
9. Reflect point ***A*** over the line of reflection and label the image ***A'***.



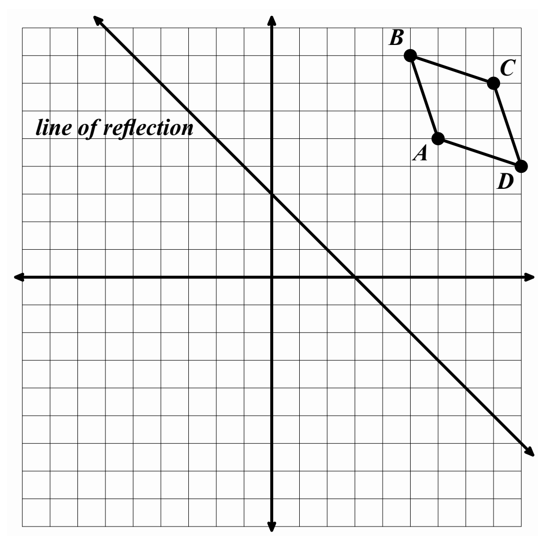
10. Reflect point ***A*** over the line of reflection and label the image ***A'***.



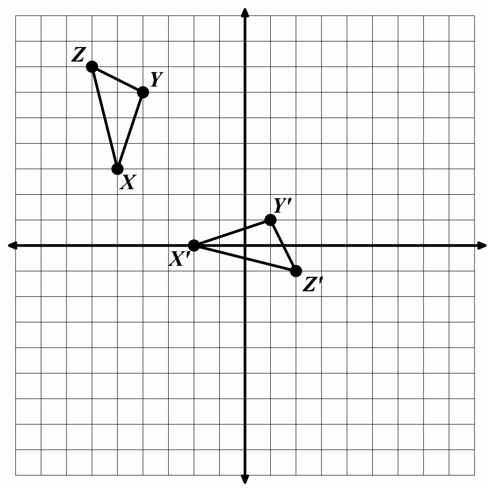
11. Reflect triangle ***ABC*** over the line of reflection and label the image ***A'B'C'***.



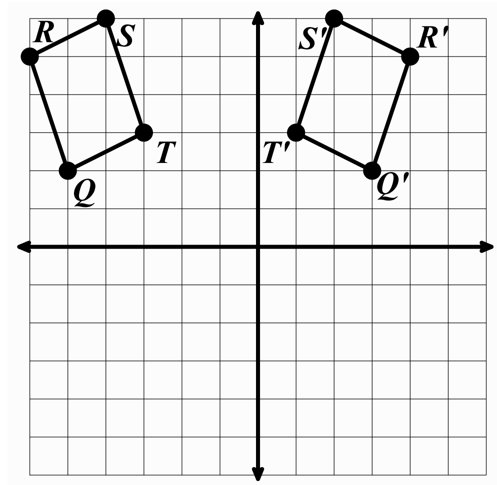
12. Reflect parallelogram ***ABCD*** over the line of reflection and label the image ***A'B'C'D'.***



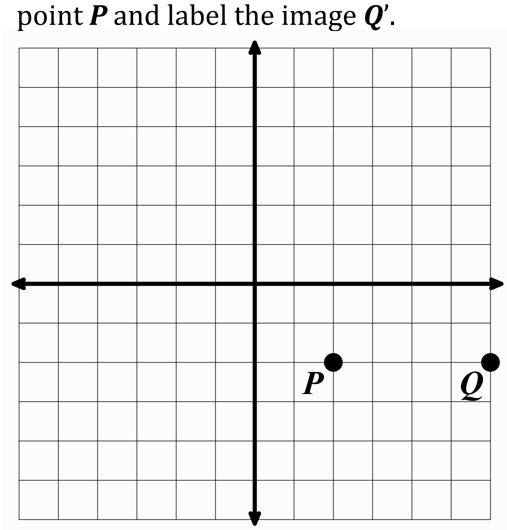
13. Given triangle ***XYZ*** and its image ***X'Y'Z'*** draw the line of reflection that was used.



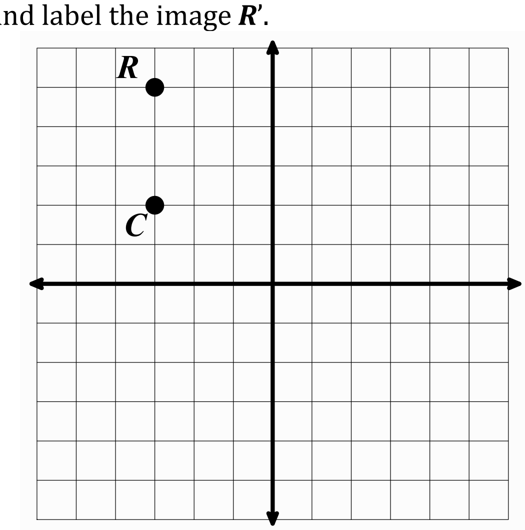
14 Given parallelogram ***QRST*** and its image ***Q'R'S'T'*** draw the line of reflection that was used.



15. Using point ***P*** as a center of rotation. Rotate point ***Q*** 120^0 clockwise about point ***P*** and label the image ***Q'***.



16. Using point ***C*** as the center or rotation. Rotate point ***R*** 270^0 counter-clockwise about point ***C*** and label the image ***R'***.



6.7 Quadrilaterals—Beyond Definition

A Practice Understanding Task

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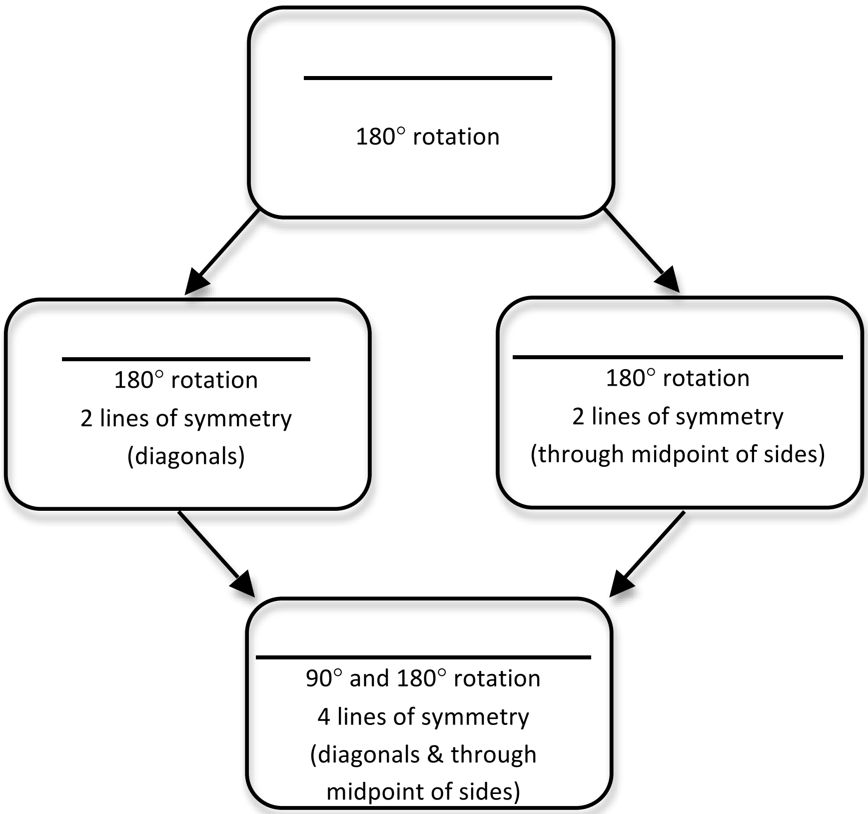
We have found that many different quadrilaterals possess line and/or rotational symmetry.

In the following chart, write the names of the quadrilaterals that are being described in terms of their symmetries.



What do you notice about the relationships between quadrilaterals based on their symmetries and highlighted in the structure of the above chart?

Based on the symmetries we have observed in various types of quadrilaterals, we can make claims about other features and properties that the quadrilaterals may possess.

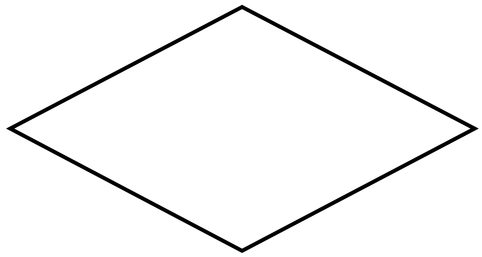
1. A **rectangle** is a quadrilateral that contains four right angles.

Based on what you know about transformations, what else can we say about rectangles besides the defining property that all four angles are right angles? Make a list of additional properties of rectangles that seem to be true based on the transformation(s) of the rectangle onto itself. You will want to consider properties of the sides, the angles, and the diagonals.

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel.

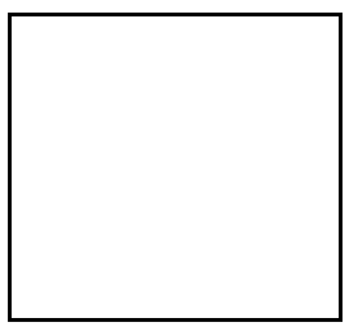
Based on what you know about transformations, what else can we say about parallelograms besides the defining property that opposite sides of a parallelogram are parallel? Make a list of additional properties of parallelograms that seem to be true based on the transformation(s) of the parallelogram onto itself. You will want to consider properties of the sides, angles and the diagonals.

3. A **rhombus** is a quadrilateral in which all four sides are congruent.



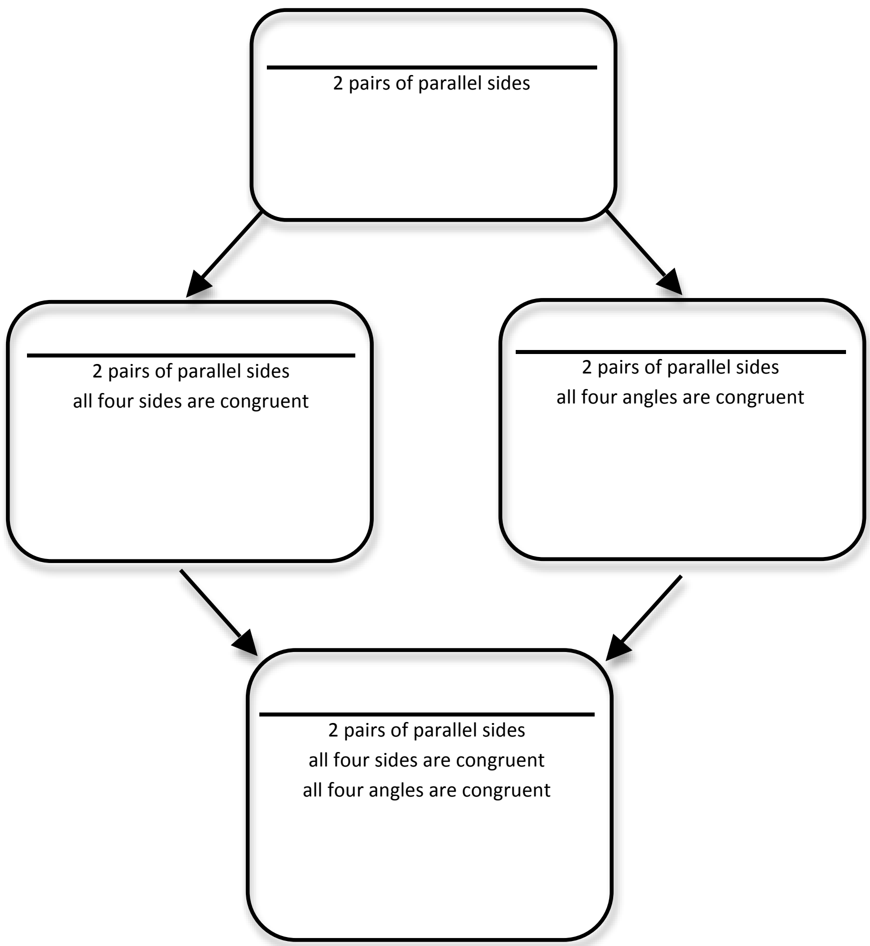
Based on what you know about transformations, what else can we say about a rhombus besides the defining property that all sides are congruent? Make a list of additional properties of rhombuses that seem to be true based on the transformation(s) of the rhombus onto itself. You will want to consider properties of the sides, angles and the diagonals.

4. A **square** is both a rectangle and a rhombus.



Based on what you know about transformations, what can we say about a square? Make a list of properties of squares that seem to be true based on the transformation(s) of the squares onto itself. You will want to consider properties of the sides, angles and the diagonals.

In the following chart, write the names of the quadrilaterals that are being described in terms of their features and properties, and then record any additional features or properties of that type of quadrilateral you may have observed. Be prepared to share reasons for your observations.



What do you notice about the relationships between quadrilaterals based on their characteristics and highlighted in the structure of the above chart?

How are the charts at the beginning and end of this task related? What do they suggest?

Ready, Set, Go!

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Ready

Topic: Defining Congruence and Similarity.

1. What do you know about two figures if they are congruent?

2. What do you need to know about two figures to be convinced that the two figures are congruent?

3. What do you know about two figures if they are similar?

4. What do you need to know about two figures to be convinced that the two figures are similar?

Set

Topic: Classifying quadrilaterals based on their properties.

Using the information given determine the most accurate classification of the quadrilateral.

5. Has 180^0 rotational symmetry.

6. Has 90^0 rotational symmetry.

7. Has two lines of symmetry that are diagonals.

8. Has two lines of symmetry that are not diagonals.

9. Has congruent diagonals.

10. Has diagonals that bisect each other.

11. Has diagonals that are perpendicular.

12. Has congruent angles.

Go

Topic: Slope and distance

Find the *slope* between each pair of points. Then, using the Pythagorean Theorem, find the *distance* between each pair of points.

13. (-3 , -2) , (0 , 0)

a. Slope: b. Distance:

14. (7 , -1) , (11 , 7)

a. Slope: b. Distance:

15. (-10 , 13) , (-5 , 1)

a. Slope: b. Distance:

16. (-6 , -3) , (3 , 1)

a. Slope: b. Distance:

17. (5 , 22) , (17 , 28)

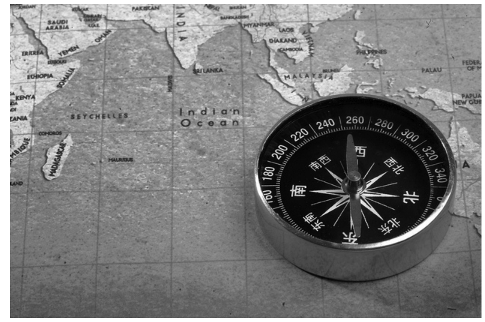
a. Slope: b. Distance:

18. (1 , -7) , (6 , 5)

a. Slope: b. Distance:

6. 8 Can You Get There From Here?

A Develop Understanding Task



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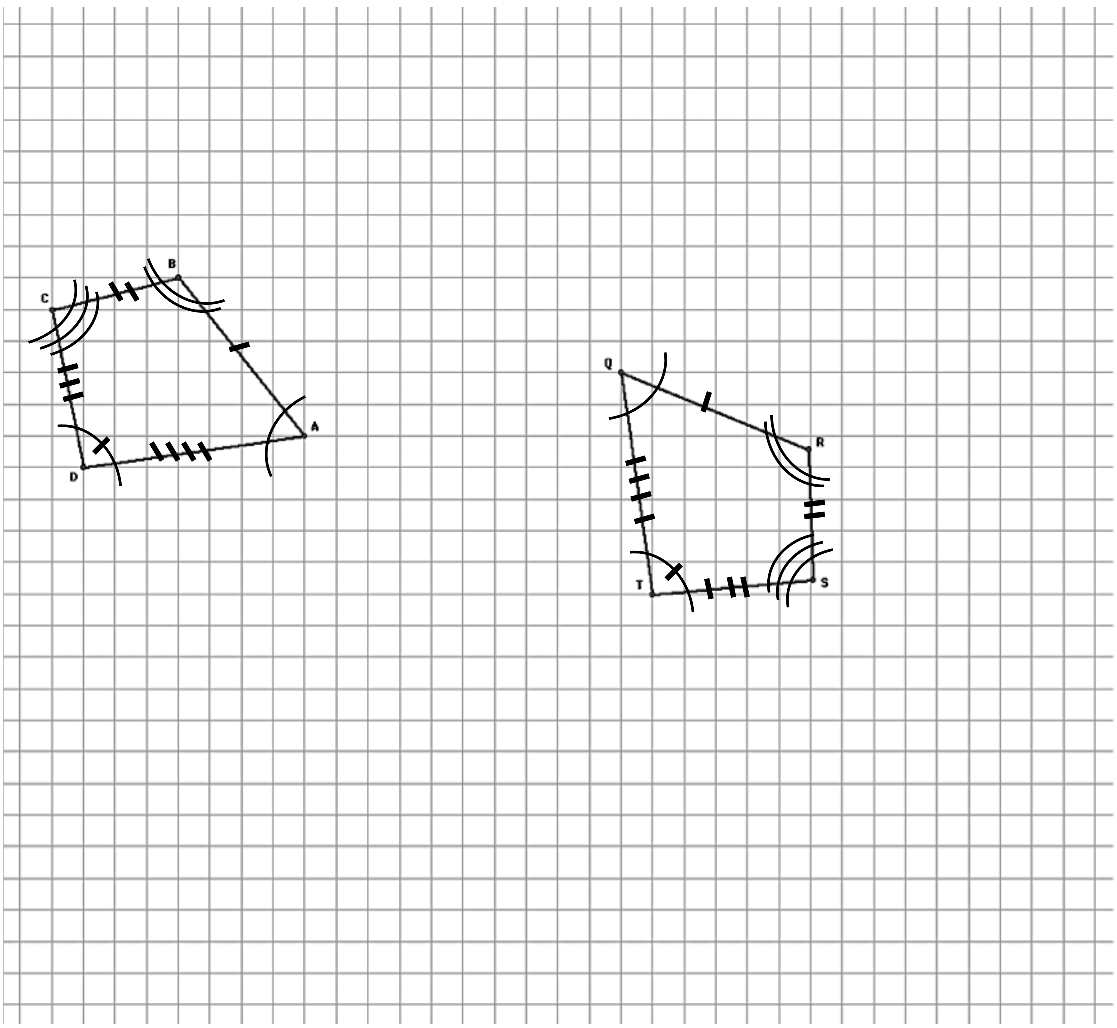
The two quadrilaterals shown below, quadrilateral *ABCD* and quadrilateral *QRST* are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral *ABCD* onto quadrilateral *QRST*. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



Ready, Set, Go!

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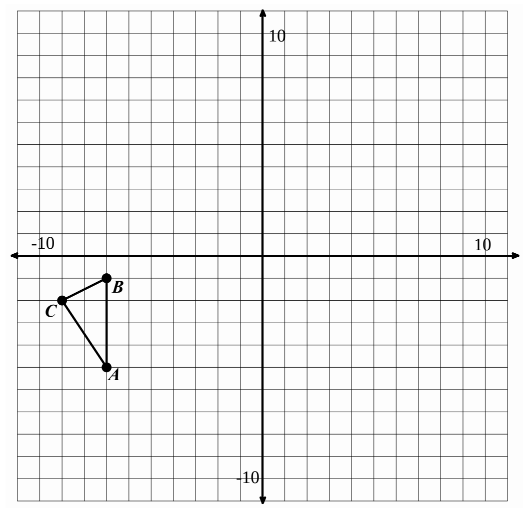
Ready

Topic: Performing a sequence of transformations.

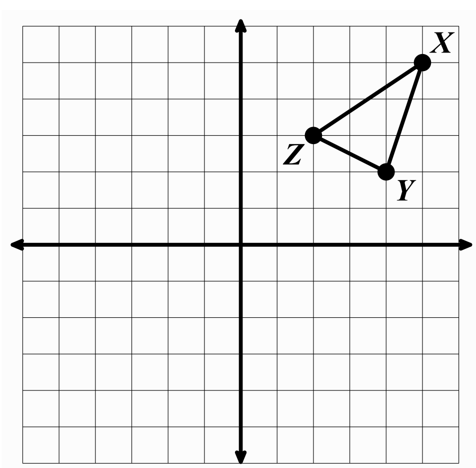
The given figures are to be used as pre-images. Perform the stated transformations to obtain an image. Label the corresponding parts of the image in accordance with the pre-image.

1. Reflect triangle ***ABC*** over the line $y = x$ and label the image ***A'B'C'***.

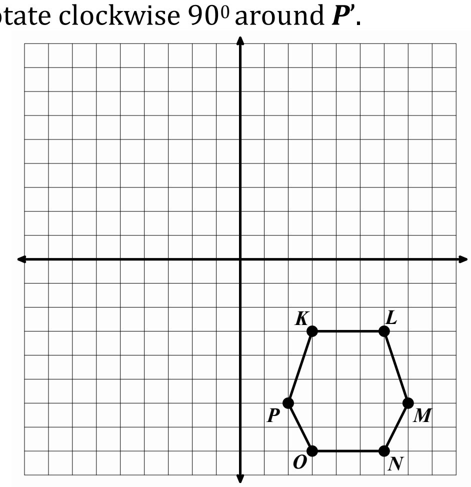
   Rotate triangle ***A'B'C'*** 180^0 counter clockwise around the origin and label the image ***A''B''C''***.



2. Reflect over the line $y = -x$.

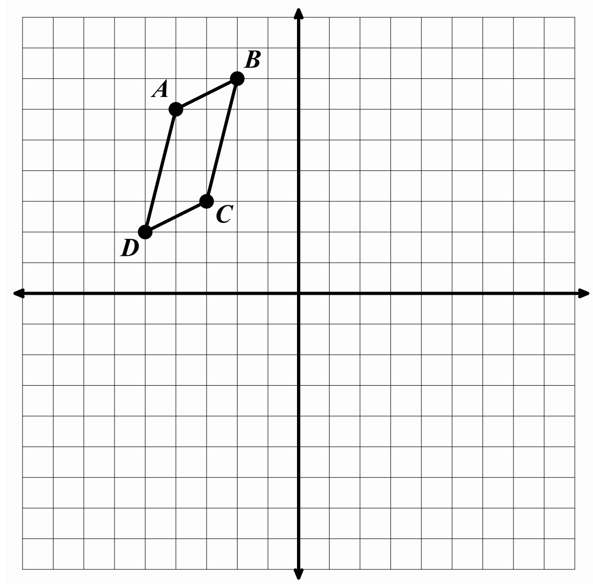


3. Reflect over y-axis and then Rotate clockwise 90^0 around ***P'***.



4. Reflect quadrilateral ABCD over the line $y = 2 + x$ and label the image A'B'C'D'.

Rotate quadrilateral A'B'C'D' counter-clockwise 90^0 around (-2, -3) as the center of rotation label the image A''B''C''C''.

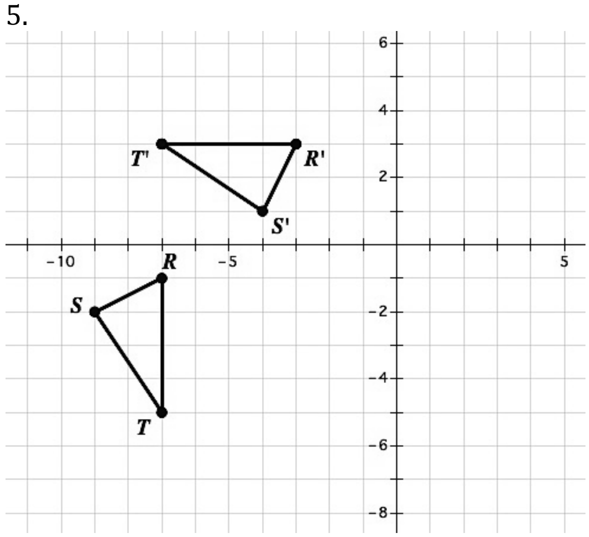


Set

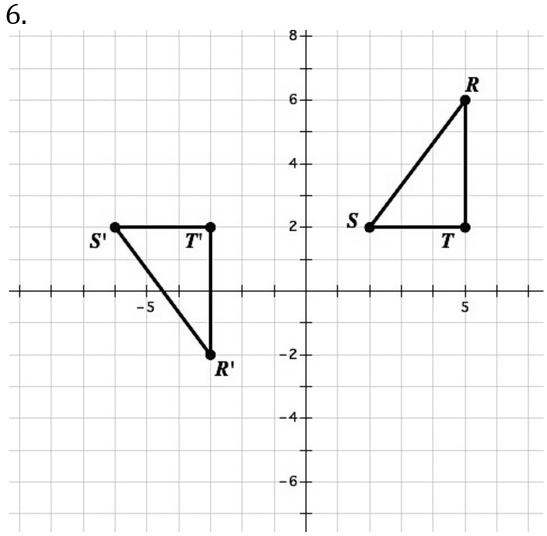
Topic: Find the sequence of transformations.

Find the sequence of transformations that will carry triangle ***RST*** onto triangle ***R'S'T'***. Clearly describe the sequence of transformations below each grid.

5.



6.



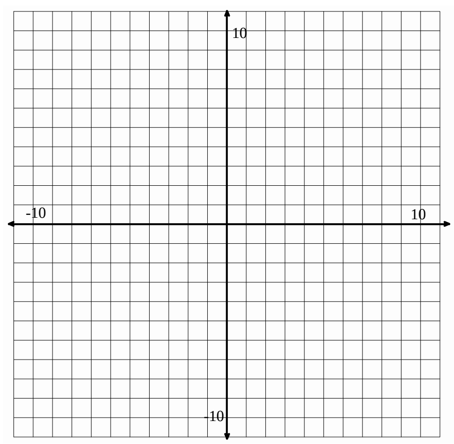
Go

Topic: Graphing functions and making comparisons.

Graph each pair of functions and make an observation about how the functions compare to one another.

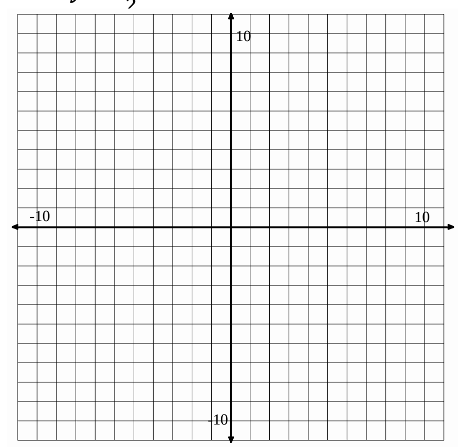
7. $y = \frac{1}{3}x - 1$

   $y = -3x - 1$



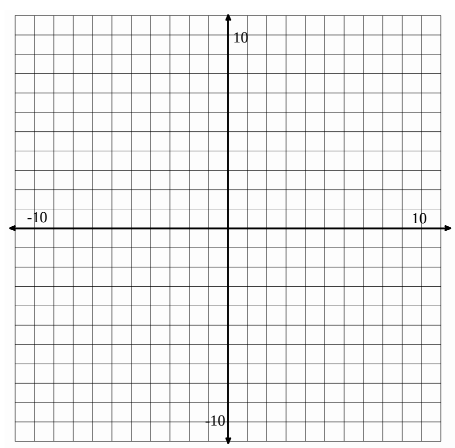
8. $y = -\frac{2}{3}x + 5$

   $y = \frac{3}{2}x + 5$



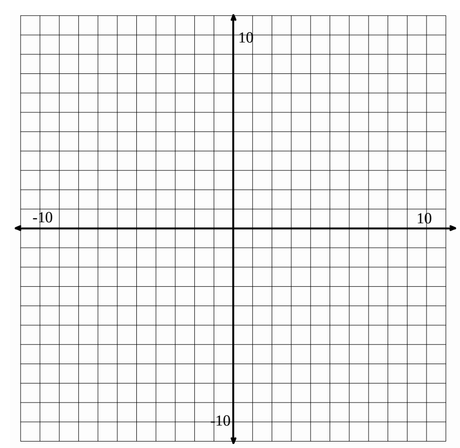
9. $y = \frac{1}{4}x + 2$

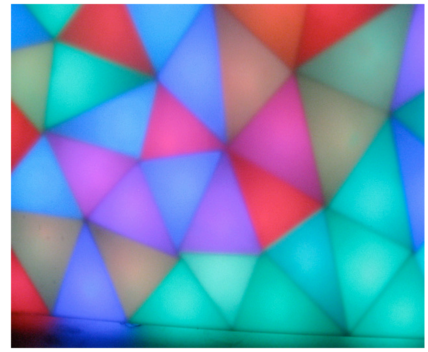
   $y = -\frac{1}{4} + 2$



10. $y = 2^x$

    $y = -2^x$





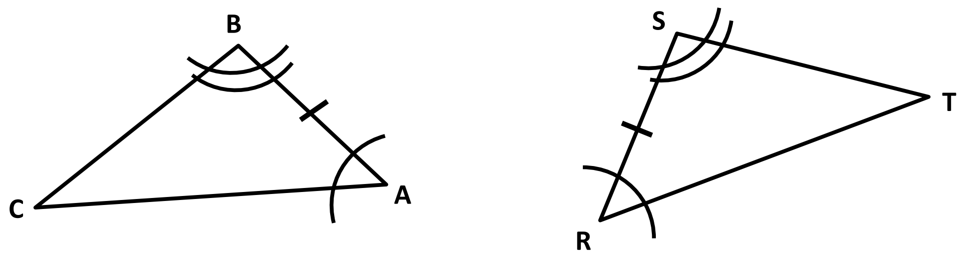
6.9 Congruent Triangles

A Solidify Understanding Task

Zac and Sione are trying to decide how much information they need to know about two triangles before they can convince themselves that the two triangles are congruent.

They are wondering if knowing that two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle—a set of criteria their teacher refers to as ASA—is enough to know that the two triangles are congruent. They are trying to justify that this would be so.

To start reasoning about the congruence of the two triangles, Zac and Sione have created the following diagram in which they have marked an ASA relationship between the triangles.



1. Based on the diagram, which angles have Zac and Sione indicated are congruent? Which sides?

2. To convince themselves that the two triangles are congruent, what else would Zac and Sione need to know?

<u>Zac's Argument</u>

"I know what to do," said Zac. "We can translate point A until it coincides with point R, then rotate $\overline{AB}$ about point R until it coincides with $\overline{RS}$. Finally, we can reflect ΔABC across $\overleftrightarrow{RS}$ and then everything coincides so the triangles are congruent." [Zac and Sione's teacher has suggested they use the word "coincides" when they want to say that two points or line segments occupy the same position on the plane. They like the word, so they plan to use it a lot.]

What do you think about Zac's argument? Does it convince you that the two triangles are congruent? Does it leave out any essential ideas that you think need to be included?

3. Write a paragraph explaining your reaction to Zac's argument:

Sione isn't sure that Zac's argument is really convincing. He asks Zac, "How do you know point C coincides with point T after you reflect the triangle?"

4. How do you think Zac might answer Sione's question?

While Zac is trying to think of an answer to Sione's question he adds this comment, "And you really didn't use all of the information about the corresponding congruent parts of the two triangles."

"What do you mean?" asked Zac.

Sione replied, "You started using the fact that $\angle A \cong \angle R$ when you translated ΔABC so that vertex A coincides with vertex R. And you used the fact that $\overline{AB} \cong \overline{RS}$ when you rotated $\overline{AB}$ to coincide with $\overline{RS}$, but where did you use the fact that $\angle B \cong \angle S$?"

"Yeah, and what does it really mean to say that two angles are congruent?" Zac added. "Angles are more than just their vertex points."

5. How might thinking about Zac and Sione's questions help improve Zac's argument?

<u>Sione's Argument</u>

"I would start the same way you did, by translating point A until it coincides with point R, rotating $\overline{AB}$ about point R until it coincides with $\overline{RS}$, and then reflecting ΔABC across $\overleftrightarrow{RS}$," Sione said. "But then I would want to convince myself that points C and T coincide. I know that an angle is made up of two rays that share a common endpoint. Since I know that $\overrightarrow{AB}$ coincides with $\overrightarrow{RS}$ and $\angle A \cong \angle R$, that means that $\overrightarrow{AC}$ coincides with $\overrightarrow{RT}$. Likewise, I know that $\overrightarrow{BA}$ coincides with $\overrightarrow{SR}$ and $\angle B \cong \angle S$, so $\overrightarrow{BC}$ must coincide with $\overrightarrow{ST}$. Since $\overrightarrow{AC}$ and $\overrightarrow{BC}$ intersect at point C, and $\overrightarrow{RT}$ and $\overrightarrow{ST}$ intersect at point *T*, points *C* and *T* must also coincide because the corresponding rays coincide. Therefore, $\overline{BC} \cong \overline{ST}$, $\overline{CA} \cong \overline{TR}$, and $\angle C \cong \angle T$ because both angles are made up of rays that coincide!"

At first Zac was confused by Sione's argument, but he drew diagrams and carefully marked and sketched out each of his statements until it started to slowly make sense.

6. Do the same kind of work that Zac did to make sense of Sione's argument. What parts of his argument are unclear to you? What ideas did sketching out the words of his proof help you to clarify?

Sione's argument suggests that ASA is sufficient criteria for determining if two triangles are congruent. Now Zac and Sione are wondering about other criteria, such as SAS or SSS, or perhaps even AAA (which Zac immediately rejects because he thinks two triangles can have the same angle measures but be different sizes).

7. Draw two triangles that have SAS congruence. Be sure to mark you triangles to show which sides and which angles are congruent.

8. Write out a sequence of transformations to show that the two triangles potentially coincide.

9. If Sione were to examine your work in #8, what questions would he wonder about?

10. How can you use the given congruence criteria (SAS) to resolve Simone's wonderings?

Repeat 7-10 for SSS congruence.

Ready, Set, Go!

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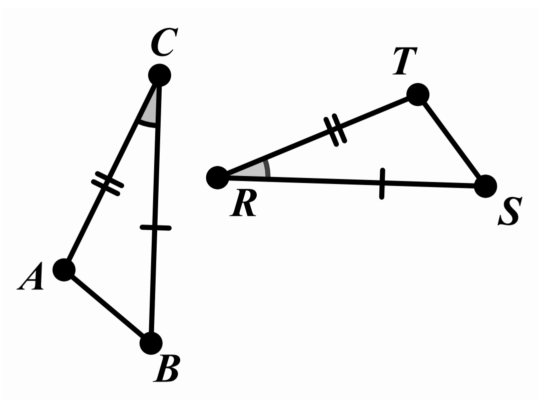
Ready

Topic: Corresponding parts of figures and transformations

Given the figures in each sketch with congruent angles and sides marked, first list the parts of the figures that correspond (For example, in #1, $\angle C \cong \angle R$) Then determine a reflection occurred as part of the sequence of transformations that was used to create the image.

1.

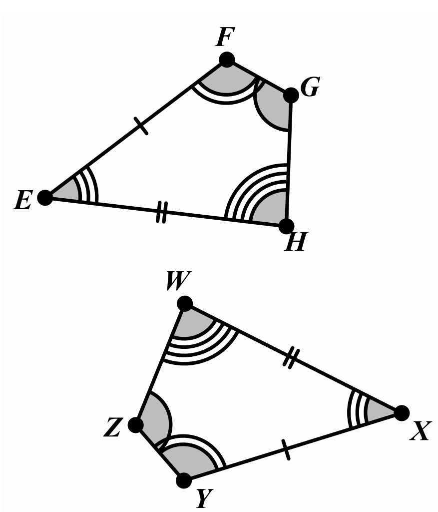


Congruencies

$\angle C \cong \angle R$

Reflected? Yes or No

2.



Congruencies

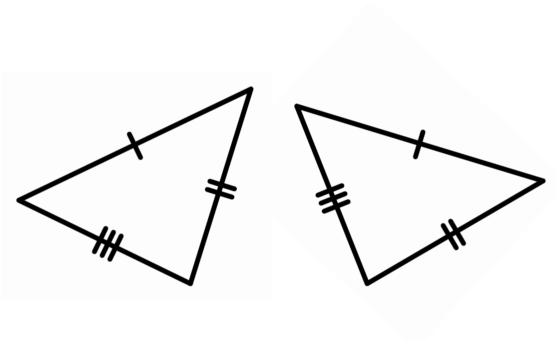
Reflected? Yes or No

Set

Topic: Triangle Congruencies

Explain whether or not the triangles are congruent, similar, or neither based on the markings that indicate congruence.

3.



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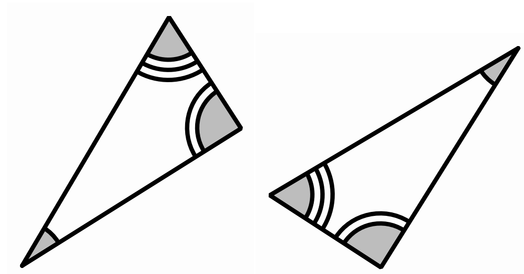
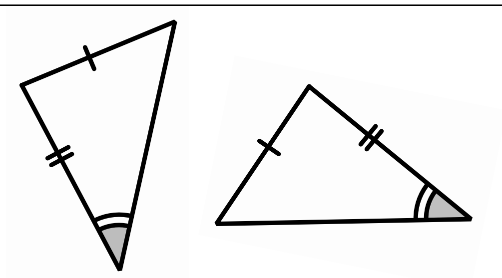


5.



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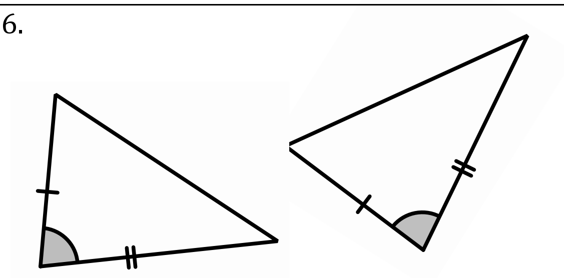
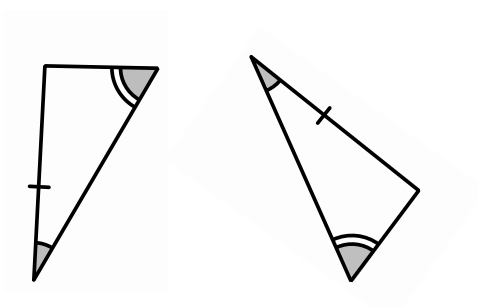
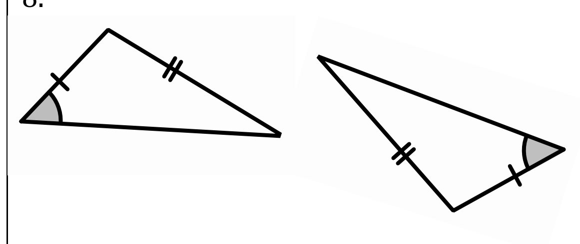


7.



8.



Use the given congruence statement to draw and label two triangles that have the proper corresponding parts congruent to one another.

8. $\Delta ABC \cong \Delta PQR$

9. $\Delta XYZ \cong \Delta KLM$

Go

Topic: Review of solving equations and finding recursive rules for sequences.

Solve each equation for *t*.

10. $\frac{3t-4}{5} = 5$

11. $10 - t = 4t + 12 - 3t$

12. $P = 5t - d$

13. $xy - t = 13t + w$

Use the given sequence of number to write a recursive rule for the *n*th value of the sequence.

14. 5, 15, 45, …

15. $\frac{1}{2}$, 0, -$\frac{1}{2}$, -1, …

16. 3, -6, 12, -24, …

17. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, …

6.10 Congruent Triangles to the Rescue

A Practice Understanding Task

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Part 1

Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent.

Zac: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up of the two congruent sides, and the midpoint of the third side.

Sione: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

Zac: But I've folded lots of isosceles triangles in half, and it always seems to work.

Sione: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.

1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement?

2. What else would Zac need to know about the line through the vertex point of the angle made up of the two congruent sides and the midpoint of the third side in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.)

3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could she use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent.

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)?

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the “crease line”? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)

<u>Part 2</u>

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral *A*, *B*, *C*, and *D*, and label the point of intersection of the two diagonals as point *N.*

2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent.

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$

3. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

- the diagonals bisect each other
- the diagonals are congruent
- the diagonals are perpendicular to each other
- the diagonals bisect the angles of the quadrilateral

Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

Ready, Set, Go!

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Ready

Topic: Defining bisectors of angles and perpendicular bisectors.

1. Based on the meaning of "bisect", which means to split into two equal parts, what would it mean to *bisect* an angle? Describe in words and also provide visuals to communicate the meaning of angle bisector.

2. What does it mean if you have a *perpendicular bisector* of a line segment? Provide both written explanation and visual sketches to communicate the meaning of perpendicular bisector.

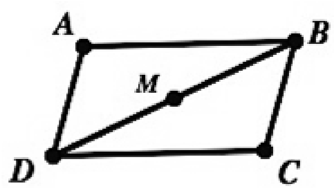
Set

Topic: Use congruent triangle criteria and transformations to justify conjectures.

In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

3. True statements:

Point $\boldsymbol{M}$ is the midpoint of $\overline{DB}$
$\angle ABD \cong \angle BDC$
$\overline{AB} \cong \overline{DC}$

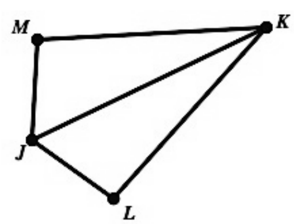


Conjecture: ÐA @ÐC
a. Is the conjecture correct?

b. Argument to prove you are right:

4. True statements

$\angle KJL \cong \angle KJM$
$\overline{JL} \cong \overline{JM}$

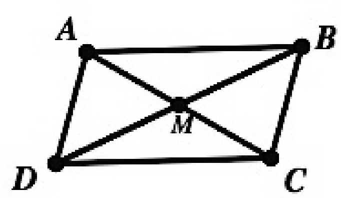


Conjecture: $\overline{JK}$ bisects $\angle MKL$
a. Is the conjecture correct?

b. Argument to prove you are right:

5. True statements

$\Delta\, ADM$ is a 180°
rotation of $\Delta\, CMB$



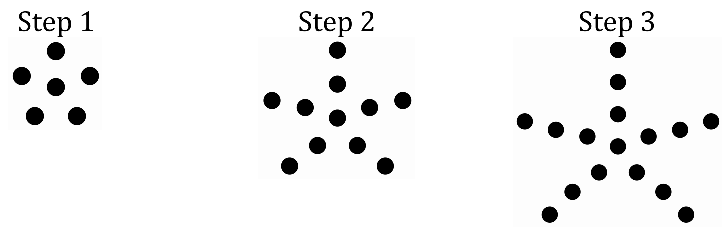
Conjecture: $\Delta\, ABM \cong \Delta CDM$
a. Is the conjecture correct?

b. Argument to prove you are right:

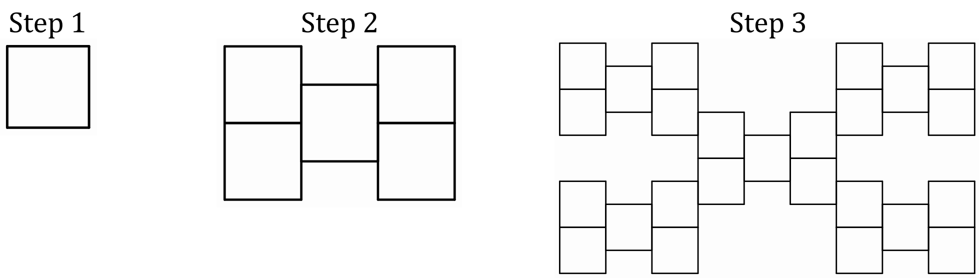
Go

Topic: Create both explicit and recursive rules for the visual patterns.

6. Find an explicit function rule and a recursive rule for dots in step *n*.



7. Find an explicit function rule and a recursive rule for squares in step *n.*



Find an explicit function rule and a recursive rule for the values in each table.

8.

Step	Value
1	1
2	11
3	21
4	31

9.

n	f(n)
2	16
3	8
4	4
5	2

10.

n	f(n)
1	-5
2	25
3	-125
4	625

6.11 Under Construction
A Develop Understanding Task

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Anciently, one of the only tools builders and surveyors had for laying out a plot of land or the foundation of a building was a piece of rope.

There are two geometric figures you can create with a piece of rope: you can pull it tight to create a line segment, or you can fix one end, and—while extending the rope to its full length—trace out a circle with the other end. Geometric constructions have traditionally mimicked these two processes using an unmarked straightedge to create a line segment and a compass to trace out a circle (or sometimes a portion of a circle called an arc). Using only these two tools you can construct all kinds of geometric shapes.

Suppose you want to construct a rhombus using only a compass and straightedge. You might begin by drawing a line segment to define the length of a side, and drawing another ray from one of the endpoints of the line segment to define an angle, as in the following sketch.

Now the hard work begins. We can't just keep drawing line segments, because we have to be sure that all four sides of the rhombus are the same length. We have to stop drawing and start constructing.

Constructing a rhombus

Knowing what you know about circles and line segments, how might you locate point C on the ray in the diagram above so the distance from B to C is the same as the distance from B to A?

1. Describe how you will locate point C and how you know $\overline{BC} \cong \overline{BA}$, then construct point C on the diagram above.

Now that we have three of the four vertices of the rhombus, we need to locate point D, the fourth vertex.

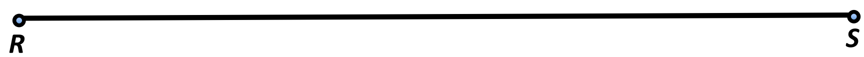
2. Describe how you will locate point D and how you know $\overline{CD} \cong \overline{DA} \cong \overline{AB}$, then construct point D on the diagram above.

Constructing a Square (A rhombus with right angles)

The only difference between constructing a rhombus and constructing a square is that a square contains right angles. Therefore, we need a way to construct perpendicular lines using only a compass and straightedge.

We will begin by inventing a way to construct a perpendicular bisector of a line segment.

3. Given $\overline{RS}$ below, fold and crease the paper so that point R is reflected onto point S. Based on the definition of reflection, what do you know about this "crease line"?



You have “constructed” a perpendicular bisector of $\overline{RS}$ by using a paper-folding strategy. Is there a way to construct this line using a compass and straightedge?

4. Experiment with the compass to see if you can develop a strategy to locate points on the “crease line”. When you have located at least two points on the “crease line” use the straightedge to finish your construction of the perpendicular bisector. Describe your strategy for locating points on the perpendicular bisector of $\overline{RS}$.

Now that you have created a line perpendicular to $\overline{RS}$ we will use the right angle formed to construct a square.

5. Label the midpoint of $\overline{RS}$ on the diagram above as point *M.* Using segment $\overline{RM}$ as one side of the square, and the right angle formed by segment $\overline{RM}$ and the perpendicular line drawn through point *M* as the beginning of a square. Finish constructing this square on the diagram above. (Hint: Remember that a square is also a rhombus, and you have already constructed a rhombus in the first part of this task.)

Ready, Set, Go!

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Ready

Topic: Tools for construction and geometric work.

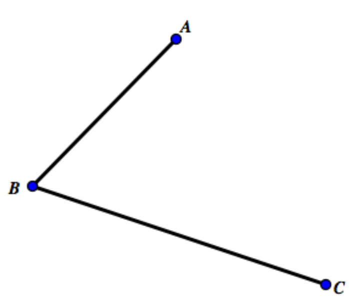
1. Using your compass draw several concentric circles that have point A as a center and then draw those same sized concentric circles that have B as a center. What do you notice about where all the circles with center A intersect all the corresponding circles with center B?

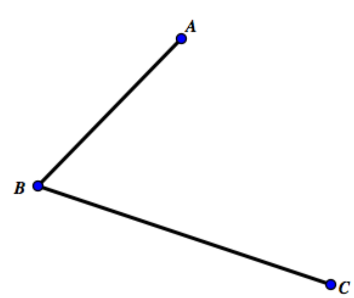
2. In the problem above you have demonstrated one way to find the midpoint of a line segment. Explain another way that a line segment can be bisected without the use of circles.

Set

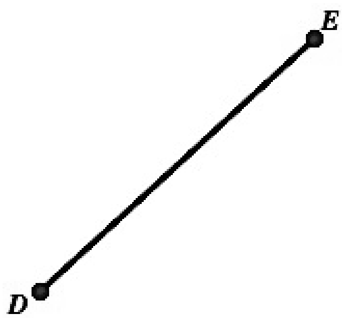
Topic: Constructions with compass and straight edge.

3. Bisect the angle below do it with compass and straight edge as well as with paper folding.

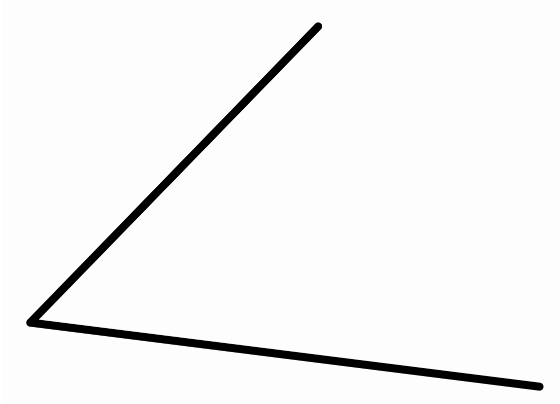




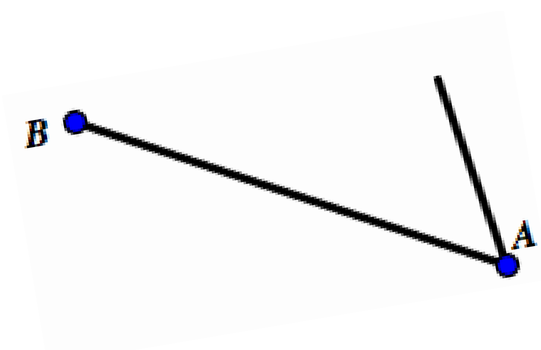
4. Copy the segment below using construction tools of compass and straight edge, label the image ***D'E'***.



5. Copy the angle below using construction tool of compass and straight edge.

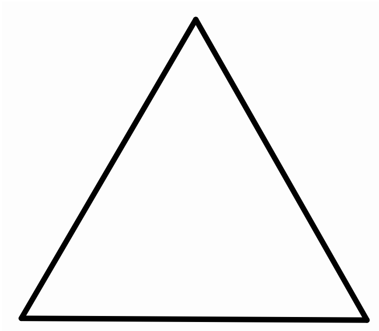


6. Construct a rhombus on the segment AB that is given below and that has point A as a vertex. Be sure to check that your final figure is a rhombus.



7. Construct a square on the segment CD that is given below. Be sure to check that your final figure is a square.

8. Given the equilateral triangle below, find the center of rotation of the triangle using compass and straight edge.



Go

Topic: Solving systems of equations review.

Solve each system of equations. Utilize substitution, elimination, graphing or matrices.

9. $\begin{cases} x = 11 + y \\ 2x + y = 19 \end{cases}$

10. $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$

11. $\begin{cases} x + 2y = 11 \\ x - 4y = 2 \end{cases}$

12. $\begin{cases} y = -x + 1 \\ y = 2x + 1 \end{cases}$

13. $\begin{cases} y = -2x + 7 \\ -3x + y = -8 \end{cases}$

14. $\begin{cases} 4x - y = 7 \\ -6x + 2y = 8 \end{cases}$

6.12 More Things Under Construction

A Develop Understanding Task

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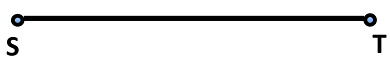
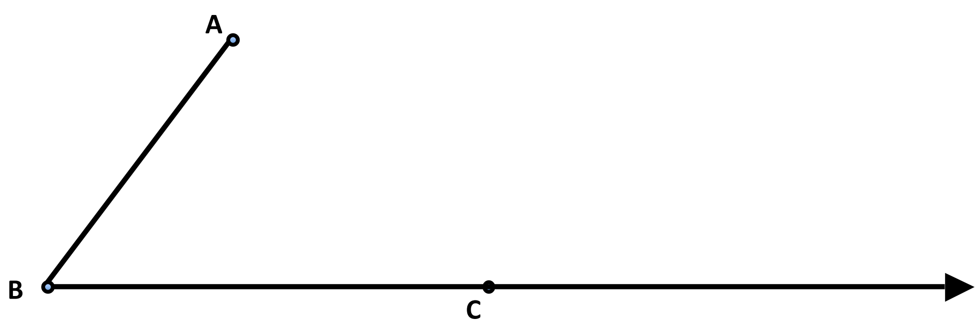
Constructing an Equilateral Triangle

Like a rhombus, an equilateral triangle has three congruent sides. Show and describe how you might locate the third vertex point on an equilateral triangle, given $\overline{ST}$ below as one side of the equilateral triangle.



Constructing a Parallelogram

To construct a parallelogram we will need to be able to construct a line parallel to a given line through a given point. For example, suppose we want to construct a line parallel to segment $\overline{AB}$ through point C on the diagram below. Since we have observed that parallel lines have the same slope, the line through point C will be parallel to $\overline{AB}$ only if the angle formed by the line and $\overrightarrow{CD}$ is congruent to $\angle ABC$. Can you describe and illustrate a strategy that will construct an angle with vertex at point C and a side parallel to $\overline{AB}$? (Hint: We know that corresponding parts of congruent triangles are congruent, so perhaps we can begin by constructing some congruent triangles.)



Constructing a Hexagon Inscribed in a Circle

Because regular polygons have rotational symmetry, they can be *inscribed* in a circle. The *circumscribed* circle has its center at the center of rotation and passes through all of the vertices of the regular polygon.

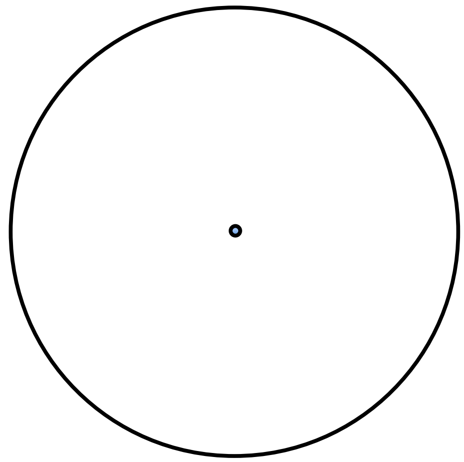
We might begin constructing a hexagon by noticing that a hexagon can be decomposed into six congruent equilateral triangles, formed by three of its lines of symmetry.

1. Sketch a diagram of such a decomposition.

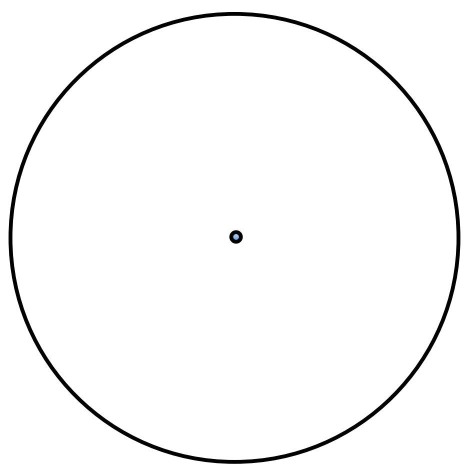
2. Based on your sketch, where is the center of the circle that would circumscribe the hexagon?

3. The six vertices of the hexagon lie on the circle in which the regular hexagon is inscribed. The six sides of the hexagon are *chords* of the circle. How are the lengths of these chords related to the lengths of the radii from the center of the circle to the vertices of the hexagon? Be able to justify how you know this is so.

4. Based on this analysis of the regular hexagon and its circumscribed circle, illustrate and describe a process for constructing a hexagon inscribed in the circle given below.



Modify your work with the hexagon to construct an equilateral triangle inscribed in the circle given below.



Describe how you might construct a square inscribed in a circle.

Ready, Set, Go!

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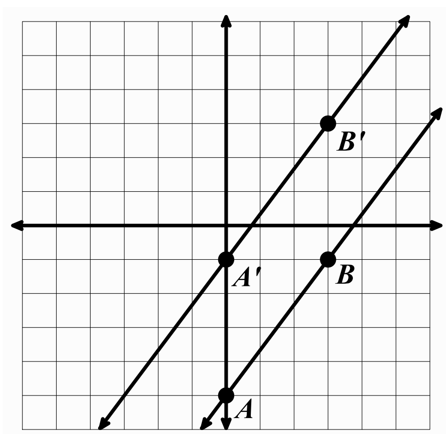
Ready

Topic: Transformations of lines, algebraic and geometric thoughts.

For each set of lines use the points on the line to determine which line is the image and which is the pre-image, label them, write image by the image line and pre image by the original line. Then define the transformation that was used to create the image. Finally find the equation for each line.

1.

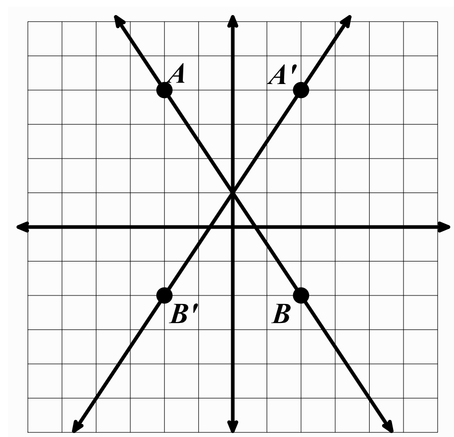


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

2.

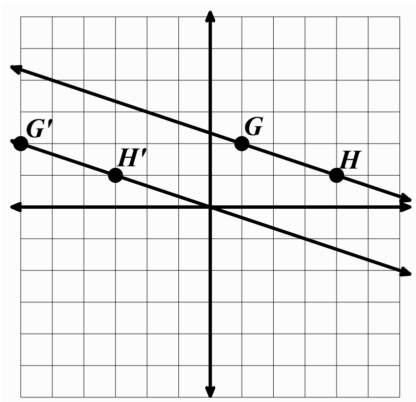


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

3.

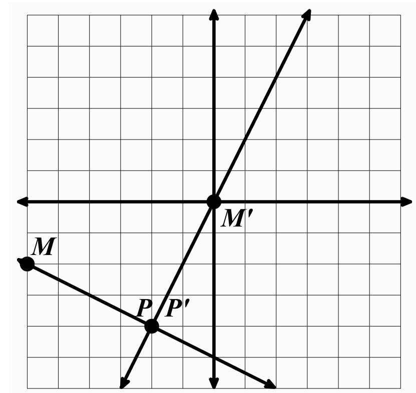


a. Description of Transformation:

b. Equation for pre-image:

c. Equation for image:

4.



a. Description of Transformation:

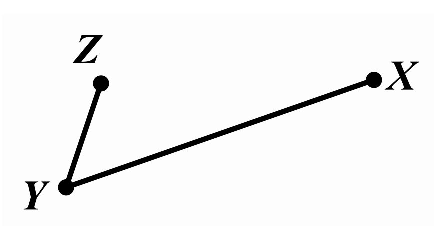
b. Equation for pre-image:

c. Equation for image:

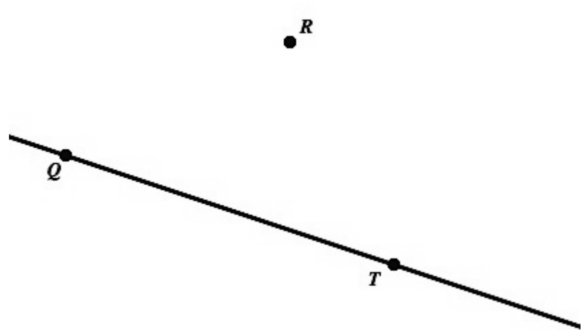
Set

Topic: Geometric Constructions using compass and straight edge.

5. Construct a parallelogram given sides $\overline{XY}$ and $\overline{YZ}$ and $\angle XYZ$.



6. Construct a line parallel to $\overline{QT}$ and through point ***R.***



7. Given segment $\overline{AB}$ show all points ***C*** such that $\triangle ABC$ is an isosceles triangle.

8. Given segment $\overline{AB}$ show all points C such that $\triangle ABC$ is a right triangle.

Go

Topic: Triangle congruence and properties of polygons.

9. What is the minimum amount of information needed to determine that two triangles are congruent? List all possible combinations of needed criteria.

10. What is a line of symmetry and what is a diagonal? Are they the same thing? Could they be the same in a polygon? If so give an example, if not explain why not.

11. How is the number of lines of symmetry for a *regular* polygon connected to the number of sides of the polygon? How is the number of diagonals for a polygon connected to the number of sides?

12. What do right triangles have to do with finding distance between points on a coordinate grid?

6.13 Justifying Constructions

A Solidify Understanding Task

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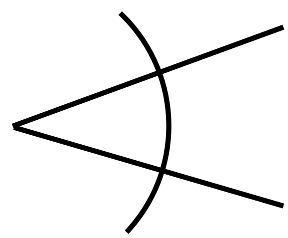
Compass and straightedge constructions can be justified using such tools as:

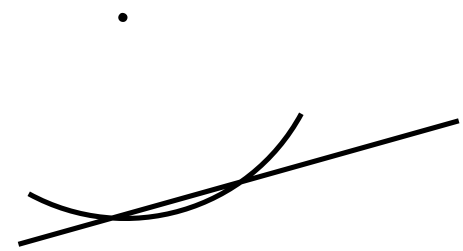
- the definitions and properties of the rigid-motion transformations
- identifying corresponding parts of congruent triangles
- using observations about sides, angles and diagonals of special types of quadrilaterals

Study the steps of the following procedure for *constructing an angle bisector*, and complete the illustration based on the descriptions of the steps.

Steps	**Illustration**
Using a compass, draw an arc (portion of a circle) that intersects each ray of the angle to be bisected, with the center of the arc located at the vertex of the angle.	
Without changing the span of the compass, draw two arcs in the interior of the angle, with the center of the arcs located at the two points where the first arc intersected the rays of the angle.	
With the straightedge, draw a ray from the vertex of the angle through the point where the last two arcs intersect.	

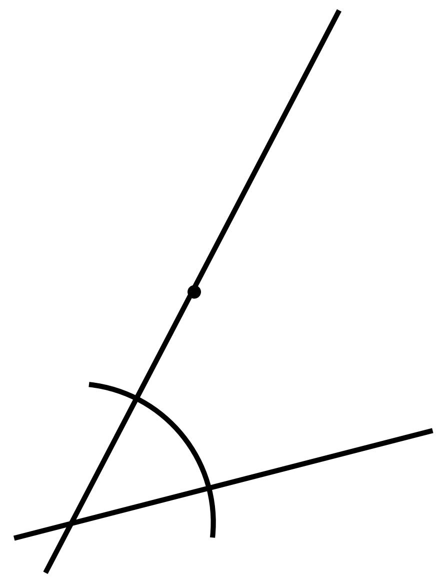
Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for *constructing a line perpendicular to a given line through a given point*, and complete the illustration based on the descriptions of the steps.

<table>
<tr><th>Steps</th><th>Illustration</th></tr>
<tr><td>Using a compass, draw an arc (portion of a circle) that intersects the given line at two points, with the center of the arc located at the given point.</td><td rowspan="3"></td></tr>
<tr><td>Without changing the span of the compass, locate a second point not on the given line, by drawing two arcs on the same side of the line, with the center of the arcs located at the two points where the first arc intersected the line.</td></tr>
<tr><td>With the straightedge, draw a line through the given point and the point where the last two arcs intersect.</td></tr>
</table>

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Study the steps of the following procedure for constructing a line parallel to a given line through a given point.

Steps	Illustration
Using a straightedge, draw a line through the given point to form an arbitrary angle with the given line.	
Using a compass, draw an arc (portion of a circle) that intersects both rays of the angle formed, with the center of the arc located at the point where the drawn line intersects the given line.	
Without changing the span of the compass, draw a second arc on the same side of the drawn line, centered at the given point. The second arc should be as long or longer than the first arc, and should intersect the drawn line.	
Set the span of the compass to match the distance between the two points where the first arc crosses the two lines. Without changing the span of the compass, draw a third arc that intersects the second arc, centered at the point where the second arc intersects the drawn line.	
With the straightedge, draw a line through the given point and the point where the last two arcs intersect.	

Explain in detail why this construction works. It may be helpful to identify some congruent triangles or a familiar quadrilateral in the final illustration. You may also want to use definitions or properties of the rigid-motion transformations in your explanation. Be prepared to share your explanation with your peers.

Ready, Set, Go!

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Ready

Topic: Rotation symmetry for regular polygons and transformations

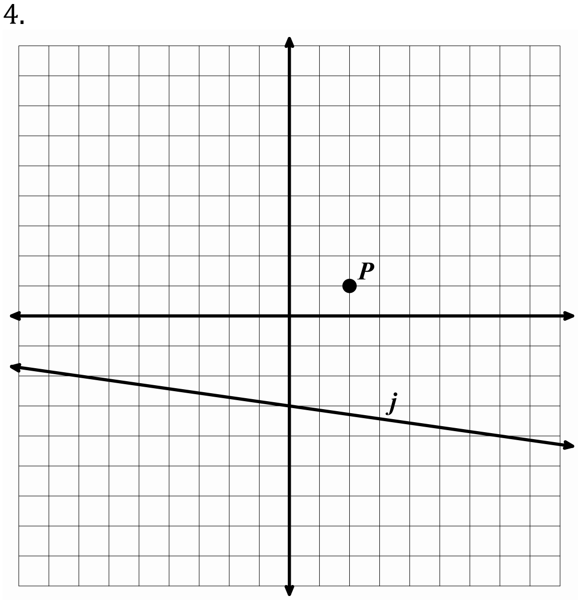
1. What angles of rotational symmetry are there for a pentagon?

2. What angles of rotational symmetry are there for a hexagon?

3. If a regular polygon has an angle of rotational symmetry that is 40^0, how many sides does the polygon have?

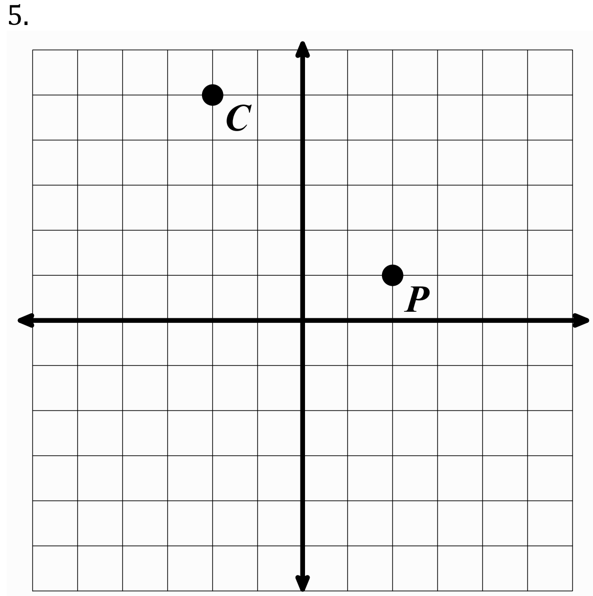
On each given coordinate grid below perform the indicated transformation.

4.



Reflect point ***P*** over line ***j***.

5.

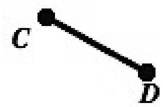


Rotate point ***P*** 90^0 clockwise around point ***C***.

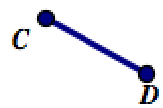
Set

Topic: Constructing regular polygons inscribed in a circle.

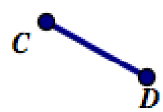
6. Construct an isosceles triangle that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the triangle.



7. Construct a hexagon that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the hexagon.



8. Construct a square that incorporates $\overline{CD}$ as one of the sides. Construct the inscribing circle around the square.



Go

Topic: Finding distance and slope.

For each pair of given coordinate points find distance between them and find the slope of the line that passes through them. Show all your work.

9. (-2, 8) , (3, -4)
   a. Slope: b. Distance:

10. (-7, -3) , (1, 5))
   a. Slope: b. Distance:

11. (3, 7) , (-5, 9)
   a. Slope: b. Distance:

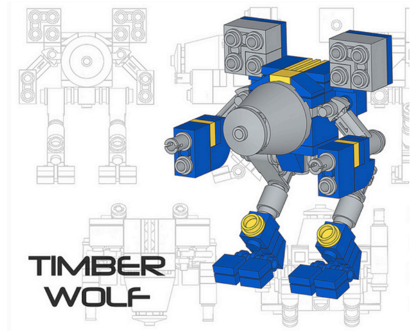
12. (1, -5) (-7, 1)
   a. Slope: b. Distance:

13. (-10, 31) (20, 11)
   a. Slope: b. Distance:

14. (16, -45) (-34, 75)
   a. Slope: b. Distance:

6.14 Construction Blueprints

A Practice Understanding Task



For each of the following straightedge and compass constructions, illustrate or list the steps for completing the construction and give an explanation for why the construction works. You explanations may be based on rigid-motion transformations, congruent triangles, or properties of quadrilaterals.

Purpose of the construction	Illustration and/or steps for completing the construction	Justification of why this construction works
Copying a segment	1. Set the span of the compass to match the distance between the two endpoints of the segment. 2. Without changing the span of the compass, draw an arc on a ray centered at the endpoint of the ray. The second endpoint of the segment is where the arc intersects the ray.	The given segment and the constructed segment are radii of congruent circles.
Copying an angle		
Bisecting a segment		
Bisecting an angle		
Constructing a perpendicular bisector of a line segment		
Constructing a perpendicular to a line through a given point		

Constructing a line parallel to a given line through a given point		
Constructing an equilateral triangle		
Constructing a regular hexagon inscribed in a circle		

Ready, Set, Go!

Ready

Topic: Connecting tables with transformations.

For each function find the outputs that fill in the table. Then describe the relationship between the outputs in each table.

1. $f(x) = 3x$ $\quad$ $g(x) = 3x - 5$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

2. $t(x) = 2x$ $\quad$ $h(x) = 2x - 5$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

Relationship between $t(x)$ and $h(x)$:

3. $f(x) = 2x$ $\quad$ $g(x) = 2(x - 3)$

x	$f(x)$
1	
2	
3	
4	

x	$g(x)$
1	
2	
3	
4	

Relationship between $f(x)$ and $g(x)$:

4. $t(x) = 4x$ $\quad$ $h(x) = 4^{(x-3)}$

x	$t(x)$
1	
2	
3	
4	

x	$h(x)$
1	
2	
3	
4	

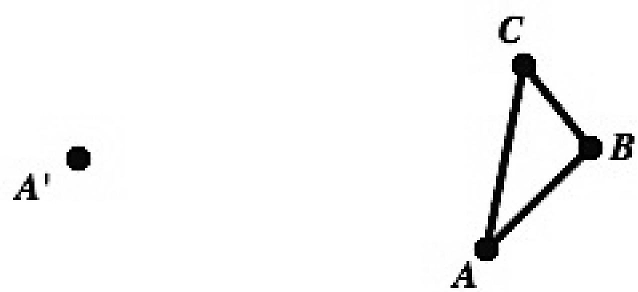
Relationship between $t(x)$ and $h(x)$:

Set

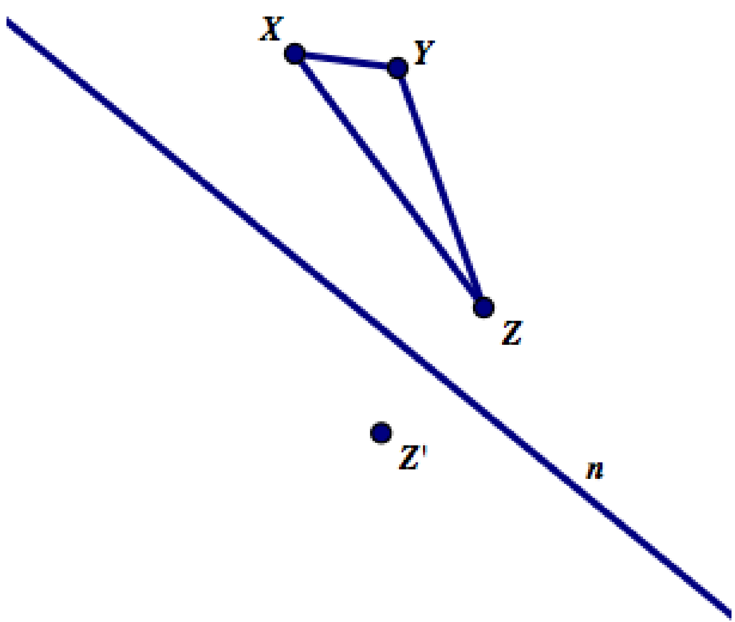
Topic: Constructing transformations

In each problem below use compass and straight edge to construct the transformation that is described.

5. Construct D A'B'C' so that it is a translation of D ABC. (Hint: parallel lines may be useful.)



6. Construct D X'Y'Z' so that it is a reflection of D XYZ over line ***m***.
(Hint: perpendicular lines may be useful.)



Go

Topic: Transformations and triangle congruence.

Determine whether or not the statement is true or false. If true, explain why. If false, explain why not or provide a counterexample.

7. If one triangle can be transformed so that one of its angles and one of its sides coincide with another triangles angle and side then the two triangles are congruent.

8. If one triangle can be transformed so that two of its sides and any one of its angles will coincide with two sides and an angle from another triangle then the two triangles will be congruent.

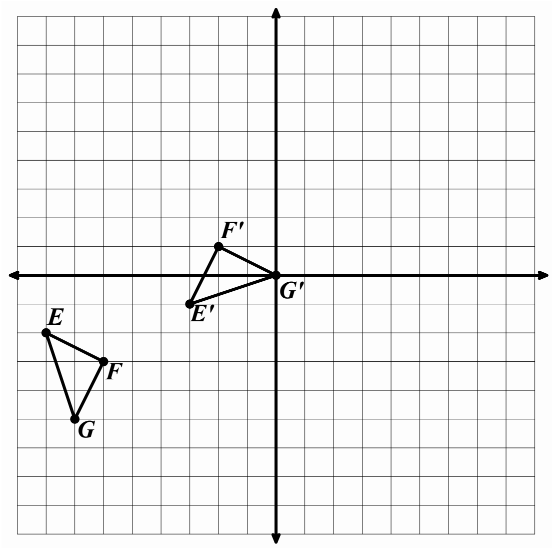
9. If all three angles of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

10. If all three sides of a triangle are congruent then there is a sequence of transformations that will transform one triangle onto the other.

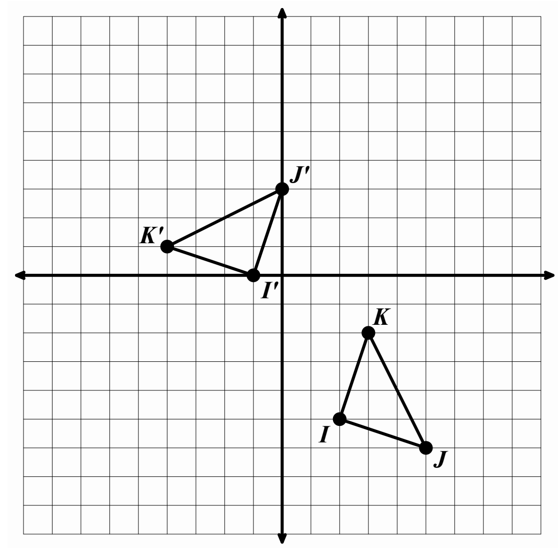
11. For any two congruent polygons there is a sequence of transformations that will transform one of the polygons onto the other.

Find the point of rotation for each of the figures below.

12.

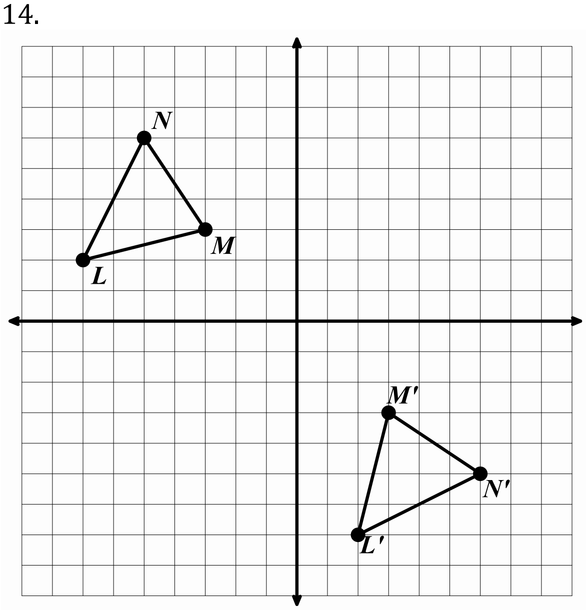


13.



Find the line of reflection for each of the figures drawn below.

14.



15.