Secondary Two Mathematics: An Integrated Approach
Module 5
Geometric Figures

By

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In partnership with the Utah State Office of Education

Module 5 – Geometric Figures

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Ready, Set, Go Homework: Geometric Figures 5.1

Classroom Task: 5.2 Do You See What I See? – A Develop Understanding Task
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Classroom Task: 5.4 Parallelism Preserved – A Develop Understanding Task
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5.1 How Do You Know That?

A Develop Understanding Task

You probably know that the sum of the interior angles of any triangle is 180°. (If you didn't know that, you do now!) But an important question to ask yourself is, "How do you know that?"

We know a lot of things because we *accept it on authority*—we believe what other people tell us; things such as the distance from the earth to the sun is 93,020,000 miles or that the population of the United States is growing about 1% each year. Other things are just defined to be so, such as the fact that there are 5,280 feet in a mile. Some things we accept as true based on experience or repeated experiments, such as the sun always rises in the east, or "I get grounded every time I stay out after midnight." In mathematics we have more formal ways of deciding if something is true.

Experiment #1

1. Cut out several triangles of different sizes and shapes. Tear off the three corners (angles) of the triangle and arrange the vertices so they meet at a single point, with the edges of the angles (rays) touching each other like pieces of a puzzle. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

2. Since you and your classmates have performed this experiment with several different triangles, does it guarantee that we will observe this same result for *all* triangles? Why or why not?

Experiment #2

Perhaps a different experiment will be more convincing. Cut out another triangle and trace it onto a piece of paper. It will be helpful to color-code each vertex angle of the original triangle with a different color. As new images of the triangle are produced during this experiment, color-code the corresponding angles with the same colors.

Locate the midpoints of each side of your cut out triangle by folding the vertices that form the endpoints of each side onto each other.

Rotate your triangle 180° about the midpoint of one of its sides. Trace the new triangle onto your paper and color-code the angles of this image triangle so that corresponding image/pre-image pairs of angles are the same color.

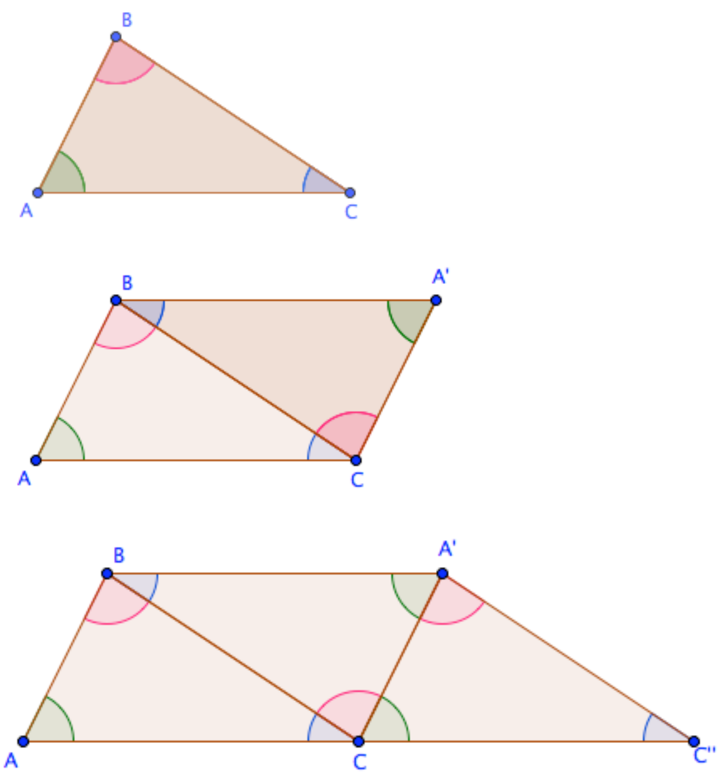
Now rotate the new "image" triangle 180° about the midpoint of one of the other two sides. Trace the new triangle onto your paper and color-code the angles of this new image triangle so that corresponding image/pre-image pairs of angles are the same color.

3. What does this experiment reveal about the sum of the interior angles of the triangles you cut out, and how does it do so?

4. Do you think you can rotate *all* triangles in the same way about the midpoints of its sides, and get the same results? Why or why not?

Examining the Diagram

Experiment #2 produced a sequence of triangles, as illustrated in the following diagram.



Here are some interesting questions we might ask about this diagram:

5. Will the second figure in the sequence always be a parallelogram? Why or why not?

6. Will the last figure in the sequence always be a trapezoid? Why or why not?

Name: Geometric Figures | 5.1

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Ready, Set, Go!

Ready

Topic: Geometric figures

One of the cool things about geometric figures is that our world is filled with them. For instance, my bathroom mirror is a perfect rectangle and the tiles on my floor are squares. Plus, the edges of these shapes are straight lines or line segments which are pieces of lines, since theoretically a line goes on forever.

1. Look around your world and make a list of the things you see that have a geometric shape. Here are some shapes to begin with. Think of all you can and be prepared to share your lists with the class.

Triangle **Trapezoid** **Parallelogram** **Cube** **Perpendicular lines**

Set

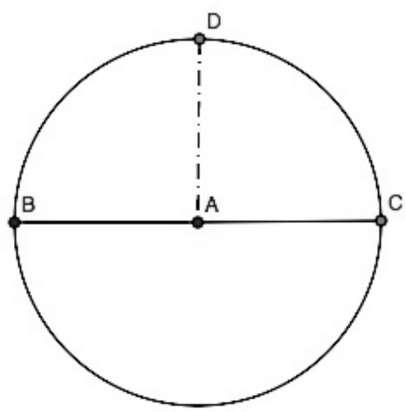
Topic: Linear pairs

2. Fold a piece of paper, making a smooth crease. Open the paper and examine the shape that you made. Is it a line? Will it always be a line? Justify your thinking.

3. Look at a wall where it meets the ceiling. How would you describe the intersection of the wall and the ceiling?

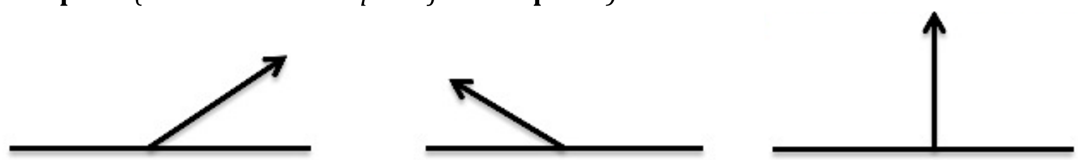
Imagine folding a circle exactly in half so that the fold passes through the center of the circle. This fold is called the diameter of the circle. It is a line segment with a length, but it is also a special kind of angle called a **straight angle**.

In order to "see" the angle, think of the center of the circle. That point is the vertex of the angle. Either side of the vertex is a radius of the circle. Whenever you draw 2 radii of the circle you make an angle. <u>*When the two radii extend in exactly opposite directions and share a common endpoint (the center), they make a line or a* **straight angle**.</u>



14. How many degrees do you think are in a *straight angle*? Use features of the diagram to justify your answer.

If two angles share a vertex and together they make a straight angle, then the two angles are called a **linear pair***. (Below are 3 examples of* **linear pairs***.)*



Examples of linear pairs in real life:

http://www.flicker.com/photos/angle_dore/6365060845

http://www.flicker.com/photos/truthlying/3845031/sizes/

5. Draw at least 2 diagrams of a real life linear pair.

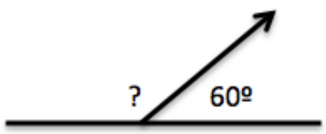
Go

Topic: The algebra of linear pairs.

For 2 angles to be a **linear pair**, *they must share a vertex and a side, and the sum of their measures must equal 180°.*

Find the measure of the missing angle.

6.



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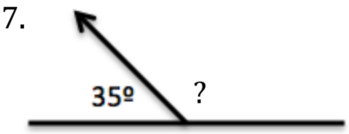

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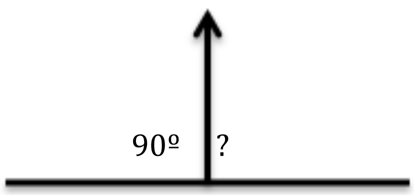

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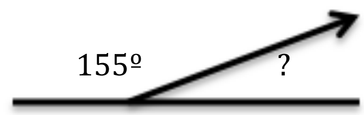


10. Linear pairs could be defined as being **supplementary angles** because they always add up to 180°. Are all supplementary angles linear pairs? Explain your answer.

Find the supplement of the given angle. Then draw the two angles as linear pairs. Label each angle with its measure.

11. $m\angle ABC = 72°$ B will be the vertex.

12. $m\angle GHK = 113°$ H will be the vertex.

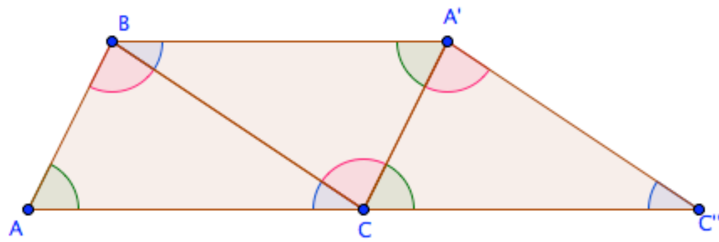
13. $m\angle XYZ = 24°$ Y will be the vertex

14. $m\angle JMS = 168°$ M will be the vertex

5.2 Do You See What I See?

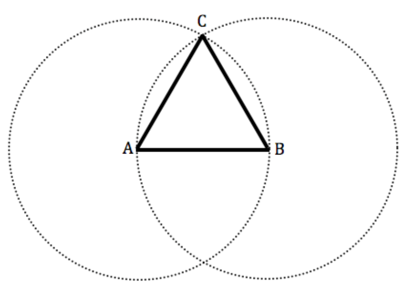
A Develop Understanding Task

In the previous task, *How Do You Know That*, we saw how the following diagram could be constructed by rotating a triangle about the midpoint of two of its sides. The final diagram suggests that the sum of the three angles of a triangle is 180°. This diagram "tells a story" because you saw how it was constructed through a sequence of steps. You may even have carried out those steps yourself.



Sometimes we are asked to draw a conclusion from a diagram when we are given the last diagram in a sequence steps. We may have to mentally reconstruct the steps that got us to this last diagram, so we can believe in the claim the diagram wants us to see.

1. For example, what can you say about the triangle in the following diagram?

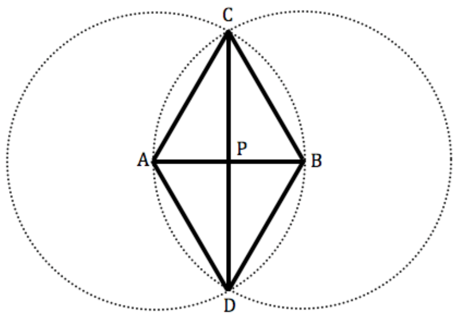


2. What convinces you that you can make this claim? What assumptions, if any, are you making about the other figures in the diagram?

3. What is the sequence of steps that led to this final diagram?

4. What can you say about the triangles, quadrilateral, or diagonals of the quadrilateral that appear in the following diagram? List several conjectures that you believe are true.

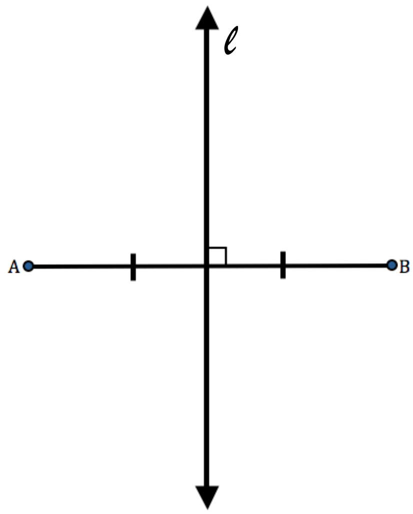
   Given: $\odot A \cong \odot B$

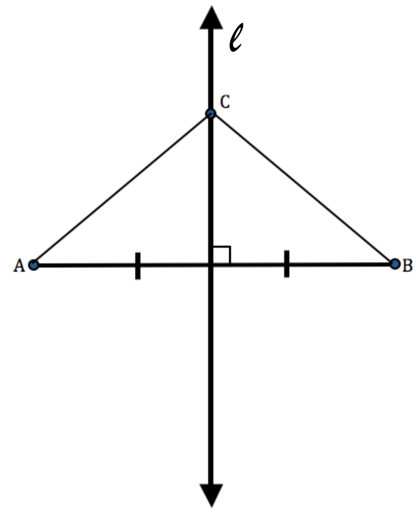


5. Select one of your conjectures and write a paragraph convincing someone else that your conjecture is true. Think about the sequence of statements you need to make to tell your story in a way that someone else can follow the steps and construct the images you want them to see.

6. Now pick a second claim and write a paragraph convincing someone else that this claim is true. You can refer to your previous paragraph, if you think it supports the new story you are trying to tell.

7. Here is one more diagram. Describe the sequence of steps that you think were used to construct this diagram beginning with the figure on the left and ending with the figure on the right.





Travis and Tehani are doing their math homework together. One of the questions asks them to prove the following statement.

The points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment?

Travis and Tehani think the diagram above will be helpful to prove this statement, but they know they will need to say more than just describe how to create this diagram. Travis starts by describing the things they know, and Tehani tries to keep a written record by jotting notes down on a piece of paper.

8. In the table below, record in symbolic notation what Tehani may have written to keep track of Travis' statements.

Tehani's Notes	Travis' Statements
	We need to start with a segment and its perpendicular bisector already drawn.
	We need to show that *any* point on the perpendicular bisector is equidistant from the two endpoints, so I can pick any arbitrary point on the perpendicular bisector. Let's call it C.
	We need to show that this point is the same distance from the two endpoints.
	If we knew the two triangles were congruent, we could say that the point on the perpendicular bisector is the same distance from each endpoint.
	So, what do we know about the two triangles that would let us say that they are congruent?
	We know that both triangles contain a right angle.
	And we know that the perpendicular bisector cuts segment AB into two congruent segments.
	Obviously, the segment from C to the midpoint of segment AB is a side of both triangles.
	So, the triangles are congruent by the SAS triangle congruence criteria.
	Since the triangles are congruent, segments AC and BC are congruent.
	And, that proves that point C is equidistant from the two endpoints!

9. Tehani thinks Travis is brilliant, but she would like the ideas to flow more easily from start to finish. Arrange Tehani's symbolic notes in a way that someone else could follow the argument and see the connections between ideas.

10. Would your justification be true regardless of where point C is chosen on the perpendicular bisector? Why?

Name: **Geometric Figures | 5.2**

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Ready, Set, Go!

Ready

Topic: Symbols in geometry

Throughout the study of mathematics, you have encountered many symbols that help you write mathematical sentences and phrases without using words. Symbols help the mathematician calculate efficiently and communicate concisely.

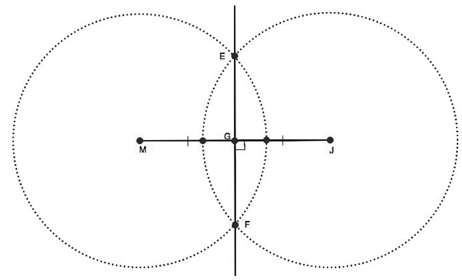
Below is a set of common mathematical symbols. Your job is to match them to their definitions. Are the symbols logical?

Symbol	Definitions
_____1. =	A. **Absolute value** – it is always equal to the positive value of the number inside the lines. It represents distance from zero.
_____2. $m\angle C$	B. **Congruent** – Figures that are the same size and shape are said to be congruent.
_____3. GH	C. **Parallel** – used between segments, lines, rays, or planes
_____4. ΔABC	D. **Line segment with endpoints G and H**. Line segments can be congruent to each other. You would not say they were equal.
_____5. $\perp$	E. **Ray GH** – The letter on the left indicates the endpoint of the ray.
_____6. $\angle ABC$	F. Used when comparing numbers of **equal value**.
_____7. $\overleftrightarrow{GH}$	G. **Plus or minus** – indicates 2 values, the positive value and the negative value
_____8. $\cong$	H. **Triangle ABC**
_____9. ~	J. Indicates the **measure of an angle**. It would be set equal to a number.
_____10. $\overline{GH}$	K. **Perpendicular** - Lines, rays, segments, and planes can all be perpendicular
_____11. $\overrightarrow{GH}$	L. **Angle ABC** – The middle letter is always the vertex of the angle.
_____12. $\|\|$	M. **Similar** – Figures that have been dilated are similar.
_____13. $\pm$	N. **The length of GH**. It would equal a number.
_____14. $\|x\|$	P. Refers to the **infinite line GH**. Lines are not equal or congruent to other lines.

Set

Topic: Construction of midpoint, perpendicular bisector, and angle bisector and using "givens" to solve problems.

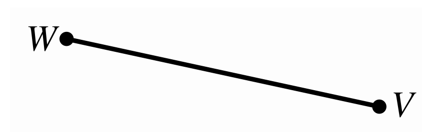
The figure on the right demonstrates the construction of a perpendicular bisector of a segment.



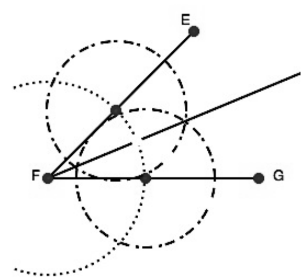
Use the diagram to guide you in constructing the perpendicular of the following line segments. Mark the right angle with the correct symbol for right angles. Indicate the segments are congruent by using slash marks.

15.

16.



The figure on the right demonstrates the construction of an angle bisector. **Use the diagram to guide you in constructing the angle bisector of the following angles. Mark your bisected angles as congruent.**



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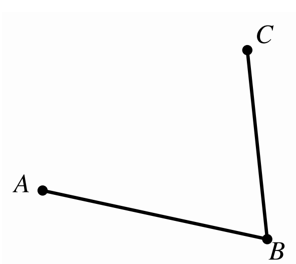

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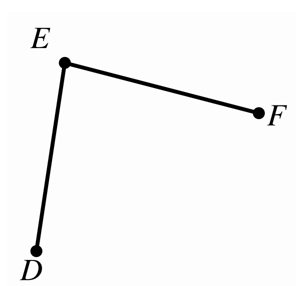

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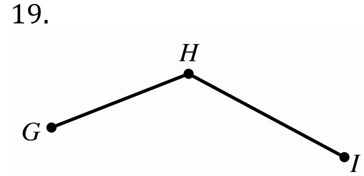
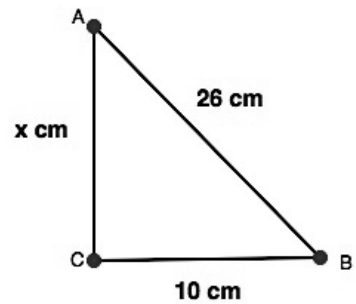
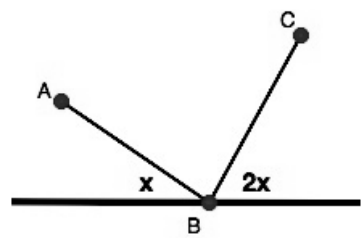


Examine the diagram and add any information that you are given. Think how you can use what you have been given and what you know to answer the question. Plan a strategy for finding the value of x. Follow your plan. Justify each step.

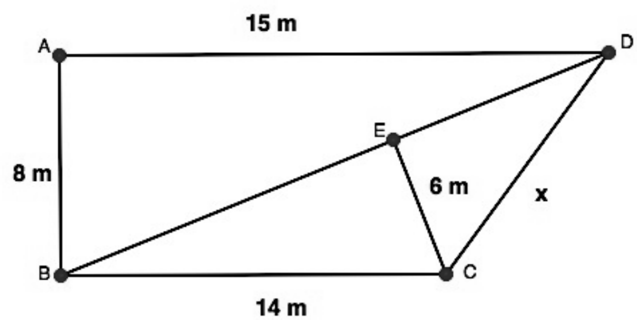
20. Given: $m\angle C = 90^\circ$



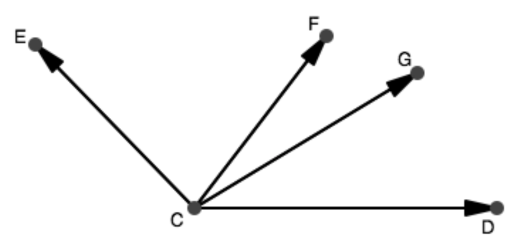
21. Given $m\angle ABC = 90^\circ$



22. Given: $\Delta BEC, \Delta CED, and\ \Delta DAB$ are right triangles.



23. Given: $\overrightarrow{CF}$ bisects $\angle ECD, m\angle ECF = 2x+10$, and $m\angle FCD = 3x-18$. Find $m\angle FCE$.



Have you answered the question?
This problem asks you to do more than find the value of x.

Go

Topic: Translations, reflections, and rotations

Perform the following transformations on the diagram below.

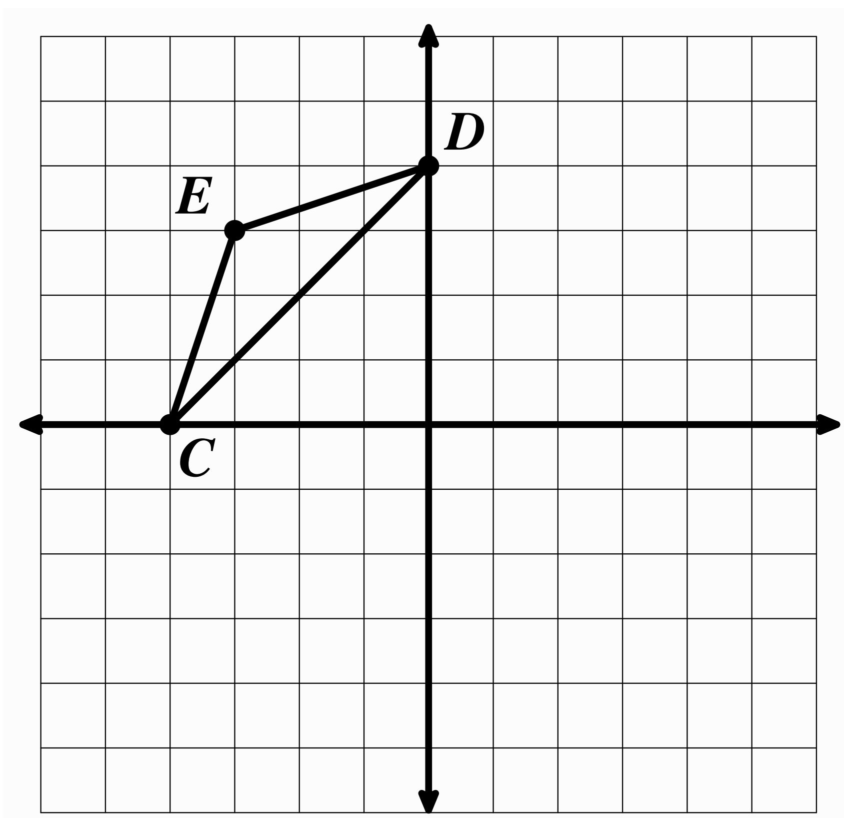
24. Label points C, E, D with the correct ordered pairs.

25. Translate ΔCED down 4 and right 6. Label the image as $\Delta C'E'D'$ and include the new ordered pairs.

26. Draw $\overline{CC'}$, $\overline{EE'}$, and $\overline{DD'}$. What is the slope of each of these line segments?

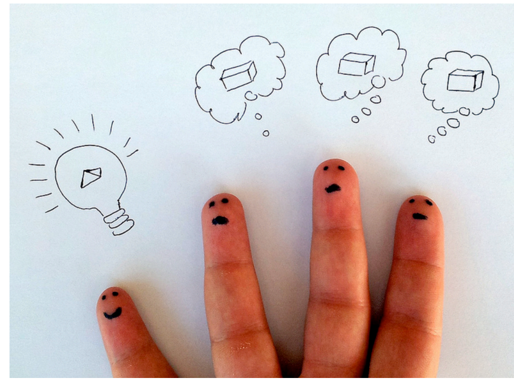
27. Reflect ΔCED across the x = 0 line. Label the image $\Delta C''E''D''$. Include the new ordered pairs. Draw $\overline{C'C''}$ *and* $\overline{E'E''}$ Why didn't you need to draw $\overline{D'D''}$?
What is the relationship between $\overline{C'C''}$ and $\overline{E'E''}$ to the $x = 0$ line?

28. Rotate ΔCED 180º about the point (-2, 0). Label the image $\Delta C'''E'''D'''$.
Include the new ordered pairs.



5.3 It's All In Your Head

A Solidify Understanding Task

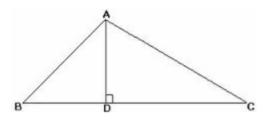


In the previous task you were asked to justify some claims by writing paragraphs explaining how various figures were constructed and how those constructions convinced you that the claims were true. Perhaps you found it difficult to say everything you felt you just knew. Sometimes we all find it difficult to explain our ideas and to get those ideas out of our heads and written down or paper.

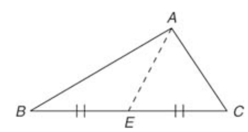
Organizing ideas and breaking complex relationships down into smaller chunks can make the task of proving a claim more manageable. One way to do this is to use a flow diagram.

First, some definitions:

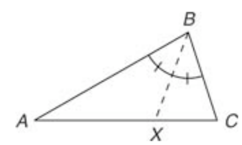
- In a triangle, an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



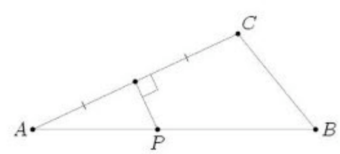
- In a triangle, a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



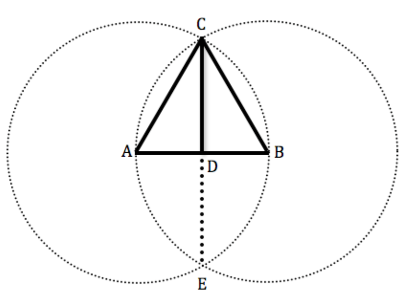
- In a triangle, an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- In a triangle, a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



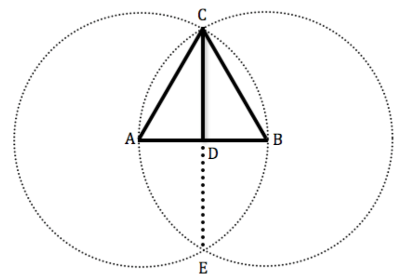
Travis used a compass and straightedge to construct an equilateral triangle. He then folded his diagram across the two points of intersection of the circles to construct a line of reflection. Travis, Tehani, Carlos and Clarita are trying to decide what to name the line segment from C to D.



Travis thinks the line segment they have constructed is also a median of the equilateral triangle. Tehani thinks it is an angle bisector. Clarita thinks it is an altitude and Carlos thinks it is a perpendicular bisector of the opposite side. The four friends are trying to convince each other that they are right.

Here is a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas.

Use four different colors to identify the statements each of the students—Travis, Tehani, Carlos and Clarita might use to make their case.



Given: ΔABC is equilateral → $\overleftrightarrow{CE}$ is a line of reflection

$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

$\angle CDA$ and $\angle CDB$ are right angles ← $\overline{CD} \perp \overline{AB}$

D is the midpoint of $\overline{AB}$

$\overline{AD} \cong \overline{DB}$

$\overline{CD} \cong \overline{CD}$

$\angle ACD \cong \angle BCD$ ← $\Delta ACD \cong \Delta BCD$

therefore, $\overline{CD}$ is an altitude

therefore, $\overline{CD}$ is a median

therefore, $\overline{CD}$ is an angle bisector

therefore, $\overline{CD}$ is a perpendicular bisector

Match each of the arrows and braces in the flow diagram with one of the following reasons that justifies why you can make the connection between the statement (or statements) previously accepted as true and the conclusion that follows:

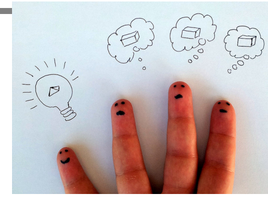
1. Definition of reflection
2. Definition of translation
3. Definition of rotation
4. Definition of an equilateral triangle
5. Definition of perpendicular
6. Definition of midpoint
7. Definition of altitude
8. Definition of median
9. Definition of angle bisector
10. Definition of perpendicular bisector
11. Equilateral triangles can be folded onto themselves about a line of reflection
12. Equilateral triangles can be rotated 60° onto themselves
13. SSS triangle congruence criteria
14. SAS triangle congruence criteria
15. ASA triangle congruence criteria
16. Corresponding parts of congruent triangles are congruent

Travis and his friends have seen their teacher write two-column proofs in which the reasons justifying a statement are written next to the statement being made. Travis decides to turn his argument into a two column proof, as follows.

Statements	**Reasons**
ΔABC is equilateral	Given
$\overleftrightarrow{CE}$ is a line of reflection	Equilateral triangles can be folded onto themselves about a line of reflection
D is the midpoint of $\overline{AB}$	Definition of reflection
$\overline{CD}$ is a median	Definition of median

Write each of Tehani's, Carlos', and Clarita's arguments in two-column proof format.

Name: Geometric Figures | 5.3



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Ready, Set, Go!

Ready

Remember that when you write a congruence statement such as $\Delta ABC \cong \Delta FGH$, the corresponding parts of the two triangles must be the parts that are congruent. For instance, $\angle A \cong \angle F, \overline{AB} \cong \overline{FG}, \angle B \cong \angle G, \overline{BC} \cong \overline{GH}$. *Also, recall that the congruence patterns for triangles, ASA. SAS, and SSS, are what we can use to justify triangle congruence.*

The segments and angles in each problem below are corresponding parts of 2 congruent triangles. Make a sketch of the two triangles. Then write a congruence statement for each pair of triangles represented. State the congruence pattern that justifies your statement.

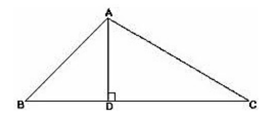
	Congruence statement	Congruence pattern
1. $\overline{ML} \cong \overline{ZJ}, \overline{LR} \cong \overline{JB}, \angle L \cong \angle J$	a.	b.
2. $\overline{WB} \cong \overline{QR}, \overline{BP} \cong \overline{RS}, \overline{WP} \cong \overline{QS}$	a.	b.
3. $\overline{CY} \cong \overline{RP}, \overline{EY} \cong \overline{BP}, \angle Y \cong \angle P$	a.	b.
4. $\overline{BC} \cong \overline{JK}, \overline{BA} \cong \overline{JM}, \angle B \cong \angle J$	a.	b.
5. $\overline{DF} \cong \overline{XZ}, \overline{FY} \cong \overline{ZW}, \angle F \cong \angle Z$	a.	b.
6. $\overline{WX} \cong \overline{AB}, \overline{XZ} \cong \overline{BC}, \overline{WZ} \cong \overline{AC}$	a.	b.

Set

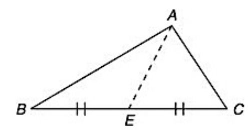
Recall the following definitions:

In a triangle:

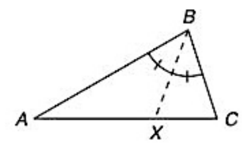
- an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).



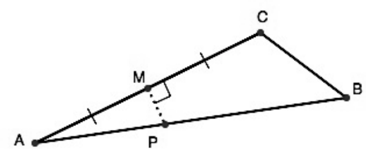
- a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.



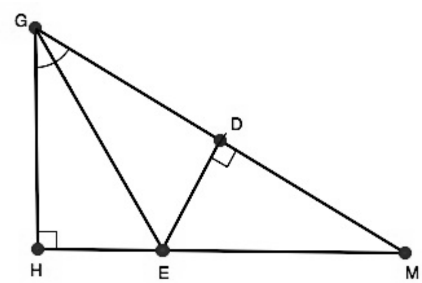
- an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.



- a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.



Be sure to use the correct notation for a segment in the following problems.



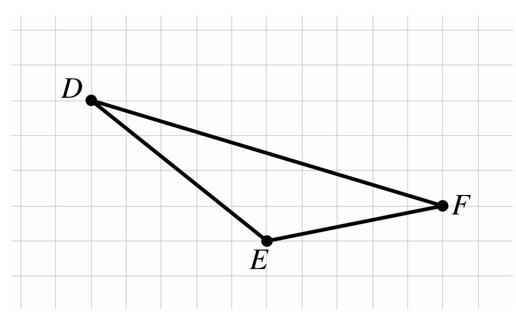
7. Name a segment in ΔGHM that is an altitude.

8. Name a segment in ΔGHM that is an angle bisector.

9. Name a segment in ΔGHM that is NOT an altitude.

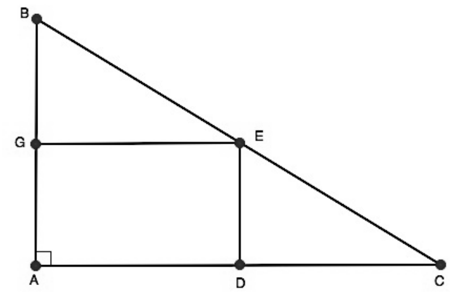
10. Create a perpendicular bisector by marking two segments congruent in ΔGHM . Name the segment that is now the perpendicular bisector.

Use ΔDEF in problems 11 – 13.



11. Construct the altitude from vertex D to $\overleftrightarrow{EF}$.
12. Construct the median from D to $\overline{EF}$.
13. Construct the perpendicular bisector of $\overline{EF}$.

Tehani has been studying the figure below. She knows that quadrilateral ADEG is a rectangle and that $\overline{ED}$ bisects $\overline{BC}$. She is wondering if with that information she can prove $\Delta BGE \cong \Delta EDC$. She starts to organize her thinking by writing what she knows and the reasons she knows it.



I know $\overline{ED}$ bisects $\overline{BC}$ because I was given that information

I know that $\overline{BE} \cong \overline{EC}$ by definition of bisect.

I know that $\overline{GE}$ must be parallel to $\overline{AD}$ because the opposite sides in a rectangle are parallel.

I know that $\overline{GA} \parallel \overline{ED}$ because they are opposite sides in a rectangle.

I know that $\overline{AD}$ is contained in $\overline{AC}$ so $\overline{AC}$ is also parallel to $\overline{GE}$.

I know that $\overline{GA}$ is contained in $\overline{BA}$ so $\overline{GA}$ is also parallel to $\overline{BA}$

I know that $\overleftrightarrow{BC}$ has the same slope everywhere because it is a line.

I know the angle that $\overline{BE}$ makes with $\overline{GE}$ must be the same as the angle that $\overline{EC}$ makes with $\overline{AC}$ since those 2 segments are parallel. So $\angle BEG \cong \angle ECD$. I think I can use that same argument for $\angle GBE \cong \angle DEC$.

I know that I now have an angle, a side, and an angle congruent to a corresponding angle, side, and angle. So $\Delta BGE \cong \Delta EDC$ by ASA.

14. Use Tehani's "I know" statements and her reasons to write a two-column proof that proves $\Delta BGE \cong \Delta EDC$. Begin your proof with the "givens" and what you are trying to prove.

Given: quadrilateral ADEG is a rectangle, $\overline{ED}$ bisects $\overline{AC}$
Prove: $\Delta BGE \cong \Delta EDC$

STATEMENTS	REASONS
1. quadrilateral ADEG is a rectangle	given
2. $\overline{ED}$ bisects $\overline{AC}$	given

Go

Topic: Transformations

Perform the following transformations on ΔABC. Use a straight edge to connect the corresponding points with a line segment. Answer the questions.

15. Reflect ΔABC over $\overleftrightarrow{JK}$. Label your new image $\Delta A'B'C'$.
16. What do you notice about the line segments $\overline{AA'}, \overline{BB'}, and\ \overline{CC'}$?

17. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A'B'}, \overline{B'C'}, \overline{C'A'}$. What is the same and what is different about these segments?

18. Translate ΔABC down 8 units and right 10 units. Label your new image $\Delta A''B''C''$.

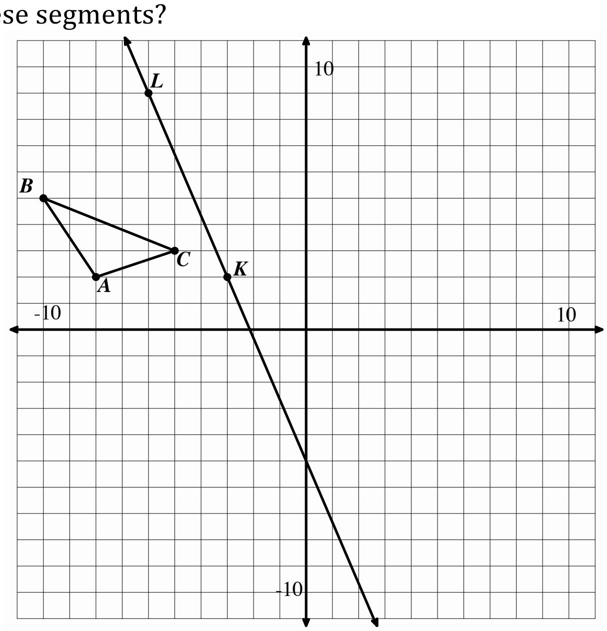
19. What do you notice about the line segments $\overline{AA''}, \overline{BB''}, and\ \overline{CC''}$?

20. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A''B''}, \overline{B''C''}, \overline{C''A''}$. What is the same and what is different about these segments?

21. Translate ΔABC down 10 units and reflect it over the Y–axis. Label your new image $\Delta A'''B'''C'''$.

22. What do you notice about the line segments $\overline{AA'''}, \overline{BB'''}, and\ \overline{CC'''}$?

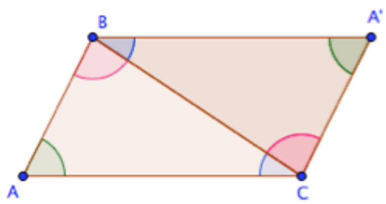
23. Compare line segments $\overline{AB}, \overline{BC}, and\ \overline{CA}\ to\ \overline{A'''B'''}, \overline{B'''C'''}, \overline{C'''A'''}$. What is the same and what is different about these segments?



5.4 Parallelism Preserved and Protected

A Develop Understanding Task

In a previous task, *How Do You Know That*, you were asked to explain how you knew that this figure, which was formed by rotating a triangle about the midpoint of one of its sides, was a parallelogram.



You may have found it difficult to explain how you knew that sides of the original triangle and its rotated image were parallel to each other except to say, "It just has to be so." There are always some statements we have to accept as true in order to convince ourselves that other things are true. We try to keep this list of statements as small as possible, and as intuitively obvious as possible. For example, in our work with transformations we have agreed that distance and angle measures are preserved by rigid motion transformations since our experience with these transformations suggest that sliding, flipping and turning figures do not distort the images in any way. Likewise, parallelism within a figure is preserved by rigid motion transformations: for example, if we reflect a parallelogram the image is still a parallelogram—the opposite sides of the new quadrilateral are still parallel.

Mathematicians call statements that we accept as true without proof *postulates.* Statements that are supported by justification and proof are called *theorems.*

Knowing that lines or line segments in a diagram are parallel is often a good place from which to start a chain of reasoning. Almost all descriptions of geometry include a *parallel postulate* among the list of statements that are accepted as true. In this task we develop some parallel postulates for rigid motion transformations.

<u>Translations</u>

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a translation? That is, which word best completes this statement?

> *After a translation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

Give reasons for your answer. If you choose "sometimes", be very clear in your explanation how to tell when the corresponding line segments before and after the translation are parallel and when they are not.

<u>Rotations</u>

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a rotation? That is, which word best completes this statement?

> *After a rotation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

Give reasons for your answer. If you choose "sometimes", be very clear in your explanation how to tell when the corresponding line segments before and after the rotation are parallel and when they are not.

<u>Reflections</u>

Under what conditions are the corresponding line segments in an image and its pre-image parallel after a reflection? That is, which word best completes this statement?

> *After a reflection, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.*

Give reasons for your answer. If you choose "sometimes" be very clear in your explanation how to tell when the corresponding line segments before and after the reflection are parallel and when they are not.

Name: Geometric Figures | 5.4

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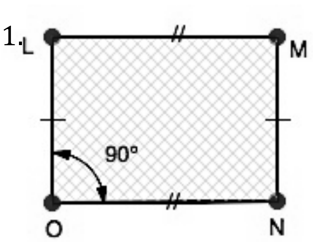
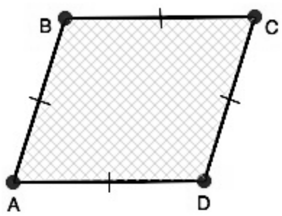
Ready, Set, Go!

Ready

Topic: Special quadrilaterals

Identify each quadrilateral as a trapezoid, parallelogram, rectangle, rhombus, square, or none of these. List ALL that apply.

1.


2.


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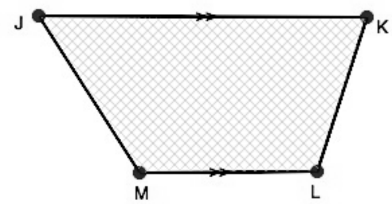
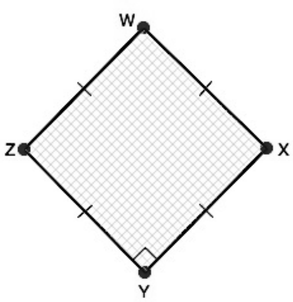
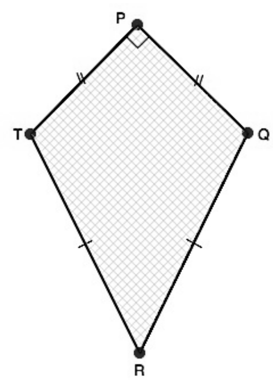


4.


5.


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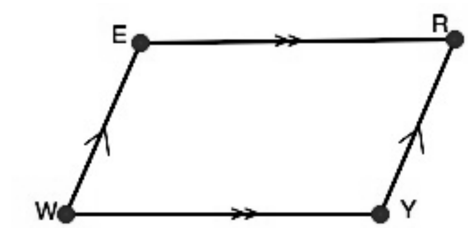
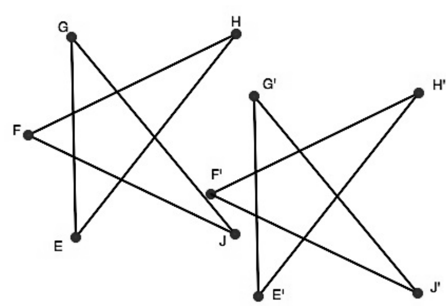
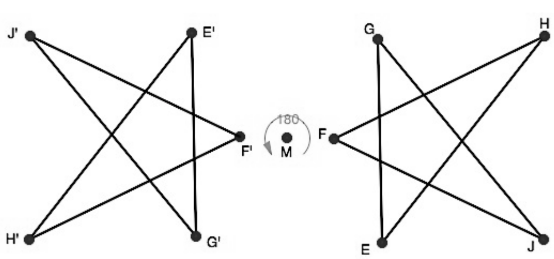


Set

7. Verify the parallel postulates below by naming the line segments in the pre-image and its image that are still parallel. Use correct mathematical notation.

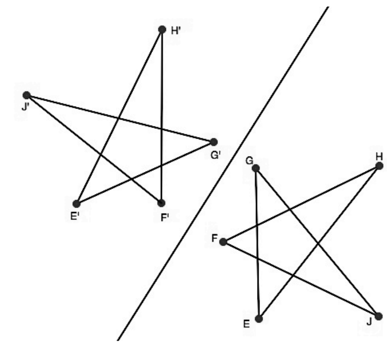
a. After a translation, corresponding line segments in an image and its pre-image are always parallel or lie along the same line.





b. After a rotation of 180°, corresponding line segments in a pre- image and its image are parallel or lie on the same line.

c. After a reflection, line segments in the pre-mage that are parallel to the line of reflection will be parallel to the corresponding line segments in the image.

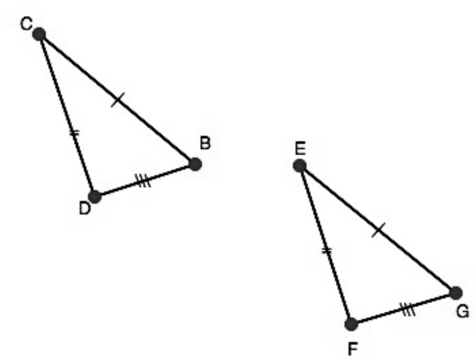


Go

Topic: Identifying congruence patterns in triangles

For each pair of triangles write a congruence statement and justify your statement by identifying the congruence pattern you used. Then justify that the triangles are congruent by connecting corresponding vertices of the pre-image and image with line segments.
How should those line segments look?

8.



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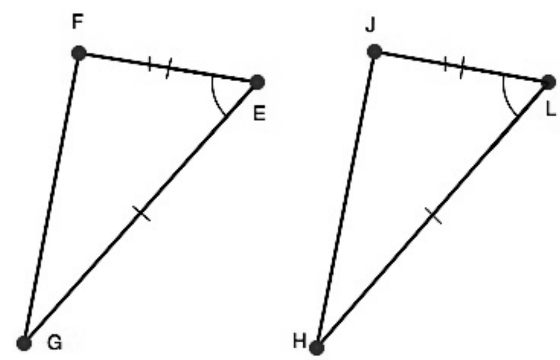

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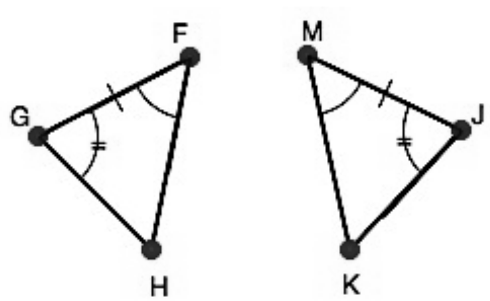

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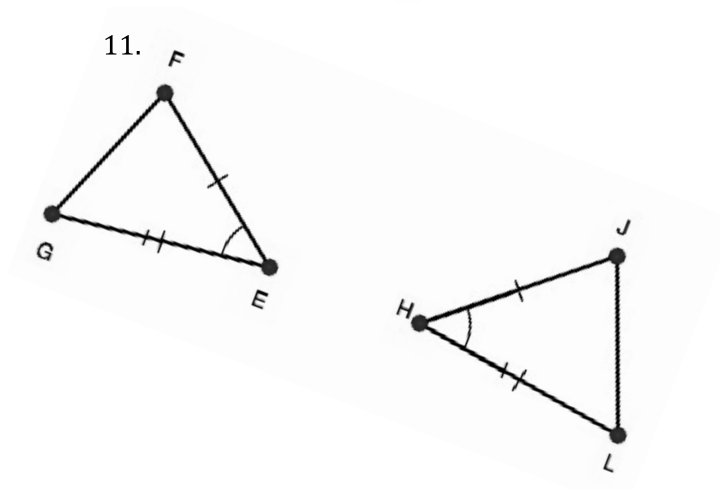
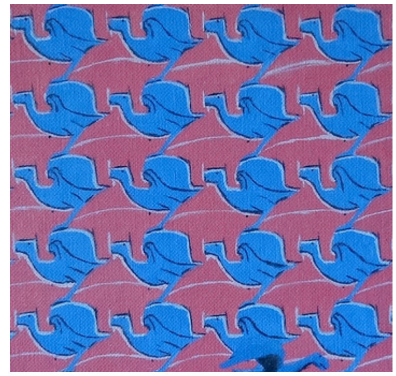


5.5 Conjectures and Proof

A Practice Understanding Task



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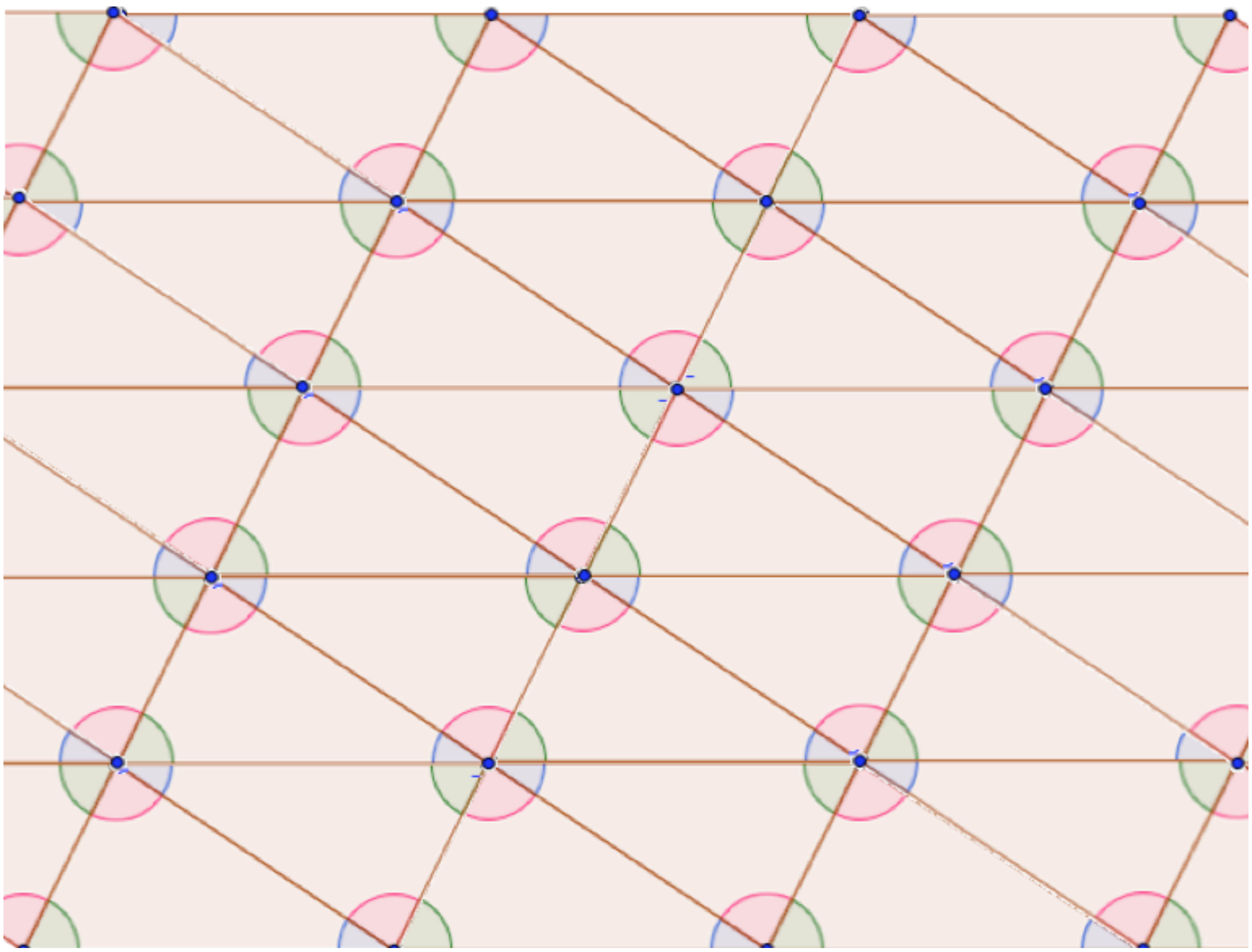
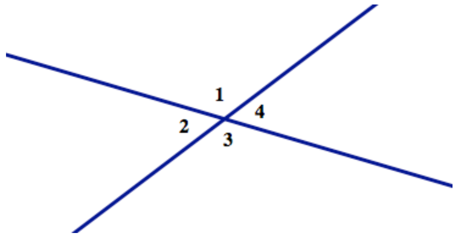
The diagram from *How Do You Know That?* has been extended by repeatedly rotating the image triangles around the midpoints of their sides to form a tessellation of the plane, as shown below.



Using this diagram, we will make some conjectures about lines, angles and triangles and then write proofs to convince ourselves that our conjectures are always true.

Vertical Angles

When two lines intersect, the opposite angles formed at the point of intersection are called *vertical angles*. In the diagram below, ∠ 1 and ∠ 3 form a pair of vertical angles, and ∠ 2 and ∠ 4 form another pair of vertical angles.



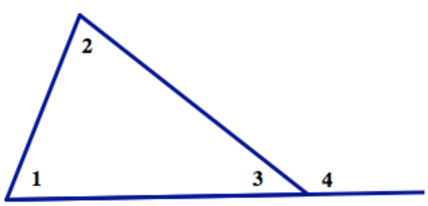
Examine the tessellation diagram above, looking for places where vertical angles occur. (You may have to ignore some line segments and angles in order to focus on pairs of vertical angles. This is a skill we have to develop when trying to see specific images in geometric diagrams.)

Based on several examples of vertical angles in the diagram, write a conjecture about vertical angles.

My conjecture:

Exterior Angles of a Triangle

When a side of a triangle is extended, as in the diagram below, the angle formed on the exterior of the triangle is called an *exterior angle*. The two angles of the triangle that are not adjacent to the exterior angle are referred to as the *remote interior angles*. In the diagram, ∠ 4 is an exterior angle, and ∠ 1 and ∠ 2 are the two remote interior angles for this exterior angle



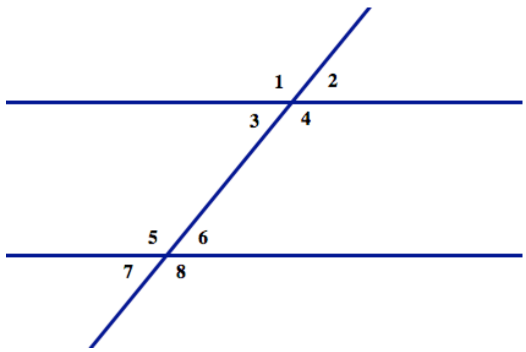
Examine the tessellation diagram above, looking for places where exterior angles of a triangle occur. (Again, you may have to ignore some line segments and angles in order to focus on triangles and their vertical angles.)

Based on several examples of exterior angles of triangles in the diagram, write a conjecture about exterior angles.

My conjecture:

Parallel Lines Cut By a Transversal

When a line intersects two or more other lines, the line is called a *transversal* line. When the other lines are parallel to each other, some special angle relationships are formed. To identify these relationships, we give names to particular pairs of angles formed when lines are crossed (or cut) by a transversal. In the diagram below, ∠ and ∠ are called *corresponding angles*, ∠ and ∠ are called *alternate interior angles*, and ∠ and ∠ are called *same side interior angles*.



Examine the tessellation diagram above, looking for places where parallel lines are crossed by a transversal line.

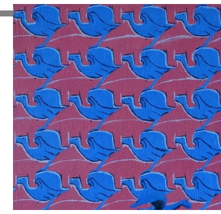
Based on several examples of parallel lines and transversals in the diagram, write some conjectures about corresponding angles, alternate interior angles and same side interior angles.

My conjectures:

Proving Our Conjectures

For each of the conjectures you wrote above, write a proof that will convince you and others that the conjecture is always true. You can use ideas about transformations, linear pairs, congruent triangle criteria, etc. to support your arguments. A good way to start is to write down everything you know in a flow diagram, and then identify which statements you might use to make your case.

Name: Geometric Figures | 5.5

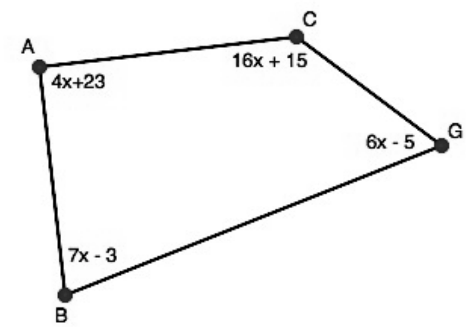


Ready, Set, Go!

Ready

Topic: Properties of quadrilaterals.

1. Use what you know about triangles to write a paragraph proof that proves that the sum of the angles in a quadrilateral is 360º.



2. Find the measure of x in quadrilateral *ABGC*.

M**atch the equation with the correct line in the graph of lines *p, q, r,* and *s*.**

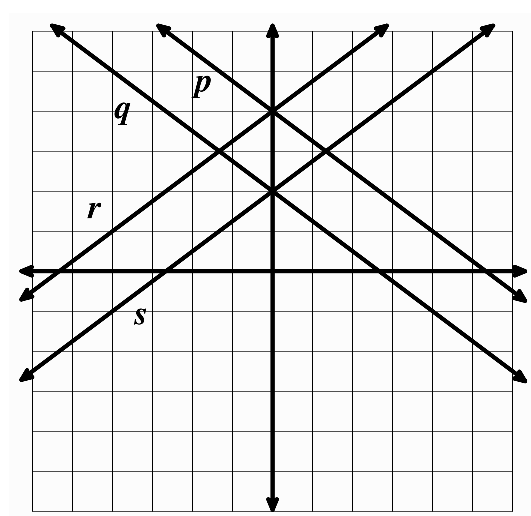
3. $y = \frac{3}{4}x + 2$

4. $y = -\frac{3}{4}x + 2$

5. $y = \frac{3}{4}x + 4$

6. $y = -\frac{3}{4}x + 4$

7. Describe the shape made by the intersection of the 4 lines. List as many observations as you can about the shape and its features.



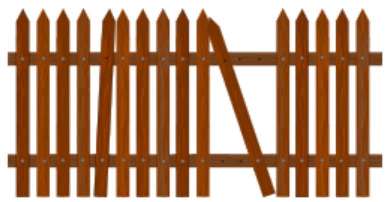
Set

Topic: Parallel lines with a transversal, vertical angles, and the exterior angle of a triangle

Label each picture as showing *parallel lines with a transversal, vertical angles*, or an *exterior angle of a triangle*. Highlight the geometric feature you identified. Can you find all 3 features in 1 picture? Where?

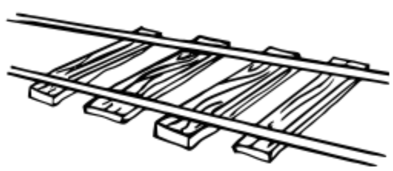
8.

9.



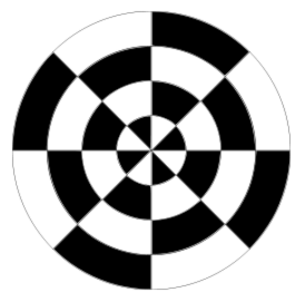
10.

11.



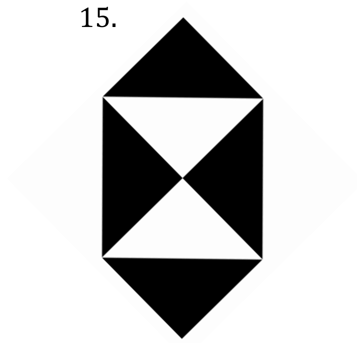
12.

13.



14.

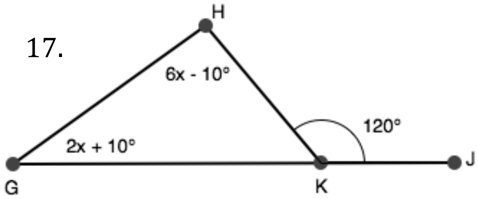
15.



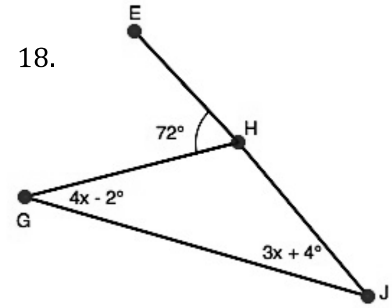
16.

Find the value of the 2 remote interior angles in the figures below.

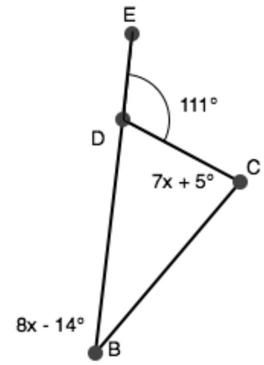
17.



18.



19.



Indicate whether each pair of angles is *congruent* or *supplementary* by trusting how they look. Lines *p* and *q* are parallel.

20. $\angle 5$ *and* $\angle 8$

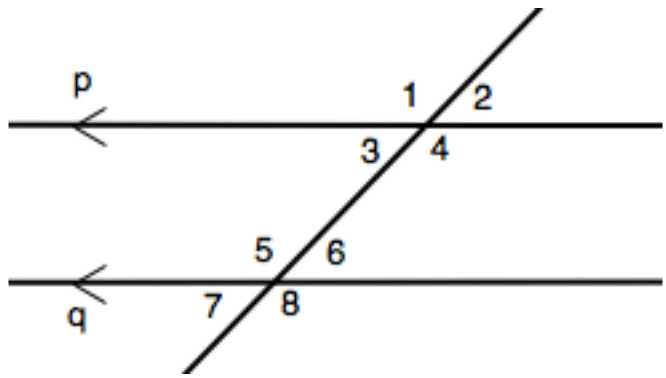
21. $\angle 2$ *and* $\angle 6$

22. $\angle 2$ *and* $\angle 8$

23. $\angle 4$ *and* $\angle 6$

24. $\angle 3$ *and* $\angle 5$

25. $\angle 1$ *and* $\angle 3$



Go

Topic: Complementary and supplementary angles.

Find the complement and the supplement of the given angles. It is possible for the complement or supplement not to exist.

26. 37°

27. 59°

28. 89°

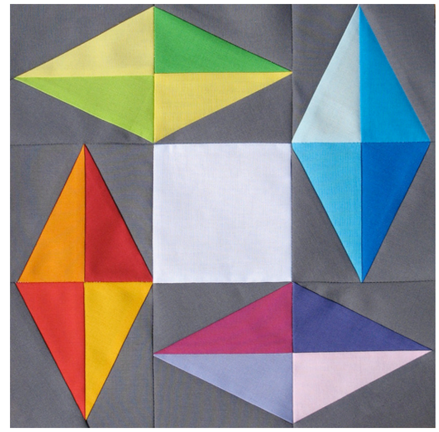
29. 111°

30. 3°

31. 90°

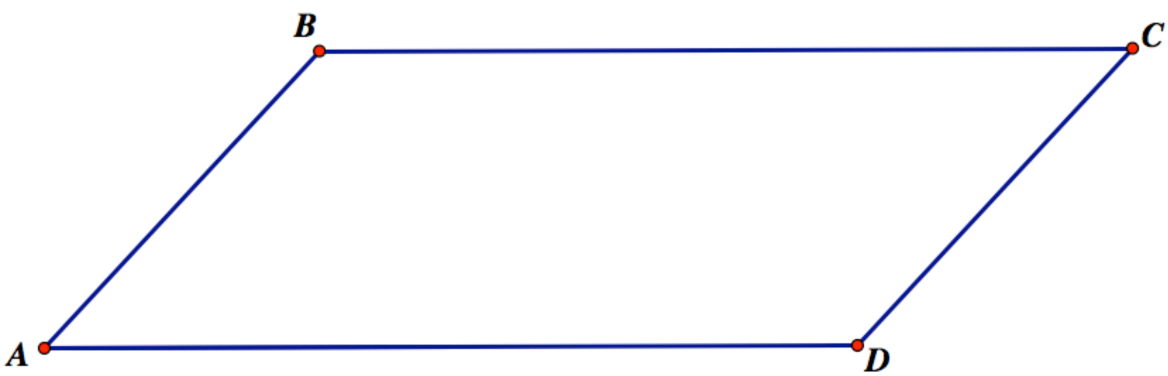
5.6 Parallelogram Conjectures and Proof

A Solidify Understanding Task



In Mathematics I you made conjectures about properties of parallelograms based on identifying lines of symmetry and rotational symmetry for various types of parallelograms. Now that we have additional knowledge about the angles formed when parallel lines are cut by a transversal, and we have criteria for convincing ourselves that two triangles are congruent, we can more formally prove some of the things we have noticed about parallelograms.

1. Explain how you would locate the center of rotation for the following parallelogram. What convinces you that the point you have located is the center of rotation?



2. If you haven't already, draw one or both of the diagonals in the above parallelogram. Use this diagram to prove this statement: *opposite sides of a parallelogram are congruent*

3. Use this diagram to prove this statement: *opposite angles of a parallelogram are congruent*

4. Use this diagram to prove this statement: *the diagonals of a parallelogram bisect each other*

The statements we have proved above extend our knowledge of properties of all parallelograms: not only are the opposites sides parallel, they are also congruent; opposite angles are congruent; and the diagonals of a parallelogram bisect each other. A parallelogram has 180° rotational symmetry around the point of intersection of the diagonals—the center of rotation for the parallelogram.

If we have a quadrilateral that has some of these properties, can we convince ourselves that the quadrilateral is a parallelogram? How many of these properties do we need to know before we can conclude that a quadrilateral is a parallelogram?

5. Consider the following statements. If you think the statement is true, create a diagram and write a convincing argument to prove the statement.

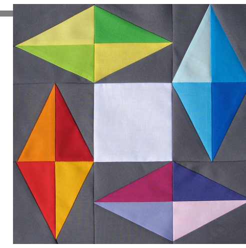
   a. If opposite sides and angles of a quadrilateral are congruent, the quadrilateral is a parallelogram.

   b. If opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram.

   c. If opposite angles of a quadrilateral are congruent, the quadrilateral is a parallelogram.

   d. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.

Name: Geometric Figures | 5.6



Ready, Set, Go!

Ready

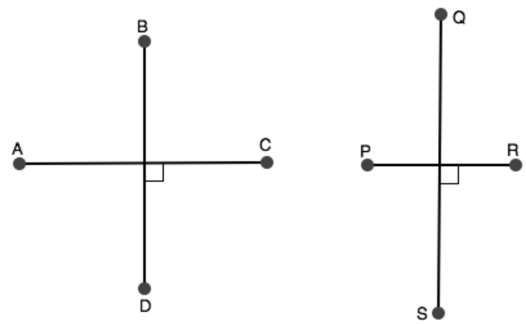
Topic: Sketching quadrilaterals based on specific features.

Sketch the quadrilateral by connecting the points in alphabetical order. Close the figure.

1. In both figures, the lines are perpendicular bisectors of each other.

   a. Are the quadrilaterals you sketched congruent?

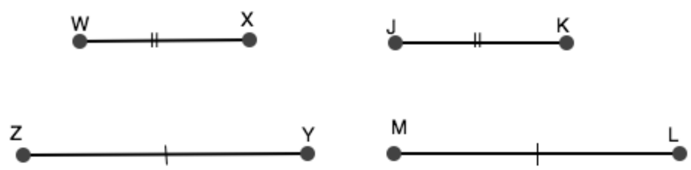
   b. What additional requirement(s) is/are needed to make the figures congruent?



2. In both figures one set of opposite sides are parallel and congruent.

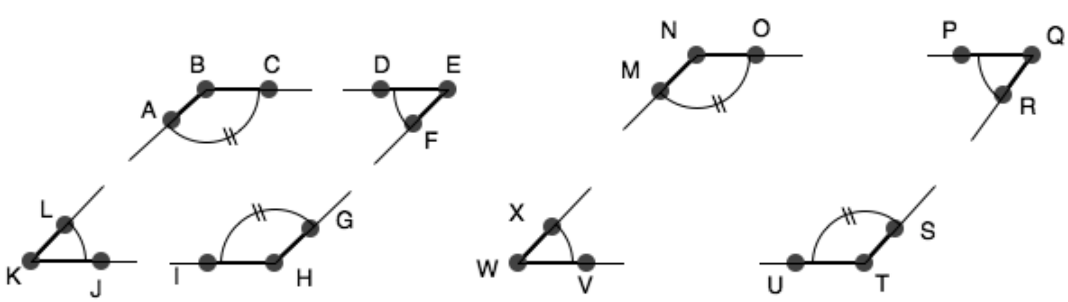
   a. Are the quadrilaterals you sketched congruent?

   b. What additional requirement(s) is/are needed to make the figures congruent?



3. In both figures corresponding angles are congruent.

   a. Are the quadrilaterals you sketched congruent?
   b. What additional requirement(s) is/are needed to make the figures congruent?



Set

Topic: Properties of parallelograms

4. Quadrilateral BCDE below was formed by 2 sets of intersecting parallel lines. Figure 2 is the image of figure 1. It has been rotated 180º. Find the center of rotation for figure 1. Make a list of everything that has been preserved in the rotation. Then make a list of anything that has changed. Is quadrilateral BCDE a parallelogram? How do you know?

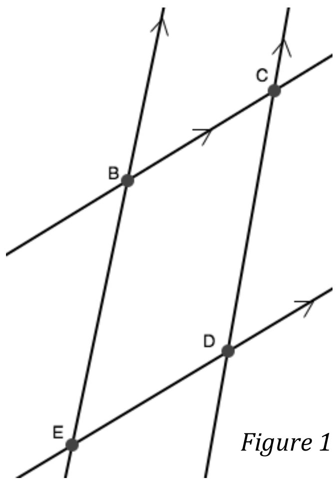


Figure 1

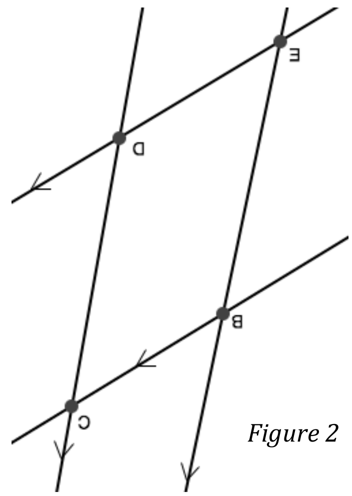


Figure 2

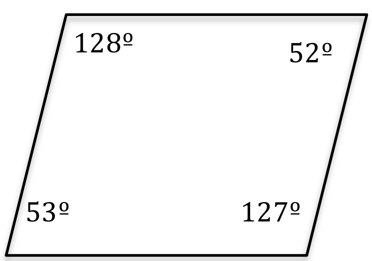
Go

The following theorems all concern parallelograms:

- ❖ Opposite sides of a parallelogram are congruent.
- ❖ Opposite angles of a parallelogram are congruent.
- ❖ Consecutive angles of a parallelogram are supplementary.
- ❖ The diagonals of a parallelogram bisect each other.

Give a reason from the list above that explains why it is NOT possible for each figure below to be a parallelogram. List ALL that apply.

5.



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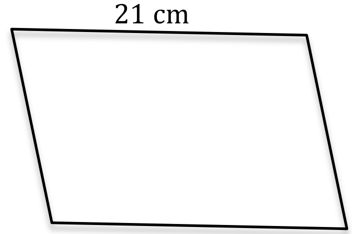
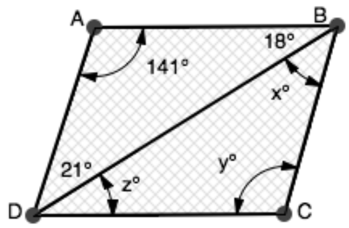


Each quadrilateral below is a parallelogram. Find the values of x, y, and z.

7.



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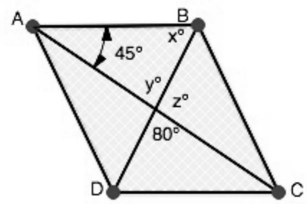

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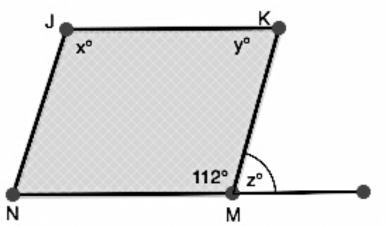

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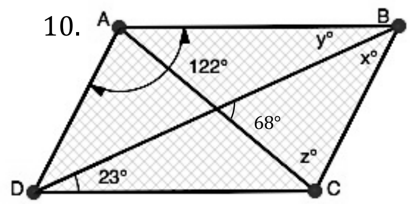

Go

Topic: Using correct mathematical symbols

Rewrite the phrases below using correct mathematical symbols.

Example: Eleven plus eight is nineteen. *11 + 8 = 19*

11. Triangle ABC is congruent to triangle GHJ. ______________________

12. Segment BV is congruent to segment PR. ______________________

13. Three feet are equal to one yard. ______________________

14. Line TR is parallel to line segment WQ. ______________________

15. Ray VP is perpendicular to segment GH. ______________________

16. Angle 3 is congruent to angle 5. ______________________

17. The distance between W and X is 7 feet. ______________________

18. The length of segment AB is equal to the length of TR. ______________________

19. The measure of angle SRT is equal to the measure of angle CDE.______________________

20. Explain when it is proper to use an equal sign and when it is proper to use the congruent symbol.

5.7 Guess My Parallelogram
A Practice Understanding Task

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Tehani and Tia are playing a guessing game in which one person describes some of the features of a quadrilateral they have drawn and the other person has to name the type of quadrilateral.

Here are some of the clues they gave each other. Decide what type of quadrilateral they are describing, and explain how you know.

1. The diagonals of this quadrilateral are perpendicular to each other.

2. The diagonals of this quadrilateral are congruent.

3. When rotated 90°, each diagonal of this quadrilateral gets superimposed on top of the other.

4. Consecutive angles of this quadrilateral are supplementary (that is, they add to 180°).

5. Consecutive angles of this quadrilateral are congruent.

6. The diagonals of this quadrilateral are congruent and perpendicular to each other.

Name: Geometric Figures | 5.7

Ready, Set, Go!

Ready

Topic: Constructing perpendicular bisectors and angle bisectors

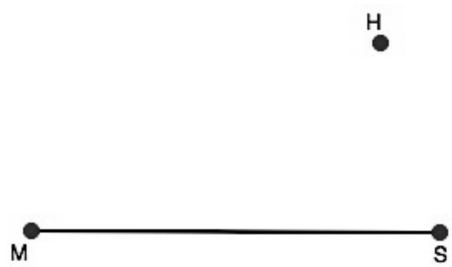
Use a compass and a straightedge to bisect the following line segments.

1. A ______________________ B

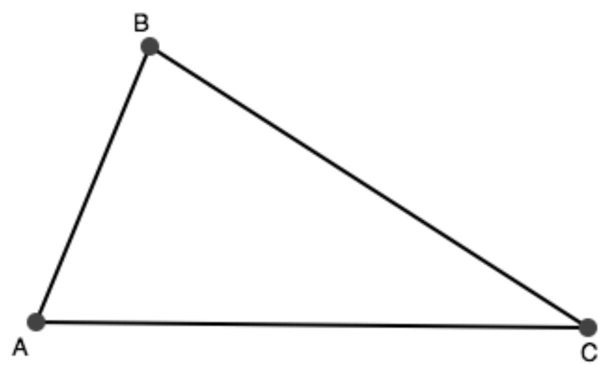
2. T ______________________________ S

3. When we construct the bisector of a segment, we are also constructing the perpendicular bisector. Must a bisector of a segment always be a perpendicular line?

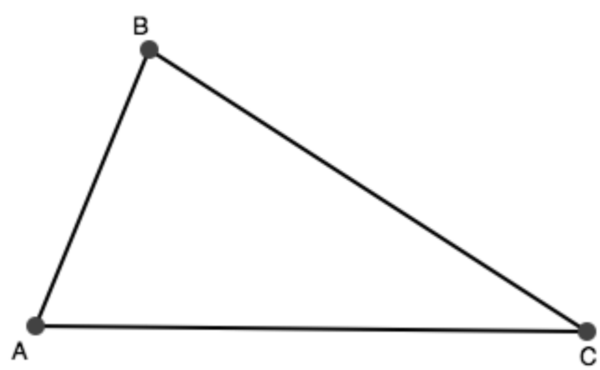
4. Construct the midpoint B of $\overline{MS}$. Then connect point B to point H.



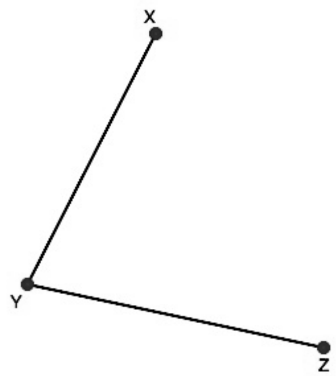
5. Construct the 3 medians of ΔABC.



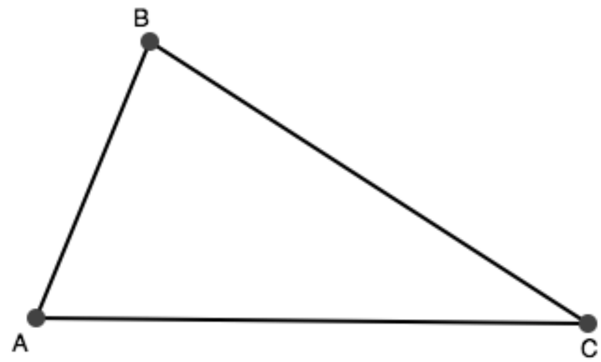
6. Construct the 3 perpendicular bisectors of ΔABC.



7. Construct the angle bisector of $\angle XYZ$.



8. Construct the 3 angle bisectors of ΔABC.



Set

Topic: Tests for parallelograms

Determine whether each quadrilateral is a parallelogram. Write YES if it is. If it is NOT a parallelogram, make a sketch of a quadrilateral that has the given features.

9. 1 pair of opposite sides is parallel and it has 2 consecutive right angles

10. The quadrilateral has 4 right angles.

11. 1 pair of opposite sides is parallel and congruent

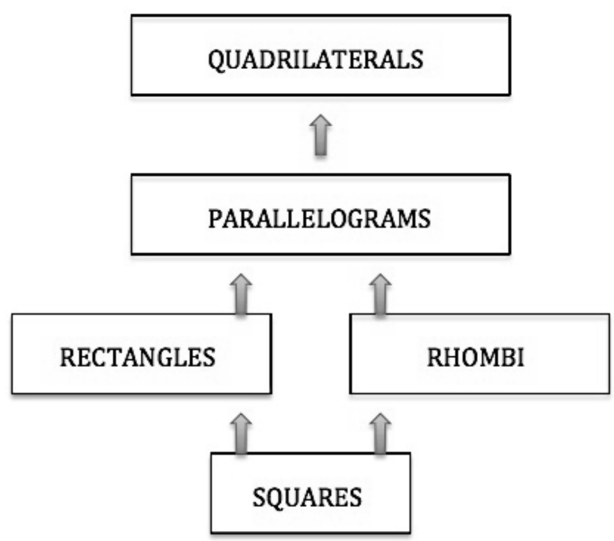
12. 1 pair of opposite sides is parallel. The other pair of opposite sides is congruent.

13. Consecutive angles are supplementary.

14. The diagonals are perpendicular.

15. The flowchart on the right has the most general 4-sided polygon at the top and the most specific one at the bottom. Around each box, write in the details that make the specific quadrilateral unique.

    Explain why the arrows point up instead of down.



Go

Topic: Features of triangles and quadrilaterals

State whether each statement is *true* or *false*. If it is false, explain why or rewrite the statement to make it true.

16. If a triangle is equilateral, then the median and the altitude are the same segments.

17. The perpendicular bisectors of the sides of a triangle also bisect the angles.

18. Some of the angles in a triangle equal 180º.

19. An altitude of a triangle may fall on the exterior of the triangle.

20. The 3rd angle in a triangle is always the supplement to the sum of the other 2 angles.

21. In a right triangle, the 2 acute angles are always complementary.

22. All squares are also rectangles.

23. A rhombus is always a square.

24. If a figure is a trapezoid, then it is also a parallelogram.

25. The diagonals of a rectangle bisect the angles.

26. A parallelogram can have 3 obtuse angles.

27. The figure made by two pair of intersecting parallel lines is always a parallelogram.

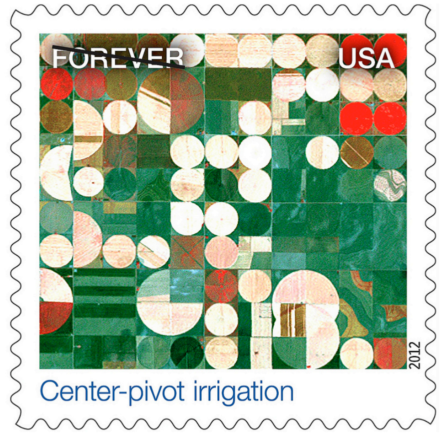
28. All of the angles in a parallelogram can be congruent.

29. A diagonal always divides a quadrilateral into 2 congruent triangles.

30. If a quadrilateral goes through a translation, the sides of the pre-image and image will remain parallel.

5.8 Centers of a Triangle

A Practice Understanding Task



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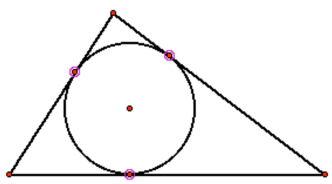
Kolton, Kevin and Kara have been asked by their fathers to help them solve some interesting geometry problems.

Problem 1

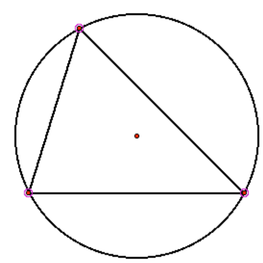
Kolton's father installs sprinkling systems for farmers. The systems he installs are called "Center Pivot Irrigation Systems" since the sprinklers are on a long pipe that rotates on wheels around a center point, watering a circular region of crops. You may have seen such "crop circles" from an airplane.



Sometimes Kolton's father has to install sprinkler systems on triangular shaped pieces of land. He wants to be able to locate the "pivot point" in the triangular field so the circle being watered will touch each of the three fences that form the boundaries of the field. He has asked for Kolton's help with this problem, since Kolton is currently studying geometry in high school.

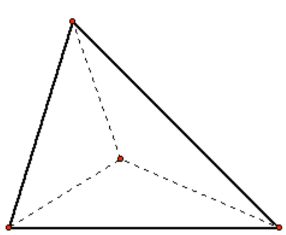
Problem 2

Kara's father installs cell towers. Since phone signals bounce from tower to tower, they have to be carefully located. Sometimes Kara's father needs to locate a new tower so that it is equidistant from three existing towers. He thinks of the three towers that are already in place as the vertices of a triangle, and he needs to be able to find a point in this triangle where he might locate the new tower so that it is equidistant from the other three. He has asked Kara to help him with this problem since she is also studying geometry in school.



Problem 3

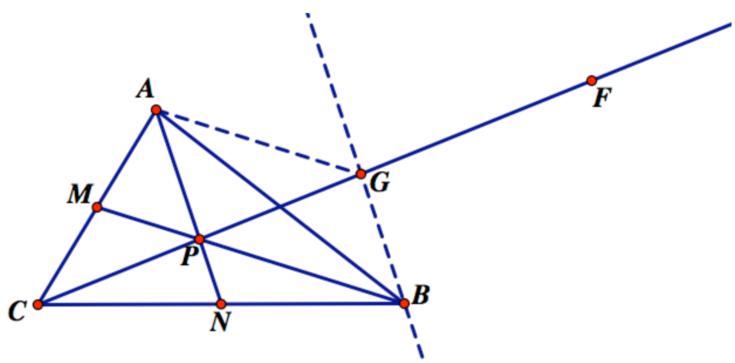
Kevin's father is an artist and has been commissioned by the city to build an art project in the park. His proposal consists of several large pyramids with different shaped triangles balanced on the vertex points of the pyramids. Kevin's father needs to be able to find the point inside of a triangle that he calls "the balancing point." He has asked Kevin to use his knowledge of geometry to help him solve this problem.



Kolton, Kevin and Kara's geometry teacher has suggested they try locating points in the interior of triangles where medians, altitudes, angle bisectors, or perpendicular bisectors of the sides intersect.

1. Try out the experiment suggested by the students' geometry teacher. Which set of line segments seem to locate a point in the triangle that best meets the needs of each of their fathers?

2. Kolton, Kevin and Kara have noticed something interesting about these sets of line segments. To their surprise, they notice that all three medians of a triangle intersect at a common point. Likewise, the three altitudes also intersect at a common point. So do the three angle bisectors, and the three bisectors of the sides. They think their fathers will find this interesting, but they want to make sure these observations are true for all triangles, not just for the ones they have been experimenting on. The diagrams and notes below suggest how each is thinking about the proof they want to show his or her father. Use these notes and diagrams to write a convincing proof.

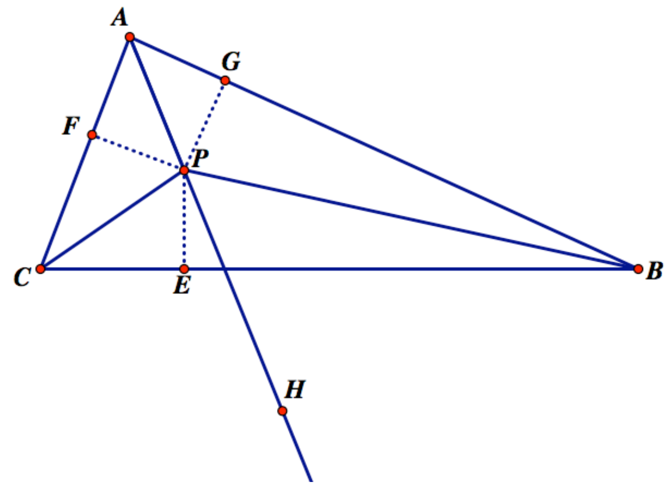


Kevin's Notes

What I did to create this diagram:

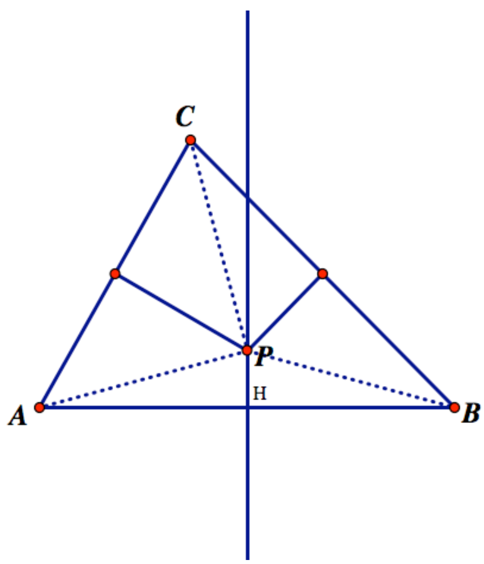
Point *M* is the midpoint of side AC, and point *N* is the midpoint of side CB. Therefore, $\overline{AN}$ and $\overline{BM}$ are medians of the triangle. I then drew ray *CF* through point *P*, the intersection of the two medians. My question is, "Does this ray contain the third median?" So, I need to find a way to answer that question. As I was thinking about this, I thought I could visualize a parallelogram with its diagonals, so I drew line *GB* to be parallel to median *AN*, and then connected vertex *A* to point *G* on the ray. Quadrilateral *AGBP* looks like a parallelogram, but I'm not so sure. And I am wondering if that will help me with my question about the third median. What do you think?

Kolton's Notes



What I did to create this diagram:

I constructed the angle bisectors of angle *ACB* and angle *ABC.* They intersected at point *P*. So I wouldn't get confused by so many lines in my diagram, I erased the rays that formed the angle bisectors past their point of intersection. I then drew ray *AH* through point *P*, the point of intersection of the two angle bisectors. My question is, "Does this ray bisect angle *CAB*?" While I was thinking about this question, I noticed that I had created three smaller triangles in the interior of the original triangle. I constructed the altitudes of these three triangles (they are drawn as dotted lines). When I added the dotted lines, I started seeing kites in my picture. I'm wondering if thinking about the smaller triangles or the kites might help me prove that ray *AH* bisects angle *CAB.*



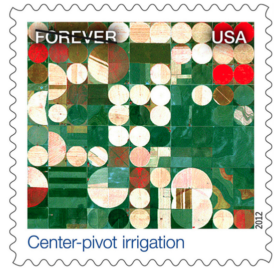
Kara's Notes

What I did to create this diagram:

I constructed the perpendicular bisectors of side *AC* and side *BC.* They intersected at point *P*. So I wouldn't get confused by so many lines in my diagram, I erased the rays that formed the perpendicular bisectors past their point of intersection. I then constructed a line perpendicular to side *AB* through point *P*, the point of intersection of the two perpendicular bisectors. I named the point where this line intersected side *AB* point *H.* My question is, "Does this perpendicular line also bisect side *AB*?" While I was thinking about this question, I noticed that I had creates some quadrilaterals in the interior of the original triangle. Since quadrilaterals in general don't have a lot of interesting properties, I decided to make some triangles by dotting in line segments drawn from *P* to each of the vertices of the original triangle. I'm wondering if thinking about these smaller triangles might help me prove that line *PH* bisects side *AB.*

Name: Geometric Figures | 5.8

Ready, Set, Go!



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Ready

Topic: Are you ready for a test on module 5?

Figure 1 has been rotated 180º about the midpoint in side BC to form figure 2. Figure 1 was then translated to the right to form figure 3.

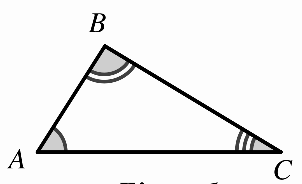


Figure 1

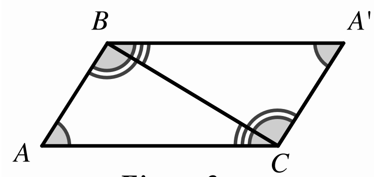


Figure 2

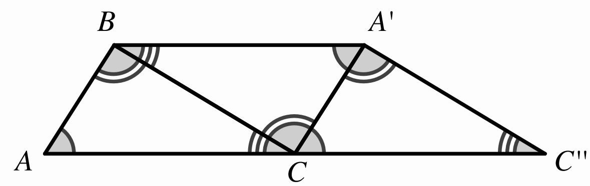


Figure 3

1. Use figure 3 to explain how you know the exterior angle $\angle B'CC''$ is equal to the sum of the 2 remote interior angles $\angle BAC$ and $\angle ABC$.

2. Use figure 3 to explain how you know the sum of the angles in a triangle is always 180º.

3. Use figure 2 to explain how you know the sum of the angles in a quadrilateral is always 360º.

4. Use figure 2 to explain how you know that the opposite angles in a parallelogram are congruent.

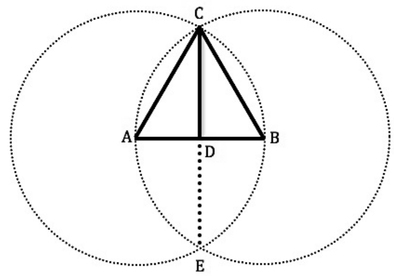
5. Use figure 2 to explain how you know that the opposite sides in a parallelogram are parallel and congruent.

6. Use figure 2 to explain how you know that when two parallel lines are crossed by a transversal, the alternate interior angles are congruent.

7. Use figure 2 and/or 3 to explain how you know that when two parallel lines are crossed by a transversal, the same-side interior angles are supplementary.

Set

Topic: Writing proofs



8. Prove that $\overline{CD}$ is an altitude of ΔABC.
   Use the diagram and write a 2 column proof.

9. Use the diagram to prove that ΔABC is an isosceles triangle. (Choose your style.)

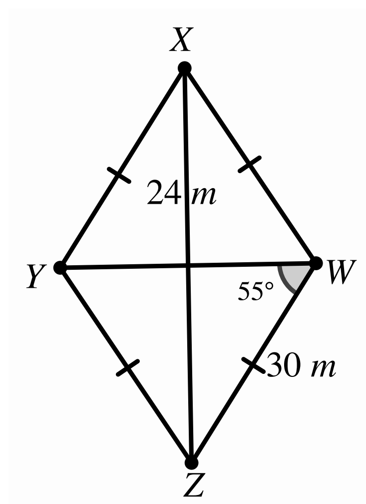
10. Use the diagram to prove that $m\angle A \cong m\angle B$. (Choose your style.)

Go

Topic: The algebra of parallelograms

Use what you know about triangles and parallelograms to find each measure.

11. $\overline{XZ}$
12. $m\angle XYZ$
13. $m\angle XYW$
14. $\overline{YX}$
15. $m\angle YXZ$
16. $\overline{YW}$



17. $\overline{LG}$

18. $\overline{HF}$

19. $m\angle EHG$

20. $m\angle FEH$

21. $m\angle ELF$

22. $\overline{FG}$

23. $\overline{EG}$

24. $m\angle FGE$