Secondary Two Mathematics: An Integrated Approach
Module 3
Quadratic Equations

By

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In partnership with the Utah State Office of Education

Module 3 – Quadratic Equations

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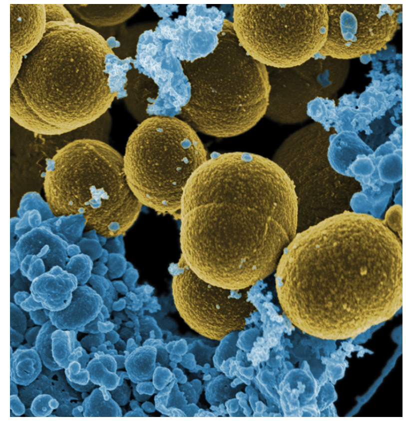
3.1 Experimenting with Exponents

A Develop Understanding Task



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[This task was adopted from the *Illustrative Mathematics Project: http://www.illustrativemathematics.org/illustrations/385*
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Travis and Miriam are studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

1. Complete the following table and plot the data on the graph at the end of this task.

Hours into the study	0	1	2	3	4
Bacteria population (in thousands)	4				

2. Write an equation for P, the population of the bacteria, as a function of time, t, and verify that it produces correct populations for t = 1, 2, 3, and 4 hours.

Travis and Miriam want to create a table with more entries; specifically, they want to fill in the population at each half hour. Unfortunately, they forgot to make these measurements so they decide to estimate the values.

Travis makes the following claim:

> "If the population doubles in 1 hour, then half that growth occurs in the first half-hour and the other half in the second half-hour. So for example, we can find the population at t = ½ by finding the average of the populations at t = 0 and t = 1."

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

Hours into the study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria population (in thousands)	4								

4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Miriam suggests they should fill in the data in the table in the following way:

> "To make the estimates, I noticed that the population increases by the same factor each hour, and I think that this property should hold over each half-hour interval as well."

4. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor in order to produce consistent results. Use this constant multiplier to complete the table. Also plot Miriam's data on the graph at the end of this task.

Hours into the study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria population (in thousands)	4								

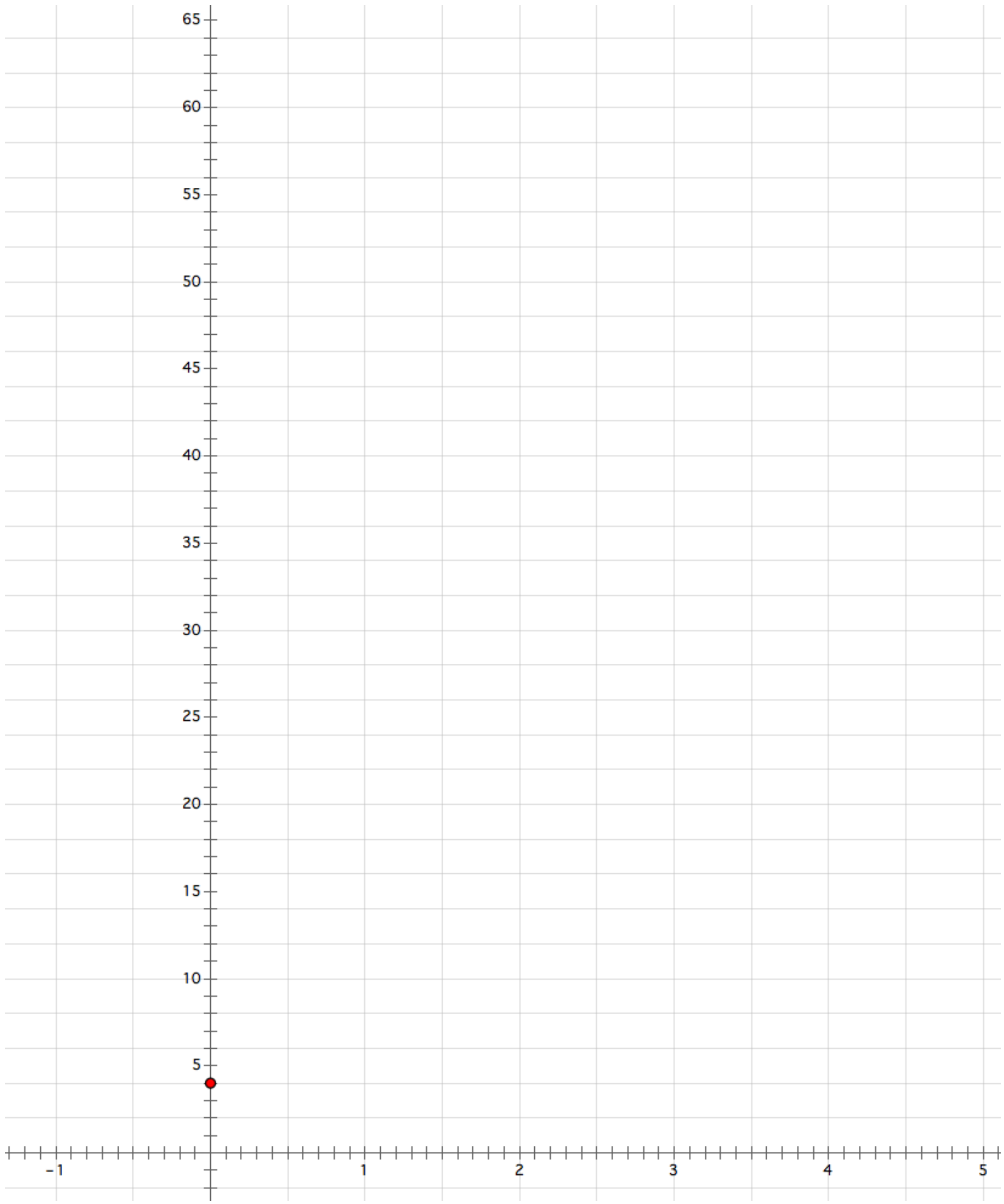
5. What if Miriam wanted to estimate the population every 20 minutes instead of every 30 minutes? What multiplier would she use for every third of an hour to be consistent with the population doubling every hour? Use this multiplier to complete the following table.

Hours into the study	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
Bacteria population (in thousands)	4									

6. What number did you use as a multiplier to complete the table in problem 4?

7. What number did you use as a multiplier to complete the table in problem 5?

8. Give a detailed description of how you would estimate the population, *P*, at $t = \frac{5}{3}$ hours.


65
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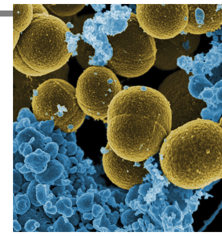
Ready, Set, Go!



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Ready

Topic: Comparing additive and multiplicative patterns.

The sequences below exemplify either an additive (arithmetic) or a multiplicative (geometric) pattern. Identify the type of sequence, fill in the missing values on the table and write an equation.

1.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32	a.	b.	c.

d. Type of Sequence: e. Equation:

2.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

3.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-3	9	-27	81	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

4.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20	a.	b.	c.	d.

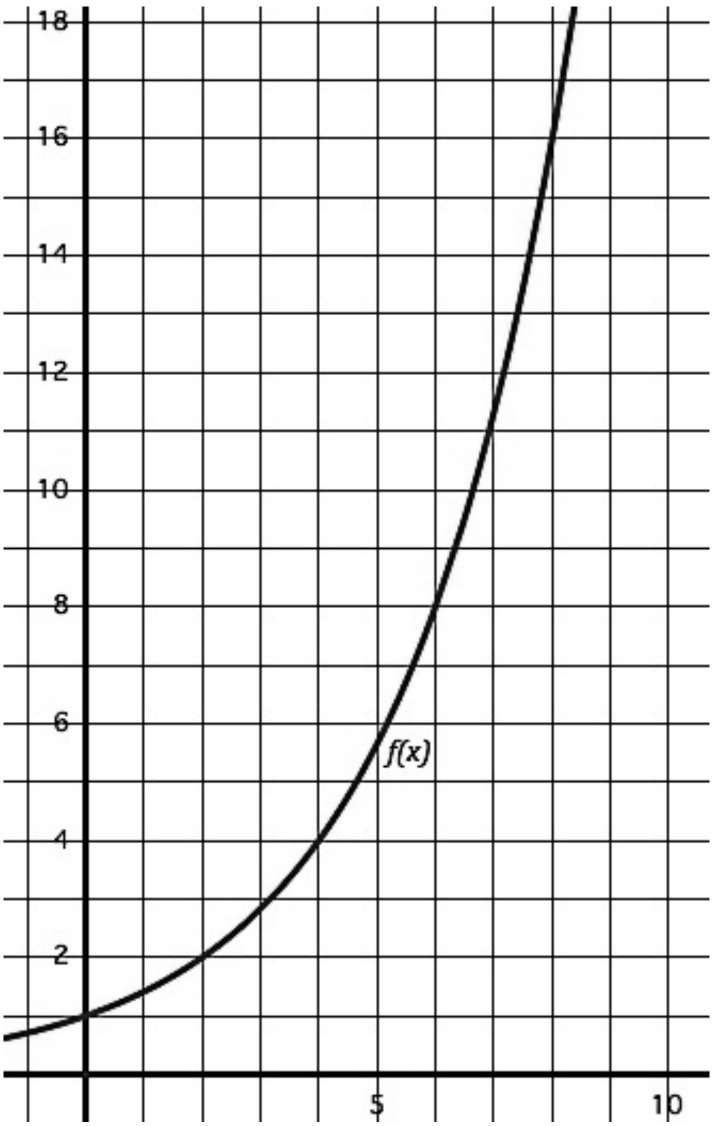
e. Type of Sequence: f. Equation:

5.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12	a.	b.	c.	d.

e. Type of Sequence: f. Equation:

Use the graph of the function to find the desired values of the function. Also create an explicit equation for the function.



6. Find the value of $f(2)$

7. Find where $f(x) = 4$

8. Find the value of $f(6)$

9. Find where $f(x) = 16$

10. What do you notice about the way that inputs and outputs for this function relate? (Create an in-out table if you need to.)

11. What is the explicit equation for this function?

Set

Topic: Evaluate the expressions with rational exponents.

Fill in the missing values of the table based on the growth that is described.

12. The growth in the table is triple at each whole year.

Years	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
bacteria	2		6						

13. The growth in the table is triple at each whole year.

Years	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$
bacteria	2			6					

14. The values in the table grow by a factor of four at each whole year.

Years	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
bacteria	2		8						

Go

Topic: Simplifying exponents

Simplify the following expressions using exponent rules and relationships, write your answers in exponential form. (For example: $2^2 \cdot 2^5 = 2^7$)

15. $3^2 \cdot 3^5$

16. $\frac{5^3}{5^2}$

17. 2^{-5}

18. 17^0

19. $\frac{7^5}{7^2} \cdot \frac{7^3}{7^4}$

20. $\frac{3^{-2} \cdot 3^5}{3^7}$

3.2 Half Interested

A Solidify Understanding Task

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Carlos and Clarita, the Martinez twins, have run a summer business every year for the past five years. Their first business, a neighborhood lemonade stand, earned a small profit that their father insisted they deposit in a savings account at the local bank. When the Martinez family moved a few months later, the twins decided to leave the money in the bank where it has been earning 5% interest annually. Carlos was reminded of the money when he found the annual bank statement they had received in the mail.

"Remember how Dad said we could withdraw this money from the bank when we are twenty years old," Carlos said to Clarita. "We have $382.88 in the account now. I wonder how much that will be five years from now?"

1. Given the facts listed above, how can the twins figure out how much the account will be worth five years from now when they are twenty years old? Describe your strategy and calculate the account balance.

2. Carlos calculates the value of the account one year at a time. He has just finished calculating the value of the account for the first four years. Describe how he can find the next year's balance, and record that value in the table.

year	amount
0	382.88
1	402.02
2	422.12
3	443.23
4	465.39
5	

3. Clarita thinks Carlos is silly calculating the value of the account one year at a time, and says that he could have written a formula for the n^{th} year and then evaluated his formula when $n = 5$. Write Clarita's formula for the n^{th} year and use it to find the account balance at the end of year 5.

4. Carlos was surprised that Clarita's formula gave the same account balance as his year-by-year strategy. Explain, in a way that would convince Carlos, why this is so.

"I can't remember how much money we earned that summer," said Carlos. "I wonder if we can figure out how much we deposited in the account five years ago, knowing the account balance now?"

5. Carlos continued to use his strategy to extend his table year-by-year back five years. Explain what you think Carlos is doing to find his table values one year at a time, and continue filling in the table until you get to -5, which Carlos uses to represent "five years ago".

year	amount
-5	
-4	
-3	
-2	
-1	364.65
0	382.88
1	402.02
2	421.12
3	443.23
4	465.39
5	

6. Clarita evaluated her formula for n = -5. Again Carlos is surprised that they get the same results. Explain why Clarita's method works.

Clarita doesn't think leaving the money in the bank for another five years is such a great idea, and suggests that they invest the money in their next summer business, *Curbside Rivalry* (which, for now, they are keeping top secret from everyone, including their friends). "We'll have some start up costs, and this will pay for them without having to withdraw money from our other accounts."

Carlos remarked, "But we'll be withdrawing our money halfway through the year. Do you think we'll lose out on this year's interest?"

"No, they'll pay us a half-year portion of our interest," replied Clarita.

"But how much will that be?" asked Carlos.

7. Calculate the account balance and how much interest you think Carlos and Clarita should be paid if they withdraw their money ½ year from now. Remember that they currently have -$382.88 in the account, and that they earn 5% annually. Describe your strategy.

Carlos used the following strategy: He calculated how much interest they should be paid for a full year, found half of that, and added that amount to the current account balance.

Clarita used this strategy: She substituted ½ for *n* in the formula $A = 382.88(1.05)^n$ and recorded this as the account balance.

8. This time Carlos and Clarita didn't get the same result. Whose method do you agree with and why?

Clarita is trying to convince Carlos that her method is correct. "Exponential rules are multiplicative, not additive. Look back at your table. We will earn $82.51 in interest during the next four years. If your method works we should be able to take half of that amount, add it to the amount we have now, and get the account balance we should have in two years, but it isn't the same."

9. Carry out the computations that Clarita suggested and compare the result for year 2 using this strategy as opposed to the strategy Carlos originally used to fill out the table.

10. The points from Carlos' table (see question 2) have been plotted on the graph at the end of this task, along with Clarita's function. Plot the value you calculated in question 9 on this same graph. What does the graph reveal about the differences in Carlos' two strategies?

11. Now plot Clarita's and Carlos' values for ½ year (see question 8) on this same graph.

"Your data point seems to fit the shape of the graph better than mine," Carlos conceded, "but I don't understand how we can use ½ as an exponent. How does that find the correct factor we need to multiply by? In your formula, writing $(1.05)^5$ means multiply by 1.05 five times, and writing $(1.05)^{-5}$ means divide by 1.05 five times, but what does $(1.05)^{1/2}$ mean?"

Clarita wasn't quite sure how to answer Carlos' question, but she had some questions of her own. She decided to jot them down, including Carlos' question:

- What numerical amount do we multiply by when we use $(1.05)^{1/2}$ as a factor?
- What happens if we multiply by $(1.05)^{1/2}$ and then multiply the result by $(1.05)^{1/2}$ again? Shouldn't that be a full year's worth of interest? Is it?
- If multiplying by $(1.05)^{1/2} \cdot (1.05)^{1/2}$ is the same as multiplying by 1.05, what does that suggest about the value of $(1.05)^{1/2}$?

12. Answer each of Clarita's questions listed above as best as you can.

As Carlos is reflecting on this work, Clarita notices the date on the bank statement that started this whole conversation. "This bank statement is three months old!" she exclaims. "That means the bank will owe us ¾ of a year's interest."

"So how much interest will the bank owe us then?", asked Carlos.

13. Find as many ways as you can to answer Carlos' question: How much will their account be worth in ¾ of a year (nine months) if it earns 5% annually and is currently worth $382.88?


465
460
455
450
445
440
435
430
425
420
415
410
405
400
395
390
385
380
(4, 465.39)
(3, 443.23)
(2, 421.12)
(1, 402.02)
(0, 382.88)
Selected: 6

Name: Solving Quadratic and Other Equations | 3.2

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Ready, Set, Go!

Ready

Topic: Simplifying Radicals

A very common radical expression is a square root. One way to think of a square root is the number that will multiply by itself to create a desired value. For example: $\sqrt{2}$ *is the number that will multiply by itself to equal 2. And in like manner* $\sqrt{16}$ *is the number that will multiply by itself to equal 16, in this case the value is 4 because 4 x 4 = 16. (When the square root of a square number is taken you get a nice whole number value. Otherwise an irrational number is produced.)*

This same pattern holds true for other radicals such as cube roots and fourth roots and so forth. For example: $\sqrt[3]{8}$ *is the number that will multiply by itself three times to equal 8. In this case it is equal to the value of 2 because* $2^3 = 2 \times 2 \times 2 = 8$.

With this in mind radicals can be simplified. See the examples below.

Example 1: Simplify $\sqrt{20}$	*Example 2*: Simplify $\sqrt[5]{96}$
$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$	$\sqrt[5]{96} = \sqrt[5]{2^5 \cdot 3} = 2\sqrt[5]{3}$

Simplify each of the radicals.

1. $\sqrt{40}$
2. $\sqrt{50}$
3. $\sqrt[3]{16}$
4. $\sqrt{72}$
5. $\sqrt[4]{81}$
6. $\sqrt{32}$
7. $\sqrt[5]{160}$
8. $\sqrt{45}$
9. $\sqrt[3]{54}$

Set

Topic: Finding arithmetic and geometric means and making meaning of rational exponents.

You may have found arithmetic and geometric means in your prior work. Finding arithmetic and geometric means requires finding values of a sequence between given values from non-consecutive terms. In each of the sequences below determine the means and show how you found them.

Find the *arithmetic* means for the following. Show your work.

10.

x	1	2	3
y	5		11

11.

x	1	2	3	4	5
y	18	a.	b.	c.	-10

12.

x	1	2	3	4	5	6	7
y	12	a.	b.	c.	d.	e.	-6

Find the *geometric* means for the following. Show your work.

13.

x	1	2	3
y	3		12

14.

x	1	2	3	4
y	7	a.	b.	875

15.

x	1	2	3	4	5	6
y	4	a.	b.	c.	d.	972

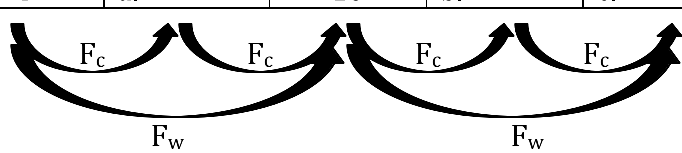
Fill in the tables of values and find the factor used to move between whole number values, F_w, as well as the factor, F_c, used to move between each column of the table.

16.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	4	a.	16	b.	c.

d. $F_w =$

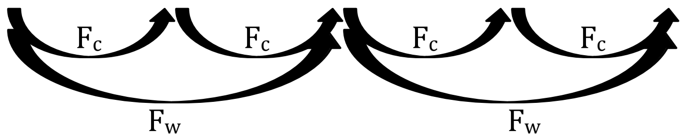
e. $F_c =$



17.

x	0	½	1	3/2	2
y	4	a.	8	b.	c.

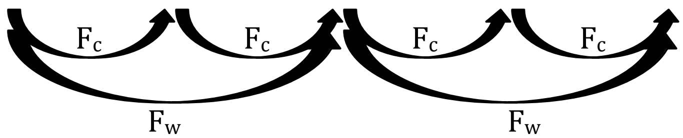
d. $F_w =$
e. $F_c =$



18.

x	0	½	1	3/2	2
y	5	a.	15	b.	c.

d. $F_w =$
e. $F_c =$



Go

Topic: Evaluating functions

Find the desired values for each function below.

19. $f(x) = 2x - 7$
   a. Find $f(-3)$
   b. Find $f(x) = 21$
   c. Find $f\left(\frac{1}{2}\right)$

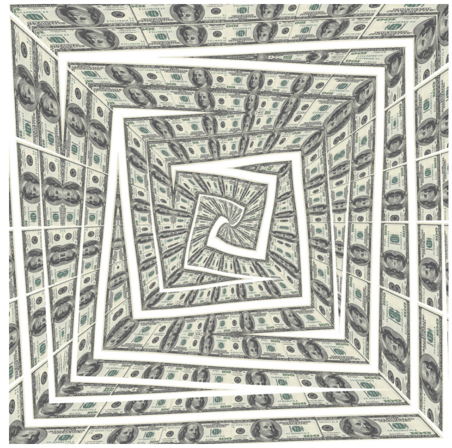
20. $g(x) = 3^x(2)$
   a. Find $g(-4)$
   b. Find $g(x) = 162$
   c. Find $g\left(\frac{1}{2}\right)$

21. $I(t) = 210(1.08^t)$
   a. Find $I(12)$
   b. Find $I(t) = 420$
   c. Find $I\left(\frac{1}{2}\right)$

22. $h(x) = x^2 + x - 6$
   a. Find $h(-5)$
   b. Find $h(x) = 0$
   c. Find $h\left(\frac{1}{2}\right)$

23. $k(x) = -5x + 9$
   a. Find $k(-7)$
   b. Find $k(x) = 0$
   c. Find $k\left(\frac{1}{2}\right)$

24. $m(x) = (5^x)2$
   a. Find $m(-2)$
   b. Find $m(x) = 1$
   c. Find $m\left(\frac{1}{2}\right)$



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3.3 More Interesting!
A Solidify Understanding Task

Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of $300.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be $315, since it increases 5% during the year.

1. Complete the table showing what amount is in the account each month during the first twelve months.

deposit											1 year
$300											$315

2. What number did you multiply the account by each month to get the next month's balance?

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$

Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300(1.05^{1/12})^{12t}$.

3. Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.

4. What is the meaning of the $12t$ in this equation?

Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $(1.05)^{1/12} \approx 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^t$ means I am earning 5% interest annually, but what does the 1.004 mean in this equation?"

5. Answer Carlos' question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

The properties of exponents can be used to explain why $[(1.05)^{1/12}]^{12t} = 1.05^t$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, simplify the expression using the properties of exponents, and explain what the expression means in terms of the context.

6. $(1.05)^{1/12} \cdot (1.05)^{1/12} \cdot (1.05)^{1/12}$

7. $[(1.05)^{1/12}]^6$

8. $(1.05)^{-1/12}$

9. $(1.05)^2 \cdot (1.05)^{1/4}$

10. $\dfrac{(1.05)^2}{(1.05)^{1/2}}$

Carlos and Clarita now have two equations representing the balance of their 5% account after *t* years: $A = 300(1.05)^t$ and $A = 300(1.05^{1/12})^{12t}$. In both of these equations *t* represents the amount of time the money has been in the account in terms of *years*.

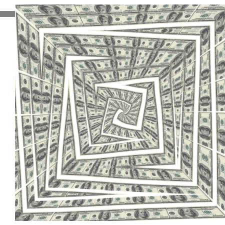
Carlos and Clarita know they can use their equations for fractions of a year by expressing *t* in terms of a portion of a year, for example, using *t = 2.5* for two and one-half years or $t = 1/12$ for one month. They are wondering if they can write an equation that would find the account balance in terms of *t* months or *t* days.

11. Write an equation that will find the account balance in terms of *t* months.

12. Write an equation that will find the account balance in terms of *t* days.

13. The account balance is currently $382.88. Write an equation that will find the account balance *t* months ago.

Ready, Set, Go!



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Ready

Topic: Meaning of Exponents

In the table below there is a column for the exponential form, the meaning of that form, which is a list of factors and the standard form of the number. Fill in the form that is missing.

Exponential form	List of factors	Standard Form
5^3	$5 \cdot 5 \cdot 5$	125
1a.	$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	b.
2. 2^{10}	a.	b.
3a.	b.	81
4. 11^5	a.	b.
5a.	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	b.
6a.	b.	625

Provide at least three other equivalent forms of the exponential expression. Use rules of exponents such as $3^5 \cdot 3^6 = 3^{11}$ and $\left(5^2\right)^3 = 5^6$ as well as division properties and others.

	1st Equivalent Form	2nd Equivalent Form	3rd Equivalent Form
7. $2^{10} =$			
8. $3^7 =$			
9. $13^{-8} =$			
10. $7^{\frac{1}{3}} =$			
11. $5^1 =$			

Set

Topic: Finding equivalent expressions and functions.

Determine whether all three expressions in each problem below are equivalent. Justify why or why they are not equivalent.

12. $5(3^{x-1})$ $\quad$ $15(3^{x-2})$ $\quad$ $\frac{3}{5}(3^{x})$

13. $64\,(2^{-x})$ $\quad$ $\frac{64}{2^{x}}$ $\quad$ $64\left(\frac{1}{2}\right)^{x}$

14. 3(x-1)+4 $\quad$ 3x - 1 $\quad$ 3(x-2) +7

15. $50(2^{x+2})$ $\quad$ $25(2^{2x+1})$ $\quad$ $50(4^{x})$

16. $30(1.05^{x})$ $\quad$ $30\left(1.05^{\frac{1}{7}}\right)^{7x}$ $\quad$ $30\left(1.05^{\frac{x}{2}}\right)^{2}$

17. $20\,(1.1^{x})$ $\quad$ $20\,(1.1^{-1})^{-1x}$ $\quad$ $20\left(1.1^{\frac{1}{5}}\right)^{5x}$

Go

Topic: Using rules of exponents

Simplify each expression. Your answer should still be in exponential form.

18. $7^3 \cdot 7^5 \cdot 7^2$

19. $(3^4)^5$

20. $(5^3)^4 \cdot 5^7$

21. $x^3 \cdot x^5$

22. x^{-b}

23. $x^a \cdot x^b$

24. $(x^a)^b$

25. $\frac{y^a}{y^b}$

26. $\frac{(y^a)^c}{y^b}$

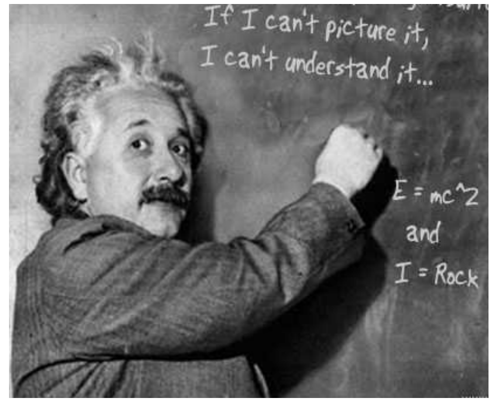
27. $\frac{(3^4)^6}{3^7}$

28. $\frac{r^5 s^3}{r s^2}$

29. $\frac{x^5 y^{12} z^0}{x^8 y^9}$

3.4 Radical Ideas

A Practice Understanding Task



Now that Tia and Tehani know that $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ they are wondering which form, radical form or exponential form, is best to use when working with numerical and algebraic expressions.

Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If n is a positive integer greater than 1 and both a and b are positive real numbers then,

1. $\sqrt[n]{a^n} = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
3. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Tehania says she prefers exponents since she understands the following properties for exponents (and there are more properities to work with):

1. $a^m \cdot a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
5. $\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
6. $a^{-n} = \dfrac{1}{a^n}$

DO THIS: Illustrate with examples and explain, using the properties of radicals and exponents, why $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ are true identities.

Using their preferred notation, Tia might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = x^{8/3} = x^{2+2/3} = x^2 \cdot x^{2/3} \text{ or } x^2 \cdot \sqrt[3]{x^2}$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original expression	What Tia and Tehani might do to simplify the expression:
$\sqrt{27}$	Tia's method
	Tehani's method
$\sqrt[3]{32}$	Tia's method
	Tehani's method
$\sqrt{20x^7}$	Tia's method
	Tehani's method
$\sqrt[3]{\dfrac{16xy^5}{x^7y^2}}$	Tia's method
	Tehani's method

Tia and Tehani continue to use their preferred notation when solving equations.

For example, Tia might solve the equation $(x+4)^3 = 27$ as follows:

$$(x+4)^3 = 27$$
$$\sqrt[3]{(x+4)^3} = \sqrt[3]{27} = \sqrt[3]{3^3}$$
$$x+4 = 3$$
$$x = -1$$

Tehani might solve the same equation as follows:

$$(x+4)^3 = 27$$
$$\left[(x+4)^3\right]^{1/3} = 27^{1/3} = (3^3)^{1/3}$$
$$x+4 = 3$$
$$x = -1$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original equation	**What Tia and Tehani might do to solve the equation:**
$(x-2)^2 = 50$	Tia's method
	Tehani's method
$9(x-3)^2 = 4$	Tia's method
	Tehani's method

Zac is showing off his new graphing calculator to Tia and Tehani. He is particularly excited about how his calculator will help him visualize the solutions to equations.

"Look," Zac says. "I treat the equation like a system of two equations. I set the expression on the left equal to y_1 and the expression of the right equal to y_2, and I know at the x value where the graphs intersect the expressions are equal to each other."

Zac shows off his new method on both of the equations Tia and Tehani solved using the properties of radicals and exponents. To everyone's surprise, both equations have a second solution.

1. Use Zac's graphical method to show that both of these equations have two solutions. Determine the exact values of the solutions you find on the calculator that Tia and Tehani did not find using their algebraic methods.

Tia and Tehani are surprised when they realize that both of these equations have more than one answer.

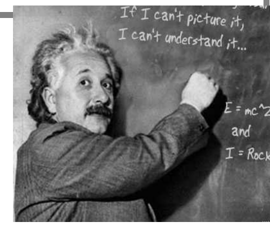
2. Explain why there is a second solution to each of these problems.

3. Modify Tia's and Tehani's algebraic approaches so they will find both solutions.

4. Use Zac's graphing calculator approach to solve the following problem.

   Carlos and Clarita deposited $300 in an account earning 5% interest. They want to take the money out of the account when it has doubled in value. To the nearest month, when can they withdraw their money?

Name: Solving Quadratic and Other Equations | 3.4



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Ready, Set, Go!

Ready

Topic: Standard form or Quadratic form

In each of the quadratic equations, $ax^2 + bx + c = 0$ **identify the values of a, b and c .**

1. $x^2 + 3x + 2 = 0$

   a =

   b =

   c =

2. $2x^2 + 3x + 1 = 0$

   a =

   b =

   c =

3. $x^2 - 4x - 12 = 0$

   a =

   b =

   c =

Write each of the quadratic expressions in factored form.

4. $x^2 + 3x + 2$
5. $2x^2 + 3x + 1$
6. $x^2 - 4x - 12$
7. $x^2 - 3x + 2$
8. $x^2 - 5x - 6$
9. $x^2 - 4x + 4$
10. $x^2 + 8x - 20$
11. $x^2 + x - 12$
12. $x^2 - 7x + 12$

Set

Topic: Radical notation and rational exponents

Each of the expressions below can be written using either radical notation, $\sqrt[n]{a^m}$ or rational exponents $a^{\frac{m}{n}}$. Rewrite each of the given expressions in the form that is missing. Express in most simplified form.

	Radical Form	Exponential Form
13.	$\sqrt[3]{5^2}$	
14.		$16^{\frac{3}{4}}$
15.	$\sqrt[3]{5^7 \cdot 3^5}$	
16.		$9^{\frac{2}{3}} \cdot 9^{\frac{4}{3}}$
17.	$\sqrt[5]{x^{13}y^{21}}$	
18.	$\sqrt[3]{27a^5b^2}$	
19.	$\sqrt[5]{\dfrac{32x^{13}}{243y^{15}}}$	
20.		$9^{\frac{3}{2}}s^{\frac{6}{3}}t^{\frac{1}{2}}$

Solve the equations below, use radicals or rational exponents as needed.

21. $(x+5)^4 = 81$

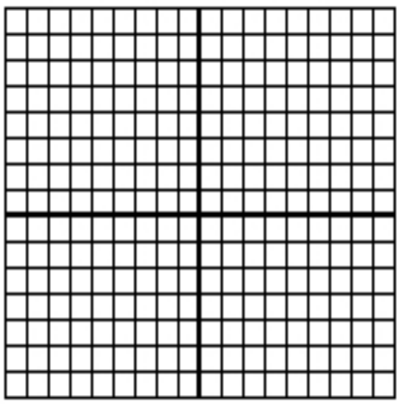
22. $2(x-7)^5 + 3 = 67$

Go

Topic: x-intercepts and y-intercepts for linear, exponential and quadratic

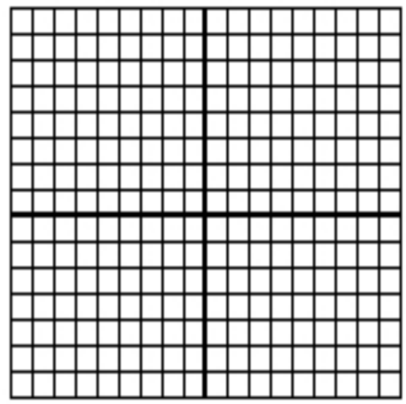
Given the function, find the x-intercept (s) and y-intercept if they exist and then use them to graph a sketch of the function.

23. $f(x) = (x+5)(x-4)$



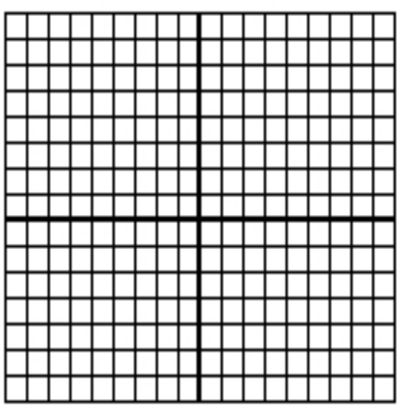
a. x-intercept(s): b. y-intercept:

24. $g(x) = 5(2^{x-1})$



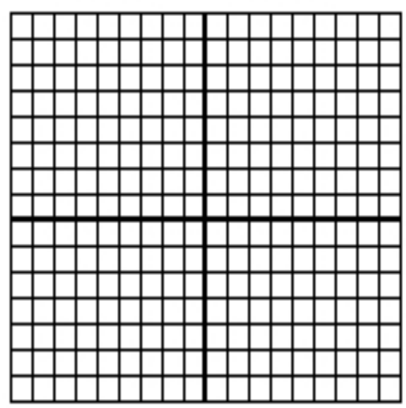
a. x-intercept(s): b. y-intercept:

25. $h(x) = -2(x+3)$



a. x-intercept(s): b. y-intercept:

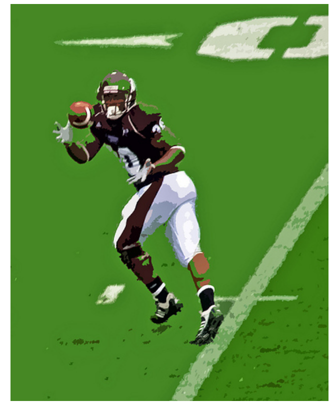
26. $k(x) = x^2 - 4$



a. x-intercept(s): b. y-intercept:

3.5 Throwing an Interception

A Develop Understanding Task



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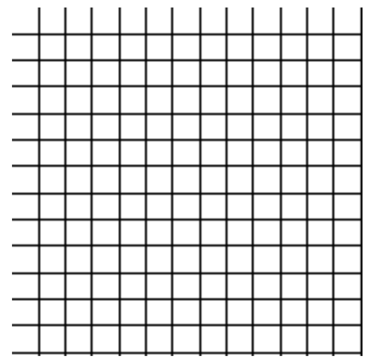
The x-intercept(s) of the graph of a function $f(x)$ are often very important because they are the solution to the equation $f(x)=0$. In past tasks, we learned how to find the x-intercepts of the function by factoring, which works great for some functions, but not for others. In this task we are going to work on a process to find the x-intercepts of any quadratic function that has them. We'll start by thinking about what we already know about a few specific quadratic functions and then use what we know to generalize to all quadratic functions with x-intercepts.

1. What can you say about the graph of the function $f(x)=x^2-2x-3$?
    a. Graph the function

    b. What is the equation of the line of symmetry?

    c. What is the vertex of the function?

2. Now let's think specifically about the x-intercepts.
    a. What are the x-intercepts of $f(x)=x^2-2x-3$?

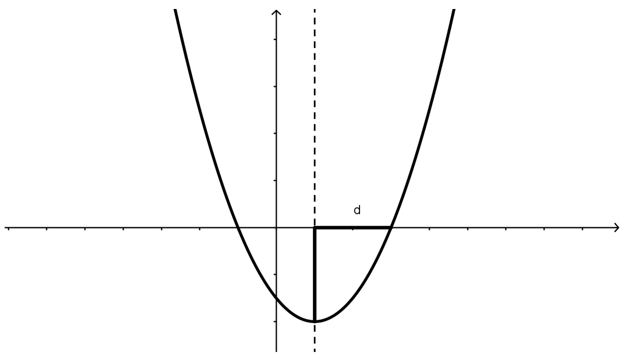
    b. How far are the x-intercepts from the line of symmetry?

    c. If you knew the line of symmetry was the line $x = h$, and you know how far the x-intercepts are from the line of symmetry, how would you find the actual x-intercepts?

    d. How far above the vertex are the x-intercepts?

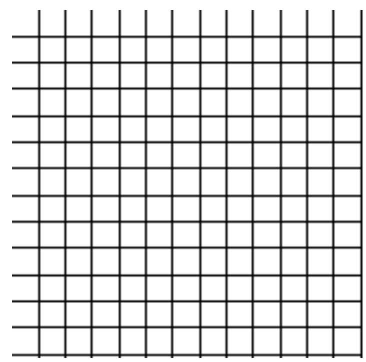
    e. What is the value of $f(x)$ at the x-intercepts?

Just to make it a little easier to talk about some of the features that relate to the intercepts, let's name them with variables. From now on, when we talk about the distance from the line of symmetry to either of the x intercepts, we'll call it *d*. The diagram below shows this feature.



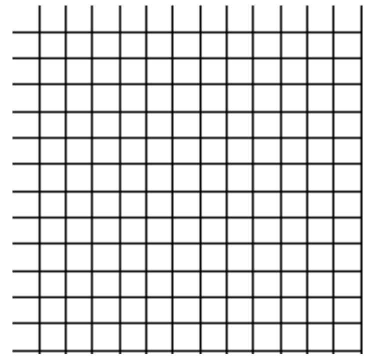
We will always refer to the line of symmetry as the line $x = h$, so the two x-intercepts will be at the points $(h - d, 0)$ and $(h + d, 0)$.

3. So, let's think about another function: $f(x) = x^2 - 6x + 4$
   a. Graph the function by putting the equation into vertex form.



   b. What is the vertex of the function?
   c. What is the equation of the line of symmetry?
   d. What do you estimate the x-intercepts of the function to be?
   e. What do you estimate d to be?
   f. What is the value of $f(x)$ at the x-intercepts?
   g. Using the vertex form of the equation and your answer to "f" above, write an equation and solve it to find the exact values of the x intercepts.
   h. What is the exact value of d?
   i. Use a calculator to find approximations for the x-intercepts. How do they compare with your estimates?

4. What about a function with a vertical stretch? Can we find exact values for the *x*-intercepts the same way? Let's try it with: $f(x) = 2x^2 - 8x + 5$.



a. Graph the function by putting the equation into vertex form.

b. What is the vertex of the function?

c. What is the equation of the line of symmetry?

d. What do you estimate the *x*-intercepts of the function to be?

e. What do you estimate *d* to be?

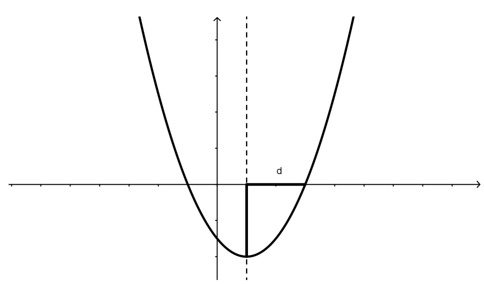
f. What is the value of $f(x)$ at the *x*-intercepts?

g. Using the vertex form of the equation and your answer to "f" above, write an equation and solve it to find the exact values of the *x*-intercepts.

h. What is the exact value of *d*?

i. Compare your solution to your estimate of the roots. How did you do?

5. Finally, let's try to generalize this process by using: $f(x) = ax^2 + bx + c$ to represent any quadratic function that has *x*-intercepts. Here's a possible graph of $f(x)$.



a. Start the process the usual way by putting the equation into vertex form. It's a little tricky, but just do the same thing with *a*, *b*, and *c* as what you did in the last problem with the numbers.

b. What is the vertex of the parabola?

c. What is the line of symmetry of the parabola?

d. Write and solve the equation for the x-intercepts just as you did previously.

6. How could you use the solutions you just found to tell what the x-intercepts are for the function $f(x) = x^2 - 3x - 1$?

7. You may have found the algebra for writing the general quadratic function $f(x) = ax^2 + bx + c$ in vertex form a bit difficult. Here is another way we can work with the general quadratic function leading to the same results you should have arrived at in 5d.

   a. Since the two x-intercepts are d units from the line of symmetry $x = h$, if the quadratic crosses the x-axis its x-intercepts are at $(h - d, 0)$ and $(h + d, 0)$. We can always write the factored form of a quadratic if we know its x-intercepts. The factored form will look like $f(x) = a(x - p)(x - q)$ where p and q are the two x-intercepts. So, using this information, write the factored form of the general quadratic $f(x) = ax^2 + bx + c$ using the fact that its x-intercepts are at h-d and h+d.

   b. Multiply out the factored form (you will be multiplying two **trinomial** expressions together) to get the quadratic in standard form. Simplify your result as much as possible by combining like terms.

c. You now have the same general quadratic function written in standard form in two different ways, one where the **coefficients** of the terms are a, b and c and one where the coefficients of the terms are expressions involving a, h and d. Match up the coefficients; that is, b, the coefficient of x in one version of the standard form is equivalent to ________ in the other version of the standard form. Likewise c, the constant term in one version of the standard form is equivalent to _________ in the other.

d. Solve the equations b = ________ and c = ________ for h and d. Work with your equations until you can express h and d with expressions that only involve a, b and c.

e. Based on this work, how can you find the x-intercepts of any quadratic using only the values for a, b and c?

f. How does your answer to "e" compare to your result in 5d?

8. All of the functions that we have worked with on this task have had graphs that open upward. Would the formula work for parabolas that open downward? Tell why or why not using an example that you create using your own values for the coefficients a, b, and c.

Name: Solving Quadratic and Other Equations | 3.5

Ready, Set, Go!

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Ready

Topic: Converting measurement of area, area and perimeter.

While working with areas it sometimes essential to convert between units of measure, for example changing from square yards to square feet and so forth. Convert the areas below to the desired measure. (Hint: area is two dimensional, for example 1 yd^2 = 9 ft^2 because 3 ft along each side of a square yard equals 9 square feet.)

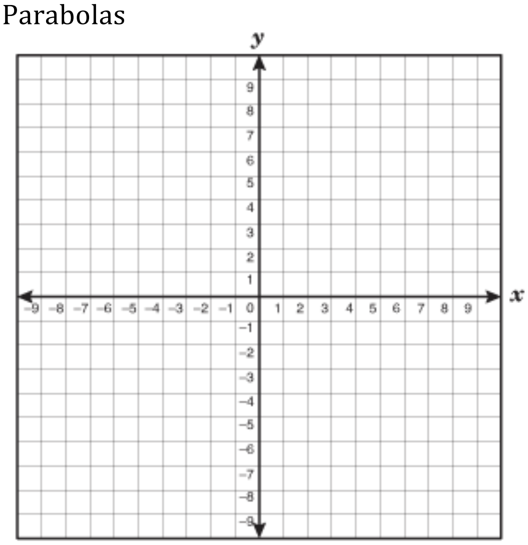
1. 7 yd^2 = ? ft^2
2. 5 ft^2 = ? in^2
3. 1 mile^2 = ? ft^2
4. 100 m^2 = ? cm^2
5. 300 ft^2 = ? yd^2
6. 96 in^2 = ? ft^2

Set

Topic: Transformations and Parabolas, Symmetry and Parabolas

7a. Graph each of the quadratic functions.

$$f(x) = x^2$$
$$g(x) = x^2 - 9$$
$$h(x) = (x+2)^2 - 9$$



b. How do the functions compare to each other?

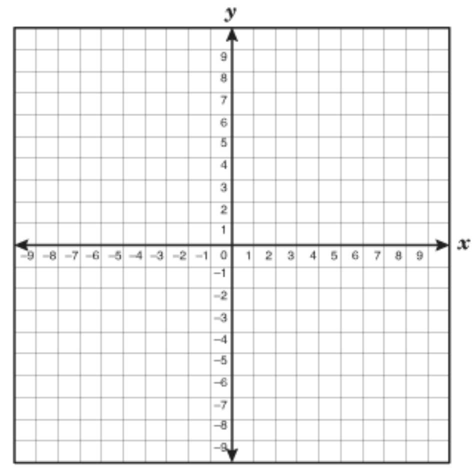
c. How do *g(x)* and *h(x)* compare to *f(x)*?

d. Look back at the functions above and identify the x-intercepts of *g(x)*. What are they?

e. What are the coordinates of the points corresponding to the x-intercepts in *g(x)* in each of the other functions? How do these coordinates compare to one another?

8a. Graph each of the quadratic functions.

$$f(x) = x^2$$
$$g(x) = x^2 - 4$$
$$h(x) = (x-1)^2 - 4$$



b. How do the functions compare to each other?

c. How do *g(x)* and *h(x)* compare to *f(x)*?

d. Look back at the functions above and identify the x-intercepts of *g(x).* What are they?

e. What are the coordinates of the points corresponding to the x-intercepts in *g(x)* in each of the other functions? How do these coordinates compare to one another?

9. How can the transformations that occur to the function $f(x) = x^2$ be used to determine where the x-intercepts of the function's image will be?

Go

Topic: Function Notation and Evaluating Functions

Use the given functions to find the missing values. (Check your work using a graph.)

10. $f(x) = x^2 + 4x - 12$

a. $f(0) =$ ______

b. $f(2) =$ ______

c. $f(x) = 0, \quad x =$ ______

d. $f(x) = 20, \quad x =$ ______

11. $g(x) = (x-5)^2 + 2$

a. $g(0) =$ ______

b. $g(5) =$ ______

c. $g(x) = 0, \quad x =$ ______

d. $g(x) = 16, \quad x =$ ______

12. $f(x) = x^2 - 6x + 9$

a. $f(0) =$ ______

b. $f(-3) =$ _____

c. $f(x) = 0$, $x =$ ______

d. $f(x) = 16$, $x =$ ______

13. $g(x) = (x - 2)^2 - 3$

a. $g(0) =$ _____

b. $g(5) =$ ______

c. $g(x) = 0$, $x =$ ______

d. $g(x) = -3$, $x =$ ______

14. $f(x) = (x + 5)^2$

a. $f(0) =$ ______

b. $f(-2) =$ _____

c. $f(x) = 0$, $x =$ ______

d. $f(x) = 9$, $x =$ ______

15. $g(x) = -(x + 1)^2 + 8$

a. $g(0) =$ _____

b. $g(2) =$ ______

c. $g(x) = 0$, $x =$ ______

d. $g(x) = 4$, $x =$ ______

3.6 Curbside Rivalry

A Solidify Understanding Task

Carlos and Clarita have a brilliant idea for how they will earn money this summer. Since the community in which they live includes many high schools, a couple of universities, and even some professional sports teams, it seems that everyone has a favorite team they like to root for. In Carlos' and Clarita's neighborhood these rivalries take on special meaning, since many of the neighbors support different teams. They have observed that their neighbors often display handmade posters and other items to make their support of their favorite team known. The twins believe they can get people in the neighborhood to buy into their new project: painting team logos on curbs or driveways.

For a small fee, Carlos and Clarita will paint the logo of a team on a neighbor's curb, next to their house number. For a larger fee, the twins will paint a mascot on the driveway. Carlos and Clarita have designed stencils to make the painting easier and they have priced the cost of supplies. They have also surveyed neighbors to get a sense of how many people in the community might be interested in purchasing their service. Here is what they have decided, based on their research.

- *A curbside logo will require 48 in^2 of paint*
- *A driveway mascot will require 16 ft^2 of paint*
- *Surveys show the twins can sell 100 driveway mascots at a cost of \$20, and they will sell 10 fewer mascots for each additional \$5 they charge*

1. If a curbside logo is designed in the shape of a square, what will its dimensions be?

A square logo will not fit nicely on a curb, so Carlos and Clarita are experimenting with different types of rectangles. They are using a software application that allows them to stretch or shrink their logo designs to fit different rectangular dimensions.

2. Carlos likes the look of the logo when the rectangle in which it fits is 8 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

3. Clarita prefers the look of the logo when the rectangle in which it fits is 13 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

Your quadratic equations on the previous two problems probably started out looking like this: $x(x+n)=48$ where *n* represents the number of inches the rectangle is longer than it is wide. The expression on the left of the equation could be multiplied out to get and equation of the form $x^2+nx=48$. If we subtract 48 from both sides of this equation we get $x^2+nx-48=0$. In this form, the expression on the left looks more like the quadratic functions you have been working with in previous tasks, $y=x^2+nx-48$.

4. Consider Carlos' quadratic equation where *n* = 8, so $x^2+8x-48=0$. How can we use our work with quadratic functions like $y=x^2+8x-48$ to help us solve the quadratic equation $x^2+8x-48=0$? Describe at least two different strategies you might use, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Carlos is trying to answer in #2.

5. Now consider Clarita's quadratic equation where *n* = 13, so $x^2+13x-48=0$. Describe at least two different strategies you might use to solve this equation, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Clarita is trying to answer in #3.

6. After much disagreement, Carlos and Clarita agree to design the curbside logo to fit in a rectangle that is 6 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write and solve a quadratic equation that represents these requirements.

7. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 6 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

8. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 10 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

Carlos and Clarita are also examining the results of their neighborhood survey, trying to determine how much they should charge for a driveway mascot. Remember, this is what they have found from the survey: *They can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.*

9. Write an equation, make a table, and sketch a graph for the number of driveway mascots the twins can sell for each $5 increment, *x*, in the price of the mascot.

10. Write an equation, make a table, and sketch a graph (on the same set of axes) for the price of the driveway mascot for each $5 increment, *x*, in the price.

11. Write an equation, make a table, and sketch a graph for the revenue the twins will collect for each $5 increment in the price of the mascot.

12. The twins estimate that the cost of supplies will be $250 and they would like to make $2000 in profit from selling driveway mascots. Therefore, they will need to collect $2250 in revenue. Write and solve a quadratic equation that represents collecting $2250 in revenue. Be sure to clearly show your strategy for solving this quadratic equation.

Ready, Set, Go!

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Ready

Topic: Finding x-intercepts for linear equations.

1. Find the x-intercept of each equation below. Write your answer as an ordered pair. Consider how the format of the given equation either facilitates for inhibits your work.

a. $3x + 4y = 12$ b. $y = 5x - 3$ c. $y - 5 = -4(x + 1)$

d. $y = -4x + 1$ e. $y - 6 = 2(x + 7)$ f. $5x - 2y = 10$

2. Which of the linear equation formats above facilitates your work in finding x-intercepts? Why?

3. Using the same equations from question 1, find the y-intercepts. Write your answers as ordered pairs

a. $3x + 4y = 12$ b. $y = 5x - 3$ c. $y - 5 = -4(x + 1)$

d. $y = -4x + 1$ e. $y - 6 = 2(x + 7)$ f. $5x - 2y = 10$

4. Which of the formats above facilitate finding the y-intercept? Why?

Set

Topic: Solve Quadratic Equations, Connecting Quadratics with Area

For each of the given quadratic equations, (a) describe the rectangle the equation fits with. (b) What constraints have been placed on the dimensions of the rectangle?

5. $x^2 + 7x - 170 = 0$

6. $x^2 + 15x - 16 = 0$

7. $x^2 + 2x - 35 = 0$

8. $x^2 + 10x - 80 = 0$

Solve the quadratic equations below.

9. $x^2 + 7x - 170 = 0$

10. $x^2 + 15x - 16 = 0$

11. $x^2 + 2x - 35 = 0$

12. $x^2 + 10x - 80 = 0$

Go

Topic: Factoring Expressions

Write each of the expressions below in factored form.

13. $x^2 - x - 132$

14. $x^2 - 5x - 36$

15. $x^2 + 5x + 6$

16. $x^2 + 13x + 42$

17. $x^2 + x - 56$

18. $x^2 - x$

19. $x^2 - 8x + 12$

20. $x^2 - 10x + 25$

21. $x^2 + 5x$

Need Assistance? Check out these additional resources:

https://www.khanacademy.org/math/trigonometry/polynomial_and_rational/quad_factoring/v/factoring-quadratic-expressions

3.7 Perfecting My Quads

A Practice Understanding Task

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Carlos and Clarita, Tia and Tehani, and their best friend Zac are all discussing their favorite methods for solving quadratic equations of the form $ax^2 + bx + c = 0$. Each student thinks about the related quadratic function $y = ax^2 + bx + c$ as part of his or her strategy.

Carlos: "I like to make a table of values for *x* and find the solutions by inspecting the table."

Clarita: "I like to write the equation in factored form, and then use the factors to find the solutions."

Tia: "I like to treat it like a quadratic function that I am trying to put in vertex form by completing the square. I can then use a square root to undo the squared expression."

Tehani: "I also like to treat it like a quadratic function, but I use the quadratic formula to find the solutions."

Zac: "I like to graph the related quadratic function and use my graph to find the solutions."

Demonstrate how each student might solve each of the following quadratic equations.

Solve: $x^2 - 2x - 15 = 0$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Solve: $2x^2 + 5x - 12 = 0$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Solve: $x^2 + 4x - 8 = 0$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Solve: $8x^2 + 2x = 3$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Describe why each strategy works.

As the students continue to try out their strategies, they notice that sometimes one strategy works better than another. Explain how you would decide when to use each strategy.

Here is an extra challenge. How might each student solve the following system of equations?

Solve the system: $y_1 = x^2 - 4x + 1$ $y_2 = x - 3$	Carlos' Strategy	Zac's Strategy
Clarita's Strategy	Tia's Strategy	Tehani's Strategy

Ready, Set, Go!

Ready

Topic: Symmetry and Distance

The given functions provide the connection between possible areas, *A(x)* , that can be created by a rectangle for a given side length, *x*, and a set amount of perimeter. You could think of it as the different amounts of area you can close in with a given amount of fencing as long as you always create a rectangular enclosure.

1. $A(x) = x\,(10 - x\,)$

Find the following:

a. A(3) = b. A(4) =

c. A(6) = d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

2. $A(x) = x\,(50 - x)$

Find the following:

a. A(10) = b. A(20) =

c. A(30) = d. A(x) = 0

e. When is A(x) at its maximum? Explain or show how you know.

3. $A(x) = x(\,75 - x)$

Find the following:

a. $A(20) =$ *b.* $A(35) =$

c. $A(40) =$ *d.* $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

4. $A(x) = x(\,48 - x)$

Find the following:

a. A(10) = b. A(20) =

c. A(28) = d. A(x) = 0

e. When is A(x) at its maximum? Explain or show how you know.

Set

Topic: Solving Quadratic Equations Efficiently

For each of the given quadratic equations find the solutions using an efficient method. State the method you are using as well as the solutions. You must use at least three different methods.

5. $x^2 + 17x + 60 = 0$

6. $x^2 + 16x + 39 = 0$

7. $x^2 + 7x - 5 = 0$

8. $3x^2 + 14x - 5 = 0$

9. $x^2 - 12x = -8$

10. $x^2 + 6x = 7$

Summarize the process for solving a quadratic by the indicated strategy. Give examples along with written explanation, also indicate when it is best to use this strategy.

11. Completing the Square

12. Factoring

13. Quadratic Formula

Go

Topic: Graphing quadratics and finding essential features of the graph. Solving systems of equations.

Graph the quadratic function and supply the desired information about the graph.

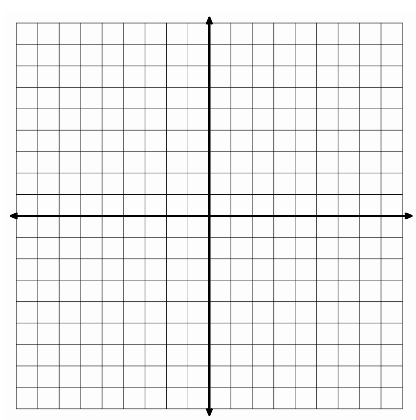
14. $f(x) = x^2 + 8x + 13$

a. Line of symmetry:

b. x-intercepts:

c. y-intercept:

d. vertex:



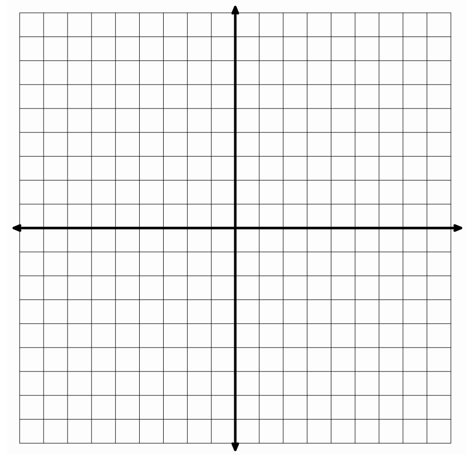
15. $f(x) = x^2 - 4x - 1$

a. Line of symmetry:

b. x-intercepts:

c. y-intercept:

d. vertex:



Solve each system of equations using an algebraic method and check your work!

16.
$$\begin{cases} 3x + 5y = 15 \\ 3x - 2y = 6 \end{cases}$$

17.
$$\begin{cases} y = -7x + 12 \\ y = 5x - 36 \end{cases}$$

18.
$$\begin{cases} y = 2x + 12 \\ y = 10x - x^2 \end{cases}$$

19.
$$\begin{cases} y = 24x - x^2 \\ y = 8x + 48 \end{cases}$$

3.8 To Be Determined . . .

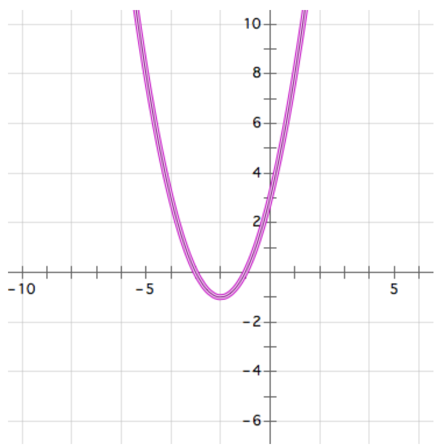
A Develop Understanding Task

Israel and Miriam are working together on a homework assignment. They need to write the equations of quadratic functions from the information given in a table or a graph. At first, this work seemed really easy. However, as they continued to work on the assignment, the algebra got more challenging and raised some interesting questions that they can't wait to ask their teacher.

Work through the following problems from Israel and Miriam's homework. Use the information in the table or the graph to write the equation of the quadratic function in all three forms. You may start with any form you choose, but you need to find all three equivalent forms. (If you get stuck, your teacher has some hints from Israel and Miriam that might help you.)

1.

x	y
-5	8
-4	3
-3	0
-2	-1
-1	0
0	3
1	8
2	15
3	24
4	35
5	48



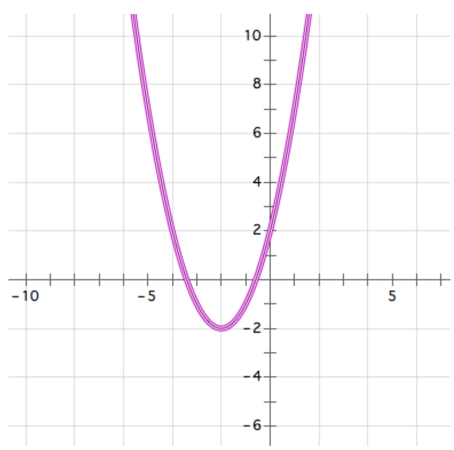
Standard form:

Factored form:

Vertex form:

2.

x	y
-5	7
-4	2
-3	-1
-2	-2
-1	-1
0	2
1	7
2	14
3	23
4	34
5	47



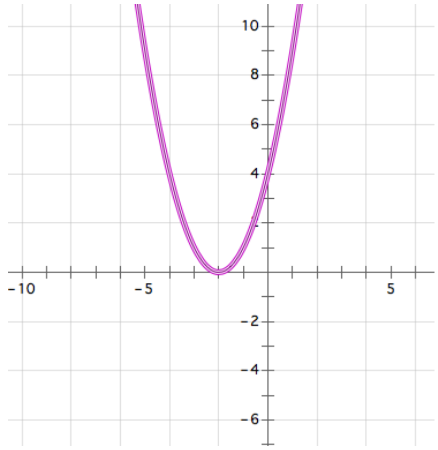
Standard form:

Factored form:

Vertex form:

3.

x	y
-5	9
-4	4
-3	1
-2	0
-1	1
0	4
1	9
2	16
3	25
4	36
5	49



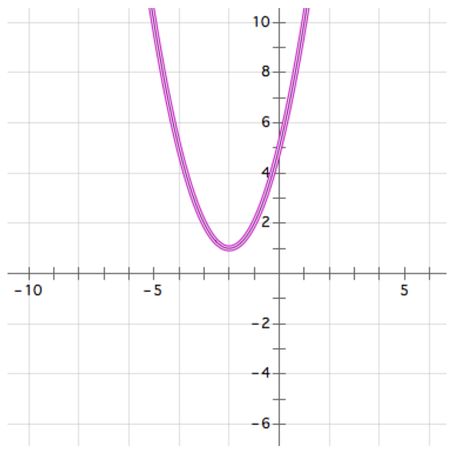
Standard form:

Factored form:

Vertex form:

4.

x	y
-5	10
-4	5
-3	2
-2	1
-1	2
0	5
1	10
2	17
3	26
4	37
5	50



Standard form:

Factored form:

Vertex form:

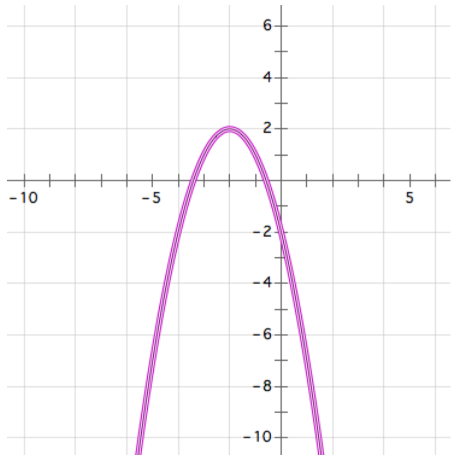
5. Israel was concerned that his factored form for the function in question 4 didn't look quite right. Miriam suggested that he test it out by plugging in some values for *x* to see if he gets the same points as those in the table. Test *your* factored form. Do you get the same values as those in the table?

6. Why might Israel be concerned about writing the factored form of the function in question 4?

Here are some more of Israel and Miriam's homework.

7.

x	y
-5	-7
-4	-2
-3	1
-2	2
-1	1
0	-2
1	-7
2	-14
3	-23
4	-34
5	-47



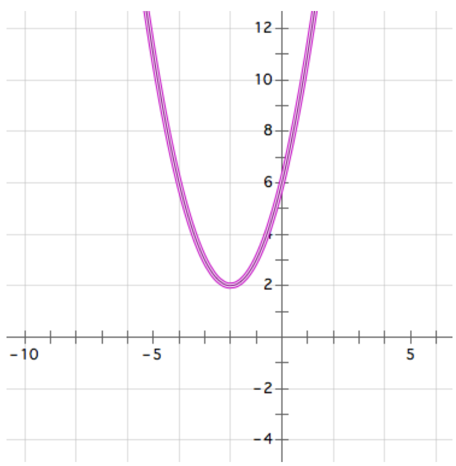
Standard form:

Factored form:

Vertex form:

8.

x	y
-5	11
-4	6
-3	3
-2	2
-1	3
0	6
1	11
2	18
3	27
4	38
5	51



Standard form:

Factored form:

Vertex form:

9. Miriam notices that the graphs of function 7 and function 8 have the same vertex point. Israel notices that the graphs of function 2 and function 7 are mirror images across the *x*-axis. What do you notice about the roots of these three quadratic functions?

<u>The Fundamental Theorem of Algebra</u>

A polynomial function is a function of the form:

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An nth degree polynomial function has n roots*.

In later math classes you will study polynomial functions that contain higher-ordered terms such as x^3 or x^5. Based on you work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots? [Examine the graphs of each of the quadratic functions you have written equations for in this task. Do they all have two roots? Why or why not?]

Name: Solving Quadratic and Other Equations | 3.8

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Ready, Set, Go!

Ready

Topic: Simplifying radicals

Simplify each of the radicals below.

1. $\sqrt{8}$ 2. $\sqrt{18}$ 3. $\sqrt{32}$

4. $\sqrt{20}$ 5. $\sqrt{45}$ 6. $\sqrt{80}$

7. What is the connection between the radicals above? Explain.

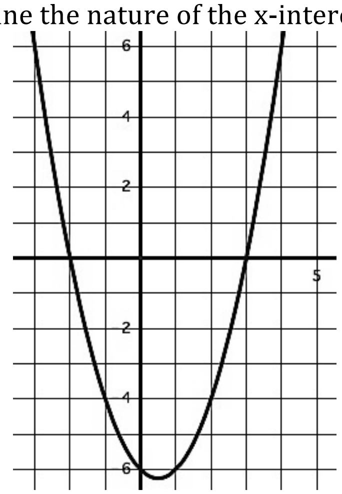
Set

Topic: Determine the nature of the x-intercepts for each quadratic below.

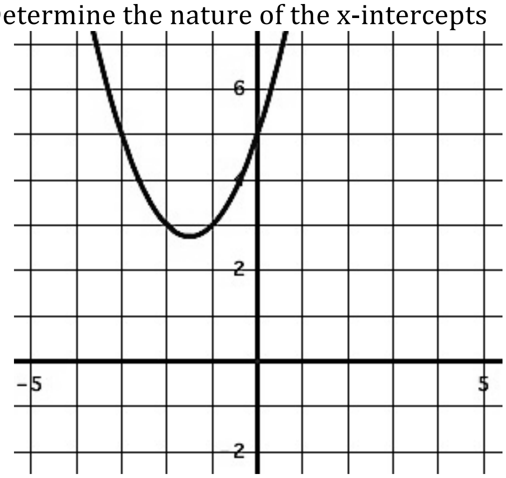
Given the quadratic function, its graph or other information below determine the nature of the x-intercepts (what type of number it is). Explain or show how you know.

(Whole numbers "$\mathbb{W}$", Integers "$\mathbb{Z}$", Rational "$\mathbb{Q}$", Irrational "$\overline{\mathbb{Q}}$", or finally, "not Real")

8. Determine the nature of the x-intercepts.



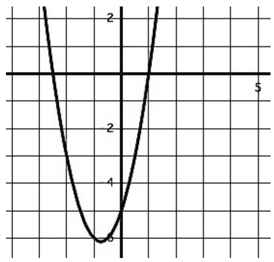
9. Determine the nature of the x-intercepts



10. Determine the nature of the x-intercepts.
$f(x) = x^2 + 4x - 24$

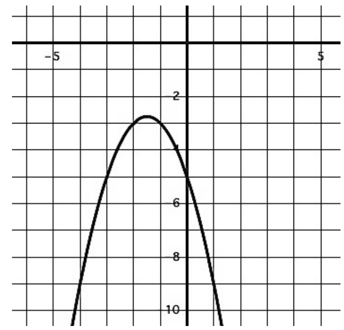
11. Determine the nature of the x-intercepts.
$g(x) = (2x - 1)(5x + 2)$

12. Determine the nature of the x-intercepts.



$f(x) = 2x^2 + 3x - 5$

13. Determine the nature of the x-intercepts.



14. Determine the nature of the x-intercepts.
$r(t) = t^2 - 8t + 16$

15. Determine the nature of the x-intercepts.
$h(x) = 3x^2 - 5x + 9$

Determine the number of roots that each polynomial will have.

16. $x^5 + 7x^3 - x^2 + 4x - 21$ 17. $4x^3 + 2x^2 - 3x - 9$ 18. $2x^7 + 4x^5 - 5x^2 + 16x + 3$

Go

Topic: Finding x-intercepts for quadratics using factoring and quadratic formula.

If the given quadratic function can be factored then factor and provide the x-intercepts. If you cannot factor the function then use the quadratic formula to find the x-intercepts.

19. $A(x) = x^2 + 4x - 21$ 20. $B(x) = 5x^2 + 16x + 3$ 21. $C(x) = x^2 - 4x + 1$

22. $D(x) = x^2 - 16x + 4$ 23. $E(x) = x^2 + 3x - 40$ 24. $F(x) = 2x^2 - 3x - 9$

25. $G(x) = x^2 - 3x$ 26. $H(x) = x^2 + 6x + 8$ 27. $K(x) = 3x^2 - 11$

Need Assistance? Check out these additional resources:

https://www.khanacademy.org/math/algebra/quadratics/quadratic_formula/v/quadratic-formula-1

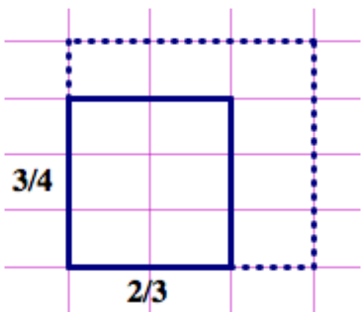
3.9 My Irrational and Imaginary Friends

A Solidify Understanding Task

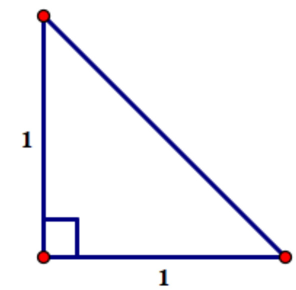
Part 1: Irrational numbers

Find the perimeter of each of the following figures. Express your answer as simply as possible.

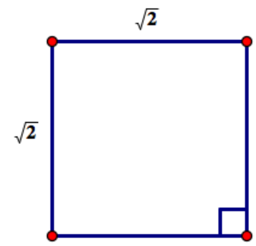
1. The 3/4 x 2/3 rectangle



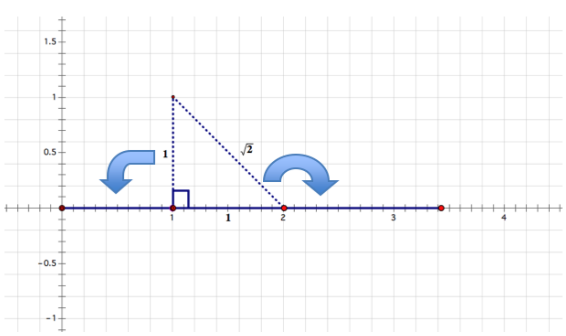
2. The isosceles right triangle



3. The √2 x √2 square



4. We might approximate the perimeter of figure 2 with a decimal number, but the exact perimeter is $2+\sqrt{2}$, which cannot be simplified any farther. Note that this notation represents a single number—the distance around the perimeter of the triangle—even though it is written as the sum of two terms. We could visualize this single number by laying the three sides of the triangle end-to-end along a number line, starting at 0, so the endpoint of the last segment would be at the number $2+\sqrt{2}$. Is the number we have located on the number line in this way a rational number or an irrational number? Explain your answer.



5. Why can we express the perimeter of figure 3 with a single term, even though the side lengths are irrational?

6. Find the area of each of the figures in 1-3.

7. Draw a representative image and find the area of the following figures:

(a) a square with sides $2+\sqrt{2}$

(b) a rectangle with sides $3+\sqrt{2}$ and $2+\sqrt{5}$

(c) a rectangle with sides $3+\sqrt{2}$ and $2+\sqrt{8}$

(d) a rectangle with sides $\sqrt{2}+\sqrt[3]{2}$ and $\sqrt{6}+\sqrt[3]{4}$

8. Are the areas of the figures in 7a, 7b, 7c and 7d rational or irrational? How do you know?

Note: The set of numbers that contains all of the *rational numbers* and all of the *irrational numbers* is called the set of *real numbers*.

Part 2: Imaginary and Complex Numbers

In the previous task, you found that the quadratic formula gives the roots of x^2+4x+5 as $-2+\sqrt{-1}$ and $-2-\sqrt{-1}$. Because the square root of a negative number has no defined value as either a rational or an irrational number, Euler proposed that a new number $i=\sqrt{-1}$ be including in what came to be known as the complex number system.

9. Based on Euler's definition of *i*, what would the value of i^2 be?

With the introduction of the number *i*, the square root of *any* negative number can be represented. For example, $\sqrt{-2}=\sqrt{2}\cdot\sqrt{-1}=\sqrt{2}\cdot i$ and $\sqrt{-9}=\sqrt{9}\cdot\sqrt{-1}=3i$.

10. Find the values of the following expressions. Show the details of your work.

(a) $\left(\sqrt{2}\cdot i\right)^2$

(b) $3i\times 3i$

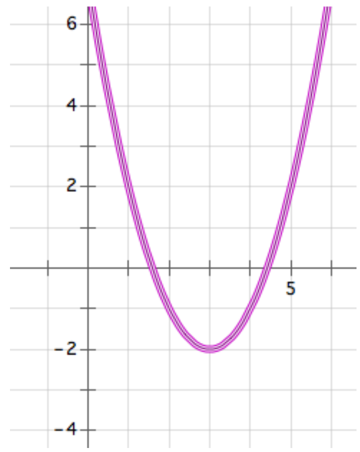
Using this new notation, the roots of x^2+4x+5 can be written as $-2+i$ and $-2-i$, and the factored form of x^2+4x+5 can be written as $(x+2-i)(x+2+i)$.

11. Verify that x^2+4x+5 and $(x+2-i)(x+2+i)$ are equivalent by expanding and simplifying the factored form. Show the details of your work.

Note: Numbers like 3*i* and $\sqrt{2}\cdot i$ are called *pure imaginary numbers*. Numbers like -2 – *i* and -2 + *i* that include a real term and an imaginary term are called *complex numbers*.

The quadratic formula is usually written in the form $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. An equivalent form is $\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$. If *a*, *b* and *c* are rational coefficients, then $\frac{-b}{2a}$ is a rational term, and $\frac{\sqrt{b^2-4ac}}{2a}$ may be a rational term, an irrational term or an imaginary term, depending on the value of the expression under the square root sign.

12. Examine the roots of the quadratic $y = x^2 - 6x + 7$ shown in the graph at the right. How do the terms $\frac{-b}{2a}$ and $\frac{\sqrt{b^2-4ac}}{2a}$ show up in this graph?

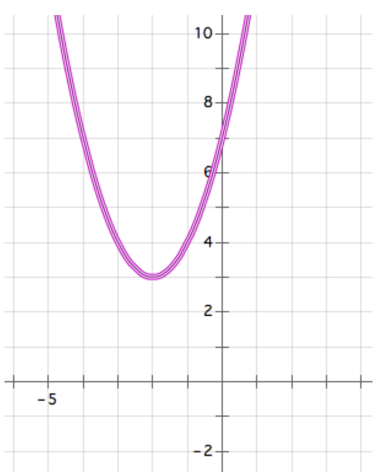


Look back at the work you did in the task *To Be Determined . . .*

13. Which quadratics in that task had complex roots?

14. How can you determine if a quadratic has complex roots from its graph?

15. Find the complex roots of the following quadratic function represented by its graph.



Note: Complex numbers are not real numbers—they do not lie on the real number line that includes all of the rational and irrational numbers; also note that the real numbers are a subset of the complex numbers since a real number results when the imaginary part of *a* + *bi* is 0, that is, *a* + 0*i*.

The Fundamental Theorem of Algebra, Revisited

Remember the following information given in the previous task:

A polynomial function is a function of the form:

$$y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

As the theory of finding roots of polynomial functions evolved, a 17th century mathematician, Girard (1595-1632) made the following claim which has come to be known as the Fundamental Theorem of Algebra: *An nth degree polynomial function has n roots*.

Based on you work in this task, do you believe this theorem holds for quadratic functions? That is, do all functions of the form $y = ax^2 + bx + c$ always have two roots?

Name: Solving Quadratic and Other Equations | 3.9

Ready, Set, Go!

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Ready

Topic: Classifying numbers according to set.

Classify each of the numbers represented below according to the sets to which they belong. If a number fits in more than one set then list all that apply.
(Whole numbers "$\mathbb{W}$", Integers "$\mathbb{Z}$", Rational "$\mathbb{Q}$", Irrational "$\overline{\mathbb{Q}}$", Real "$\mathbb{R}$", Complex "$\mathbb{C}$")

1. π
2. -13
3. $\sqrt{-16}$
4. 0
5. $\sqrt{75}$
6. $\frac{9}{3}$
7. $\sqrt{\frac{4}{9}}$
8. $5+\sqrt{2}$
9. $\sqrt{-40}$

Set

Topic: Simplifying radicals, imaginary numbers

Simplify each radical expression below.

10. $3 + \sqrt{2} - 7 + 3\sqrt{2}$
11. $\sqrt{5} - 9 + 8\sqrt{5} + 11 - \sqrt{5}$
12. $\sqrt{12} + \sqrt{48}$
13. $\sqrt{8} - \sqrt{18} + \sqrt{32}$
14. $11\sqrt{7} - 5\sqrt{7}$
15. $7\sqrt{7} + 5\sqrt{3} - 3\sqrt{7} + \sqrt{3}$

Simplify. Express as a complex number using "*i*" if necessary.

16. $\sqrt{-2}\cdot\sqrt{-2}$
17. $7+\sqrt{-25}$
18. $(4i)^2$
19. $i^2 \cdot i^3 \cdot i^4$
20. $\left(\sqrt{-4}\right)^3$
21. $(2i)(5i)^2$

Solve each quadratic equation over the set of complex numbers.

22. $x^2 + 100 = 0$

23. $t^2 + 24 = 0$

24. $x^2 - 6x + 13 = 0$

25. $r^2 - 2r + 5 = 0$

Go

Topic: Solve quadratic equations.

Use the discriminant to determine the nature of the roots to the quadratic equation.

26. $x^2 - 5x + 7 = 0$

27. $x^2 - 5x + 6 = 0$

28. $2x^2 - 5x + 5 = 0$

29. $x^2 + 7x + 2 = 0$

30. $2x^2 + 7x + 6 = 0$

31. $2x^2 + 7x + 7 = 0$

32. $2x^2 - 7x + 6 = 0$

33. $2x^2 + 7x - 6 = 0$

34. $x^2 + 6x + 9 = 0$

Solve the quadratic equations below using an appropriate method.

35. $m^2 + 15m + 56 = 0$

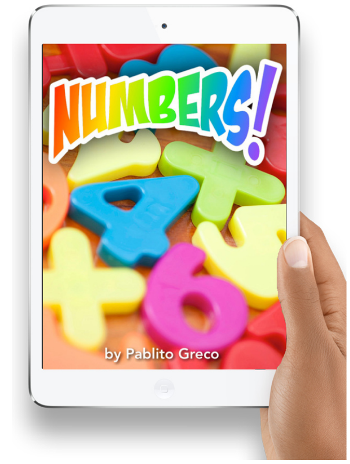
36. $5x^2 - 3x + 7 = 0$

37. $x^2 - 10x + 21 = 0$

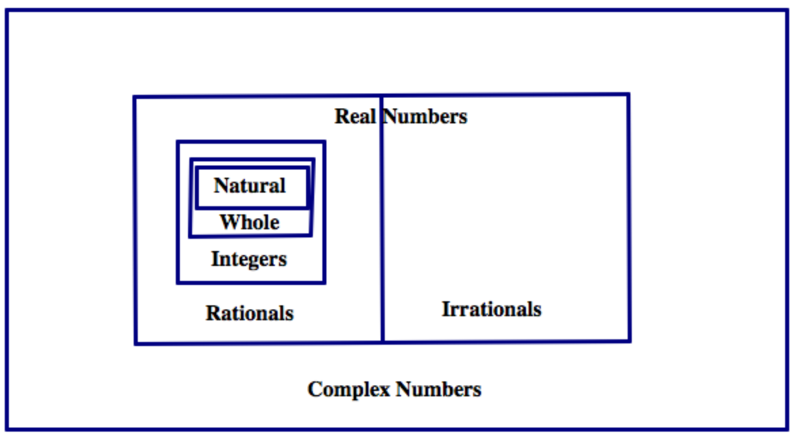
38. $6x^2 + 7x - 5 = 0$

3.10 iNumbers

A Practice Understanding Task



In order to find solutions to all quadratic equations, we have had to extend the number system to include complex numbers.



Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #1: The sum of two integers is [always, sometime, never] an integer.

Conjecture #2: The sum of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #3: The sum of two irrational numbers is [always, sometimes, never] an irrational number.

Conjecture #4: The sum of two real numbers is [always, sometimes, never] a real number.

Conjecture #5: The sum of two complex numbers is [always, sometimes, never] a complex number.

Conjecture #6: The product of two integers is [always, sometime, never] an integer.

Conjecture #7: The quotient of two integers is [always, sometime, never] an integer.

Conjecture #8: The product of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #9: The quotient of two rational numbers is [always, sometimes, never] a rational number

Conjecture #10: The product of two irrational numbers is [always, sometimes, never] an irrational number.

Conjecture #11: The product of two real numbers is [always, sometimes, never] a real number.

Conjecture #12: The product of two complex numbers is [always, sometimes, never] a complex number.

13. The ratio of the circumference of a circle to its diameter is given by the irrational number π. Can the diameter of a circle and the circumference of the same circle both be rational numbers? Explain why or why not.

The Arithmetic of Polynomials

In the task *To Be Determined . . .* we defined polynomials to be expressions of the following form:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

Do the following for each of the problems below:

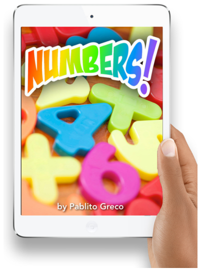
- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #P1: The sum of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P2: The difference of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P3: The product of two polynomials is [always, sometime, never] a polynomial.

Ready, Set, Go!



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Ready

Topic: Attributes of quadratics and other functions

1. Summarize what you have learned about quadratic functions to this point. In addition to your written explanation provide graphs, tables and examples to illustrate what you know.

2. In prior work you have learned a great deal about both linear and exponential functions. Compare and contrast linear and exponential functions with quadratic functions. What similarities if any are there and what differences are there between linear, exponential and quadratic functions?

Set

Topic: Operations on different types of numbers

3. The Natural numbers, $\mathbb{N}$, are just that the numbers that come naturally or the counting numbers. As any child first learns numbers they learn 1, 2, 3, … What operations on the Natural numbers would cause the need for other types of numbers? What operation on Natural numbers create a need for Integers or Rational numbers and so forth. (Give examples and explain.)

In each of the problems below use the given items to determine whether or not it is possible *always*, *sometimes* or *never* to create a new element* **that is in the desired set.**

4. Using the operation of addition and elements from the Integers, $\mathbb{Z}$, [always, sometime, never] an element of the Irrational numbers, $\overline{\mathbb{Q}}$, will be created. Explain.

5. Consider the equation $a - b = c$, where $a \in \mathbb{N}$ and $b \in \mathbb{N}$, c will be an Integer, $\mathbb{Z}$ [always, sometimes, never]. Explain.

6. Consider the equation $a \div b = c$, where $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, then is $c \in \mathbb{Z}$ [sometimes, always, never]. Explain.

**The numbers in any given set of numbers may be referred to as elements of the set. For example, the Rational number set, $\mathbb{Q}$, contains elements or numbers that can be written in the form $\frac{a}{b}$, where a and b are integer values (b≠0).*

7. Using the operation of subtraction and elements from the Irrationals, $\overline{\mathbb{Q}}$, an element of the Irrational numbers, $\overline{\mathbb{Q}}$, will be created [always, sometime, never]. Explain.

8. If two Complex numbers, $\mathbb{C}$, are subtracted the result will [always, sometimes, never] be a Complex number, $\mathbb{C}$. Explain.

Go

Topic: Solving all types of Quadratic Equations, Simplifying Radicals

Make a prediction as to the nature **of the solutions for each quadratic (Real, Complex, Integer, etc.) then solve each of the quadratic equations below using an appropriate and efficient method. Give the solutions and compare to your prediction.**

9. $-5x^2 + 3x + 2 = 0$

Prediction:

Solutions:

10. $x^2 + 3x + 2 = 0$

Prediction:

Solutions:

11. $x^2 + 3x - 12 = 0$

Prediction:

Solutions:

12. $4x^2 - 19x - 5 = 0$

Prediction:

Solutions:

Simplify each of the radical expressions. Use rational exponents if desired.

13. $\sqrt[4]{81x^8y^{12}}$

14. $\sqrt{\frac{a^7b^{10}}{a^3}}$

15. $\sqrt[5]{625x^{12}}$

16. $\left(\sqrt{n}\right)^5$

17. $\sqrt[3]{-27}$

18. $(\sqrt{8})(\sqrt{3^2})(2)$

Fill in the table so each expression is written in radical form and with rational exponents.

	Radical Form	Exponential Form
19.	$\sqrt[4]{8^3}$	
20.		$256^{\frac{3}{4}}$
21.	$\sqrt[4]{2^7 \bullet 4^5}$	
22.		$16^{\frac{3}{2}} \bullet 4^{\frac{1}{2}}$
23.	$\sqrt[10]{x^{23}y^{31}}$	
24.	$\sqrt[5]{64a^9b^{18}}$	

3.11H Quadratic Quandaries

A Develop and Solidify Understanding Task

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In the task *Curbside Rivalry* Carlos and Clarita were trying to decide how much they should charge for a driveway mascot. Here are the important details of what they had to consider.

- *Surveys show the twins can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.*
- *The twins estimate that the cost of supplies will be $250 and they would like to make $2000 in profit from selling driveway mascots. Therefore, they will need to collect $2250 in revenue.*

This information led Carlos and Clarita to write and solve the quadratic equation $(100-10x)(20+5x)=2250$.

1. Either review your work from *Curbside Rivalry* or solve this quadratic equation for *x* again.

2. What do your solutions for *x* mean in terms of the story context?

3. How would your solution change if this had been the question Carlos and Clarita had asked: "How much should we charge if we want to collect at least $2250 in revenue?"

4. What about this question: "How much should we charge if we want to maximize our revenue?"

As you probably observed, the situation represented in question 3 didn't have a single solution, since there are many different prices the twins can charge to collect more than $2250 in revenue. Sometimes our questions lead to quadratic inequalities rather than quadratic equations.

Here is another quadratic inequality based on your work on *Curbside Rivalry.*

5. Carlos and Clarita want to design a logo that requires less than 48 in^2 of paint, and fits inside a rectangle that is 8 inches longer than it is wide. What are the possible dimensions of the rectangular logo?

Again question 5 has multiple answers, and those answers are restricted by the context. Let's examine the inequality you wrote for question 5, but not restricted by the context.

6. What are the solutions to the inequality $x(x+8)<48$?

7. How might you support your answer to question 6 with a graph or a table?

Here are some more quadratic inequalities without contexts. Show how you might use a graph, along with algebra, to solve each of them.

8. $x^2+3x-10\geq 0$

9. $2x^2-5x<12$

10. $x^2 - 4 \leq 4x + 1$

Carlos and Clarita both used algebra and a graph to solve question 10, but they both did so in different ways. Illustrate each of their methods with a graph and with algebra.

11. Carlos: "I rewrote the inequality to get 0 on one side and a factored form on the other. I found the zeroes for each of my factors. To decide what values of *x* made sense in the inequality I also sketched a graph of the quadratic function that was related to the quadratic expression in my inequality. I shaded solutions for *x* based on the information from my graph."

12. Clarita: "I graphed a linear function and a quadratic function related to the linear and quadratic expressions in the inequality. From the graph I could estimate the points of intersection, but to be more exact I solved the quadratic equation $x^2 - 4 = 4x + 1$ by written an equivalent equation that had 0 on one side. Once I knew the *x*-values for the points of intersection in the graph, I could shade solutions for *x* that made the inequality true."

Carlos and Clarita have decided to create 3-D mascots out of clay for their customers who want them. They want the mascot to fit within a rectangular box with a volume that is no more than 96 in^3 and whose width is 2 inches shorter than its length, and whose height is 8 inches more than its length.

Carlos writes this inequality to represent the box's description: $x(x-2)(x+8) \le 96$

With the help of his cousin who is in advanced mathematics he is able to rewrite this inequality in an equivalent factored form that has 0 on one side of the inequality: $(x-4)(x+4)(x+8) \le 0$

Because Carlos doesn't know how to graph cubic polynomials any better than he can factor them, he is wondering how his work with quadratic inequalities might help his solve this cubic inequality.

13. Devise a strategy based on your work with quadratic inequalities that could be used to solve this cubic inequality with three factors: $(x-4)(x+4)(x+8) \le 0$

14. Use the solutions to this cubic inequality to determine the dimensions of rectangular boxes that meet their criteria.

15. Here is the algebra work produced by Carlos' cousin. Explain each step in the process that led from Carlos' inequality to his cousin's.

$$x(x-2)(x+8) \le 96$$
$$x(x^2+6x-16) \le 96$$
$$x^3+6x^2-16x \le 96$$
$$x^3+6x^2-16x-96 \le 0$$
$$x^2(x+6)-16(x+6) \le 0$$
$$(x^2-16)(x+6) \le 0$$
$$(x-4)(x+4)(x+6) \le 0$$

Name: Solving Quadratic and Other Equations | 3.11H

Ready, Set, Go!


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Ready

Topic: Factoring polynomials

Factor each of the polynomials completely.

1. $x^2 + x - 12$
2. $x^2 - 2x - 8$
3. $x^2 + 5x - 14$
4. $x^2 - x - 6$
5. $x^2 + 6x + 9$
6. $x^2 - 7x + 10$
7. $2x^2 - 9x - 5$
8. $3x^2 - 3x - 18$
9. $2x^2 + 8x - 42$

10. How is the factored form of a quadratic helpful when graphing the parabola?

Set

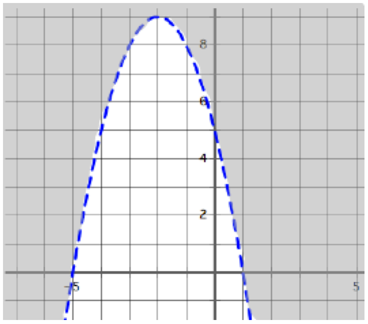
Topic: Solving quadratic inequalities

Solve each of the quadratic inequalities.

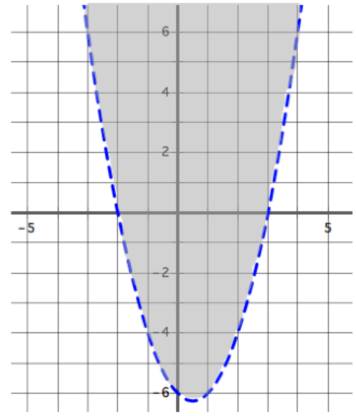
11. $x^2 + x - 12 > 0$
12. $x^2 - 2x - 8 \leq 0$
13. $x^2 + 5x - 14 \geq 0$
14. $2x^2 - 9x - 5 \geq 0$
15. $3x^2 - 3x - 18 < 0$
16. $x^2 + 4x - 21 < 0$
17. $x^2 - 4x \leq 0$
18. $x^2 \leq 25$
19. $x^2 - 4x \leq 5$

Match each graph with its inequality.

____20.



____21.



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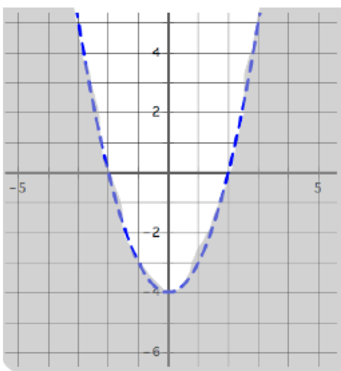
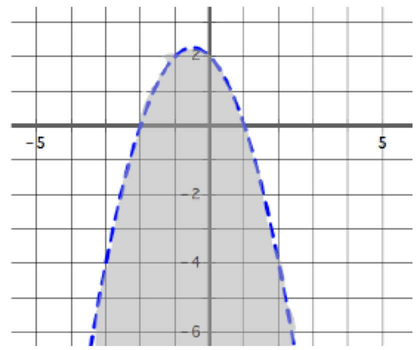


____23.



a. $y > x^2 - x - 6$ b. $y < x^2 - 4$ **c.** $y < (x+2)(1-x)$ d. $y > 5 - 4x - x^2$

Go

Topic: Vertex form of quadratic equations

Write each quadratic function below in vertex form.

24. $x^2 + 6x + 5$

25. $(x+3)(x-5)$

26. $(x-2)(x+6)$

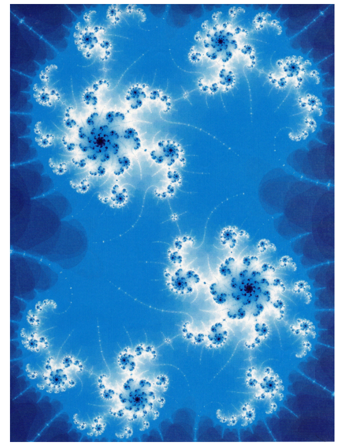
27. $x^2 - 12x + 20$

28. $2x^2 + 16x + 8$

29. $x^2 - 2x - 8$

3.12H Complex Computations

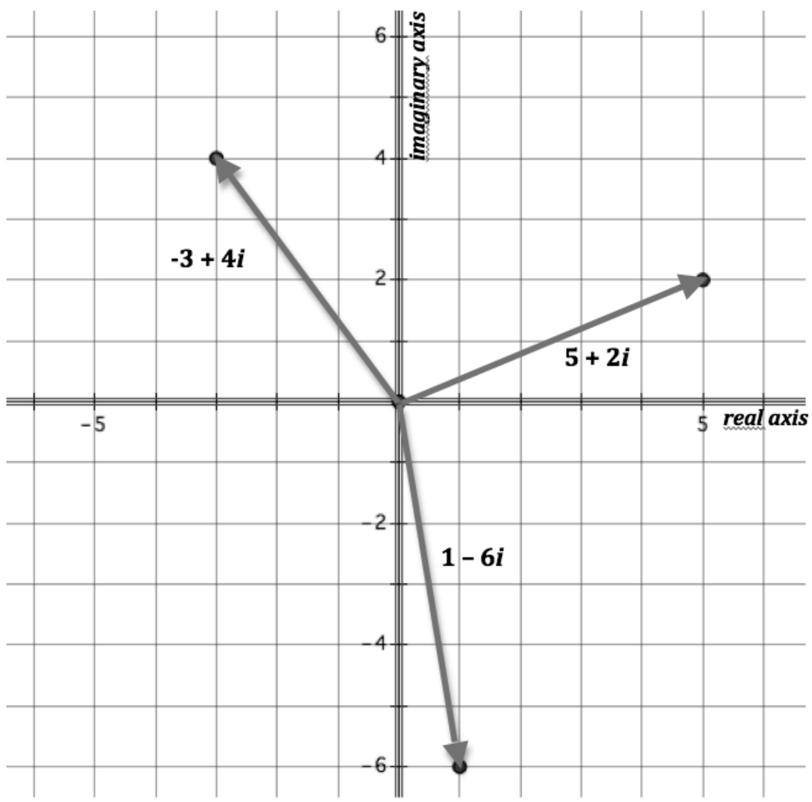
A Solidify Understanding Task



It is helpful to illustrate the arithmetic of complex numbers using a visual representation. To do so, we will introduce the complex plane.

As shown in the figure below, the complex plane consists of a horizontal axis representing the set of real numbers and a vertical axis representing the set of imaginary numbers. Since a complex number $a + bi$ has both a real component and an imaginary component, it can be represented as a point in the plane with coordinates (a, b). It can also be represented by a position vector with its tail located at the point (0, 0) and its head located at the point (a, b), as shown in the diagram. It will be useful to be able to move back and forth between both geometric representations of a complex number in the complex plane—sometimes representing the complex number as a single point, and sometimes as a vector.

You may want to review the Secondary Math 1 task, *The Arithmetic of Vectors*, so you can draw upon those ideas in the following work.



The Modulus of a Complex Number

It is often useful to be able to compare the magnitude of two different numbers. For example, collecting $25 in revenue will not pay off a $45 debt, since $|25| < |-45|$. Note that in this example we used the absolute value of signed numbers to compare the magnitude of the revenue and the debt. Since -45 lies farther from 0 along a real number line than 25, the debt is greater than the revenue. In a similar way, we can compare the relative magnitudes of complex numbers by determining how far they lie away from the origin, the point (0, 0), in the complex plane. We refer to the magnitude of a complex number as its **modulus**, and symbolize this length with the notation $|a + bi|$.

1. Find the modulus of each of the complex numbers shown in the figure above.

2. State a rule, either in words or using algebraic notation, for finding the modulus of any complex number $a + bi$.

Adding and subtracting complex numbers

3. Experiment with the vector representation of complex numbers to develop and justify an algebraic rule for adding two complex numbers: $(a + bi) + (c + di)$. How do your representations of addition of vectors on the complex plane help to explain your algebraic rule for adding complex numbers?

4. How would you represent the additive inverse of a complex number on the complex plane? How would you represent the additive inverse algebraically?

5. If we think of subtraction as adding the additive inverse of a number, use the vector representation of complex numbers to develop and justify an algebraic rule for subtracting two complex numbers: $(a + bi) - (c + di)$. How do your representations of the additive inverse of a complex number and the addition of vectors on the complex plane help to explain your algebraic rule for subtracting complex numbers?

Multiplying complex numbers

One way to think about multiplication on the complex plane is to treat the first factor in the multiplication as a scale factor.

6. Provide a few examples of multiplying a complex number by a real number scale factor: $a(c + di)$. For example, what happens to the vector representation of a complex number when the scale factor a is 4? ½? -2?

7. Provide a few examples of multiplying a complex number by an imaginary scale factor: $bi(c + di)$. For example, what happens to the vector representation when the scale factor bi is i? $2i$? $-3i$?

8. Experiment with the vector representation of complex numbers to justify the following rule for multiplying complex numbers:

$$(a + bi)(c + di) = a(c + di) + bi(c + di) = ac - bd + (ad + bc)i.$$

   How do the geometric observations you made in question 6, question 7 and question 3 show up in this work?

The conjugate of a complex number

The conjugate of a complex number $a + bi$ is the complex number $a - bi$. The conjugate of a complex number is represented with the notation $\overline{a + bi}$.

9. Illustrate an example of a complex number and its conjugate in the complex plane using vector representations.

10. Illustrate finding the sum of a complex number and its conjugate in the complex plane using vector representations.

11. Illustrate finding the product of a complex number and its conjugate in the complex plane using vector representations. (Use the geometric observations you made in questions 6-8 to guide your work.)

12. If z is a complex number and $\bar{z}$ is its conjugate, how are the moduli $|z|$ and $|\bar{z}|$ related?

13. Use either a geometric or algebraic argument to complete and justify the following statements for any complex number $a + bi$:

- The sum of a complex number and its conjugate is always the real number _________ .

- The product of a complex number and its conjugate is always the real number ________ .

The division of complex numbers

Dividing a complex number by a real number is the same as multiplying the complex number by the multiplicative inverse of the divisor. That is, $\frac{a+bi}{c} = \frac{1}{c}(a+bi) = \frac{a}{c} + \frac{b}{c}i$. Therefore, division of a complex number by a real number can be thought of in terms of multiplying the complex number by a real-valued scale factor, an idea we explored in question 6.

We have also observed that multiplying a complex number by its conjugate always gives us a real number result. We make use of this fact to change a problem involving division by a complex number into an equivalent problem in which the divisor is a real number.

14. Explain why $\frac{a+bi}{c+di}$ is equivalent to $\frac{(a+bi)}{(c+di)} \cdot \frac{(c-di)}{(c-di)}$.

15. Use this idea to find the quotient $\frac{3+5i}{4+2i}$.

We have been using a vector representation of complex numbers in the complex plane in the previous problems. In the following problems we will represent complex numbers simply as points in the complex plane.

Finding the distance between two complex numbers

To find the distance between two points on a real number line, we find the absolute value of the difference between their coordinates. (Illustrate this idea with a couple of examples.)

In a similar way, we define the distance between two complex numbers in the complex plane as the modulus of the difference between them.

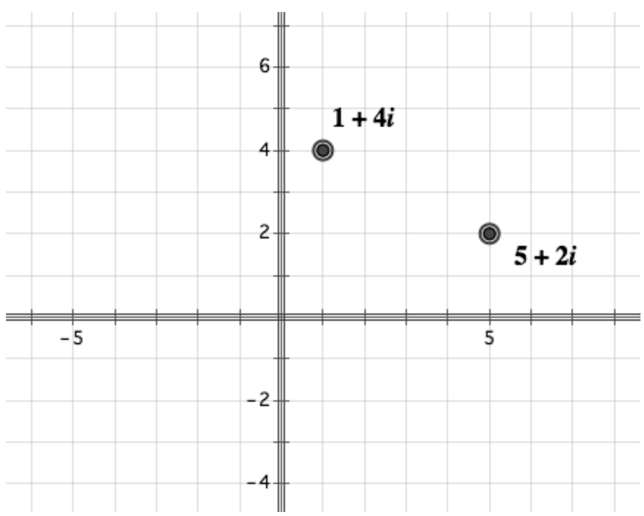
16. Find the distance between the two complex numbers plotted on the complex plane below.

Finding the average of two complex numbers

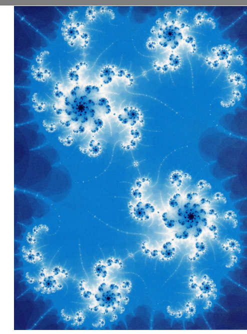
The average of two real numbers $\frac{x_1 + x_2}{2}$ is located at the midpoint of the segment connecting the two real numbers on the real number line. (Illustrate this idea with a couple of examples.)

In a similar way, we define the average of two complex numbers to be the midpoint of the segment connecting the two complex numbers in the complex plane.

17. Find the average of the two complex numbers plotted on the complex plane below.



Name: Solving Quadratic and Other Equations | 3.12H



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Ready, Set, Go!

Ready

Topic: Solving Systems of linear equations

Solve each system of equations using substitution.

1. $\begin{cases} y = 3x \\ y = -2x - 15 \end{cases}$

2. $\begin{cases} 3x + y = 21 \\ y = -2x - 15 \end{cases}$

3. $\begin{cases} 3x + 2y = 7 \\ x - 2y = -3 \end{cases}$

Solve each system of equations using elimination.

4. $\begin{cases} 5x - y = 13 \\ -2x + y = -1 \end{cases}$

5. $\begin{cases} 3x + y = 21 \\ -3x + 5y = -3 \end{cases}$

6. $\begin{cases} 3x + 2y = 7 \\ x + y = 2 \end{cases}$

Create an augmented matrix for each system of equations and then use row reductions to solve the system.

7. $\begin{cases} 2x + y = 7 \\ -2x + y = -1 \end{cases}$

8. $\begin{cases} 3x - 4y = 21 \\ -3x + 5y = -3 \end{cases}$

9. $\begin{cases} 5x - y = 13 \\ -2x + y = -1 \end{cases}$

Set

Topic: Operations with imaginary numbers

Perform the indicated operations on the complex numbers.

10. $(3+4i)+(2-5i)$

11. $(6-4i)-(7+2i)$

12. $3(5+2i)$

13. $(9-2i)(1+3i)$

14. $4(3-2i)-(5+3i)$

15. $(2-5i)(2+5i)$

Use the conjugate of each denominator to rationalize the denominators and write an equivalent fraction.

16. $\frac{3-5i}{2+5i}$

17. $\frac{6+7i}{4-3i}$

18. $\frac{2-3i}{1-6i}$

Find the modulus for each complex number.

19. $3-5i$

20. $4-3i$

21. $-4+3i$

22. If the graphical representation of the operations between two complex numbers results in a value along the y-axis or imaginary axis, what must be true about the two complex numbers?

23. If the graphical representation of the operations between two complex numbers results in a value along the x-axis or real number axis, what must be true about the two complex numbers?

Go

Topic: Solving Quadratics

24. List the strategies that can be used to solve quadratic equations. Explain when each of the strategies would be most efficient. Give an example of a quadratic that would be most efficiently solved for each.

Solve the quadratics below using an appropriate method.

25. $x^2 + 9x + 18 = 0$

26. $x^2 - 2x - 3 = 0$

27. $2x^2 - 5x + 3 = 0$

28. $(x - 2)(x + 3) = 0$

29. $10x^2 - x + 9 = 0$

30. $(x - 2)^2 = 20$

3.13H All Systems Go!

A Solidify Understanding Task


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Carlos likes to buy supplies for *Curbside Rivalry* at the *All a Dollar Paint Store* where the price of every item is a multiple of $1. This makes it easy to keep track of the total cost of his purchases. Clarita is worried that items at *All a Dollar Paint Store* might cost more, so she is going over the records to see how much Carlos is paying for different supplies. Unfortunately, Carlos has once again forgotten to write down the cost of each item he purchased. Instead, he has only recorded what he purchased and the total cost of all of the items.

Carlos and Clarita are trying to figure out the cost of a gallon of paint, the cost of a paintbrush, and the cost of a roll of masking tape based on the following purchases:

Week 1: Carlos bought 2 gallons of paint and 1 roll of masking tape for $30.
Week 2: Carlos bought 1 gallon of paint and 4 brushes for $20.
Week 3: Carlos bought 2 brushes and 1 roll of masking tape for $10.

1. Determine the cost of each item using whatever strategy you want. Show the details of your work so that someone else can follow your strategy.

You probably recognized that this problem could be represented as a system of equations. In previous math courses you have developed several methods for solving systems.

2. Which of the methods you have developed for solving systems of equations could be applied to this system? Which methods seem more problematic? Why?

In the MVP Secondary Math I tasks *To Market with Matrices* and *Solving Systems with Matrices* you learned how to solve systems of equations involving two equations and two unknown quantities using row reduction of matrices. You may want to review those two tasks before continuing.

3. Modify the "row reduction of matrices" strategy so you can use it to solve Carlos and Clarita's system of equations using row reduction. What modifications did you have to make, and why?

In the MVP Secondary Math I sequence of tasks *More Arithmetic of Matrices*, *Solving Systems with Matrices, Revisited* and *The Determinant of a Matrix* you learned how to solve these same types of systems using the multiplication of matrices. You may want to review those tasks before continuing.

4. Multiply the follow pairs of matrices:

   a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

   b. $\begin{bmatrix} 0.4 & 0.2 & -0.4 \\ -0.1 & 0.2 & 0.1 \\ 0.2 & -0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

5. What property is illustrated by the multiplication in question 4a?

6. What property is illustrated by the multiplication in question 4b?

7. Rewrite the following system of equations, which represents Carlos and Clarita's problem, as a matrix equation in the form **AX = B** where **A**, **X** and **B** are all matrices.

$$\begin{aligned} 2g + 0b + 1t &= 30 \\ 1g + 4b + 0t &= 20 \\ 0g + 2b + 1t &= 10 \end{aligned}$$

8. Solve your matrix equation by using multiplication of matrices. Show the details of your work so that someone else can follow it.

You were able to solve this equation using matrix multiplication because you were given the inverse of matrix **A.** Unlike 2×2 matrices, where the inverse matrix can be easily found by hand using the methods described in *More Arithmetic of Matrices*, the inverses of $n \times n$ in general can be difficult to find by hand. In such cases, we will use technology to find the inverse matrix so that this method can be applied to all linear systems involving n equations and n unknown quantities.

Name: Solving Quadratic and Other Equations | 3.13H

Ready, Set, Go!

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Ready

Topic: Rational Exponents Review and methods for solving quadratics

Write each exponential expression in radical form.

1. $10^{\frac{3}{2}}$ 2. $x^{\frac{1}{5}}$ 3. $3n^{\frac{1}{3}}$

4. $6^{\frac{2}{7}}$ 5. $7^{\frac{5}{3}}$ 6. $t^{\frac{4}{5}}$

Write each radical expression in exponential form.

7. $\sqrt[5]{3}$ 8. $\left(\sqrt[6]{7a}\right)^5$ 9. $\sqrt{x^3}$

10. $\sqrt[3]{n^5}$ 11. $\left(\sqrt[y]{n}\right)^x$ 12. $\sqrt[p]{n^q}$

Explain each strategy for solving quadratics and explain the circumstances in which the strategy is most efficient.

13. Graphing: ____

____

14. Factoring: ____

____

15. Completing the square: ____

____

16. What other strategies do you know for solving quadratic equations? When would you use them?

Set

Topic: Solving systems with three unknowns.

Solve the system of equations using matrices. Create a matrix equation for the system of equations that can be used to find the solution. Then find the inverse matrix and use it to solve the system.

17. $\begin{cases} 2x - 4y + z = 0 \\ 5x - 4y - 5 = 12 \\ 4x + 4y + z = 24 \end{cases}$

18. $\begin{cases} x + 2y + 5z = -15 \\ x + y - 4z = 12 \\ x - 6y + 4z = -12 \end{cases}$

19. $\begin{cases} 4p + q - 2r = 5 \\ -3p - 3q - 4r = -16 \\ 4p - 4q + 4r = -4 \end{cases}$

20. $\begin{cases} -6x - 4y + z = -20 \\ -3x - y - 3z = -8 \\ -5x + 3y + 6z = -4 \end{cases}$

Go

Topic: Solving Quadratics

Solve each of the quadratics below using an appropriate and efficient method.

21. $x^2 - 5x = -6$

22. $3x^2 - 5 = 0$

23. $5x^2 - 10 = 0$

24. $x^2 + 1x - 30 = 0$

25. $x^2 + 2x = 48$

26. $x^2 - 3x = 0$