Secondary Two Mathematics: An Integrated Approach
Module 2
Structures of Expressions

By

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In partnership with the Utah State Office of Education

Module 2 – Structures of Expressions

Classroom Task: 2.1 Shifty y's – A Develop Understanding Task
Connecting transformations to quadratic functions and parabolas (**F.IF.7, F.BF.3**)
Ready, Set, Go Homework: Structures of Expressions 2.1

Classroom Task: 2.2 Transformers: More Than Meets the y's– A Solidify Understanding Task
Working with vertex form of a quadratic, connecting the components to transformations (**F.IF.7, F.BF.3**)
Ready, Set, Go Homework: Structures of Expressions 2.2

Classroom Task: 2.3 Building the Perfect Square – A Practice Understanding Task
Visual and algebraic approaches to completing the square (**F.IF.8**)
Ready, Set, Go Homework: Structures of Expressions 2.3

Classroom Task: 2.4 Factor Fixin' – A Solidify Understanding Task
Connecting the factored and expanded or standard forms of a quadratic (**F.IF.8, F.BF.1, A.SSE.3**)
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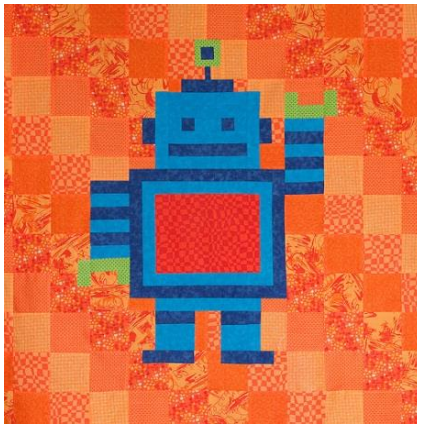
Classroom Task: 2.5 Lining Up Quadratics – A Solidify Understanding Task
Focus on the vertex and intercepts for quadratics (**F.IF.8, F.BF.1, A.SSE.3**)
Ready, Set, Go Homework: Structures of Expressions 2.5

Classroom Task: 2.6 I've Got a Fill-in – A Solidify Understanding Task
Building fluency in rewriting and connecting different forms of a quadratic (**F.IF.8, F.BF.1, A.SSE.3**)
Ready, Set, Go Homework: Structures of Expressions 2.6



2.1 Transformers: Shifty y's

A Develop Understanding Task

Optima is designing a robot quilt for her new grandson. She plans for the robot to have a square face. The amount of fabric that she needs for the face will depend on the area of the face, so Optima decides to model the area of the robot's face mathematically. She knows that the area A of a square with side length x is modeled by the function, $A(x) = x^2$.

1. What is the domain of the function $A(x)$ in this context?

2. Match each statement about the area to the function that models it:

Matching Equation (A,B, C, or D)	**Statement**	**Function Equation**
	The length of any given side is increased by 5 units.	A) $A = 5x^2$
	The length of any given side is multiplied by 5 units.	B) $A = (x+5)^2$
	The area of a square is increased by 5 square units.	C) $A = (5x)^2$
	The area of a square is multiplied by 5.	D) $A = x^2 + 5$

Optima started thinking about the graph of $y = x^2$ (in the domain of all real numbers) and wondering about how changes to the equation of the function like adding 5 or multiplying by 5 affect the graph. She decided to make predictions about the effects and then check them out.

3. Predict how the graphs of each of the following equations will be the same or different from the graph of $y = x^2$.

	Similarities to the graph of $y = x^2$	**Differences from the graph of $y = x^2$**
$y = 5x^2$		
$y = (x+5)^2$		
$y = (5x)^2$		
$y = x^2 + 5$		

4. Optima decides to test her ideas using technology. She thinks that it is always a good idea to start simple, so she decides to go with $y = x^2 + 5$. She graphs it along with $y = x^2$ in the same window. Test it yourself and describe what you find.

5. Knowing that things make a lot more sense with more representations, Optima tries a few more examples like $y = x^2 + 2$ and $y = x^2 - 3$, looking at both a table and a graph for each. What conclusion would you draw about the effect of adding or subtracting a number to $y = x^2$? Carefully record the tables and graphs of these examples in your notebook and explain why your conclusion would be true for any value of *k*, given, $y = x^2 + k$.

6. After her amazing success with addition in the last problem, Optima decides to look at what happens with addition and subtraction inside the parentheses, or as she says it, "adding to the *x* before it gets squared". Using your technology, decide the effect of *h* in the equations: $y = (x + h)^2$ and $y = (x - h)^2$. (Choose some specific numbers for *h*.) Record a few examples (both tables and graphs) in your notebook and explain why this effect on the graph occurs.

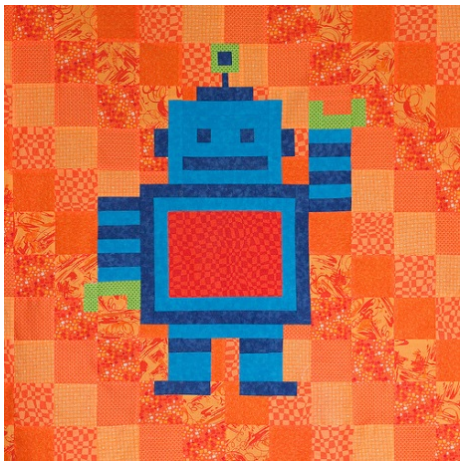
7. Optima thought that #6 was very tricky and hoped that multiplication was going to be more straightforward. She decides to start simple and multiply by -1, so she begins with

$y = -x^2$. Predict what the effect is on the graph and then test it. Why does it have this effect?

8. Optima is encouraged because that one was easy. She decides to end her investigation for the day by determining the effect of a multiplier, a, in the equation: $y = ax^2$. Using both positive and negative numbers, fractions and integers, create at least 4 tables and matching graphs to determine the effect of a multiplier.

NAME Structures of Expressions | 2.1

Ready, Set, Go!

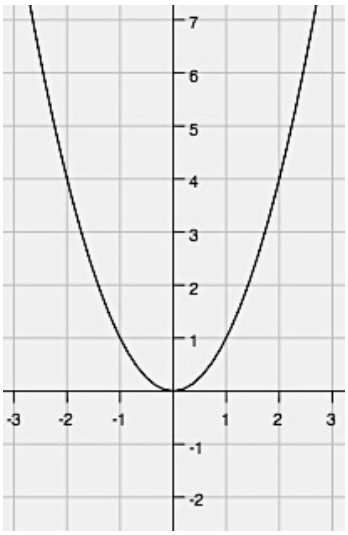


Ready

Topic: Finding key features in the graph of a quadratic equation

Make a point on the **vertex** and draw a dotted line for the **axis of symmetry**.
Label the coordinates of the vertex and state whether it's a **maximum** or a **minimum.**
Write the **equation for the axis of symmetry**.

1.



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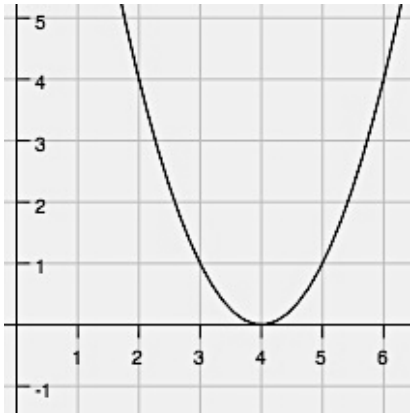

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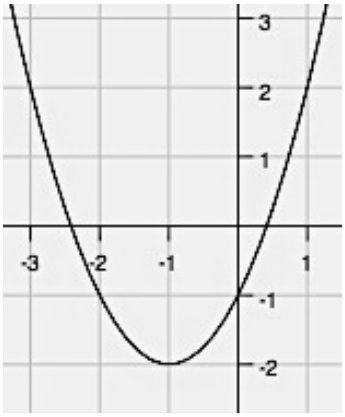

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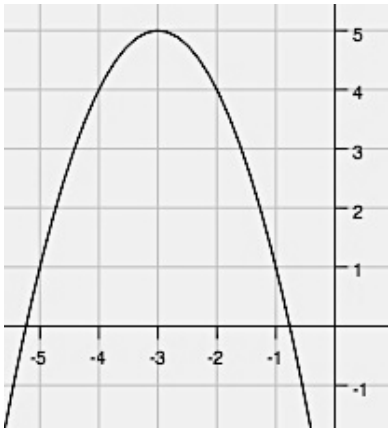

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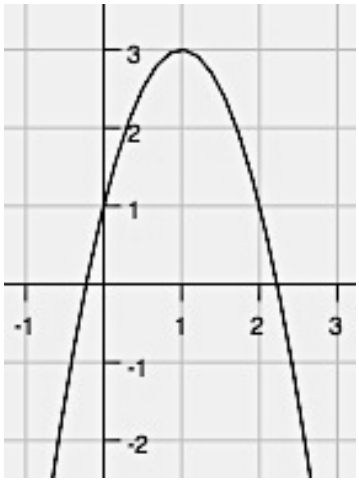
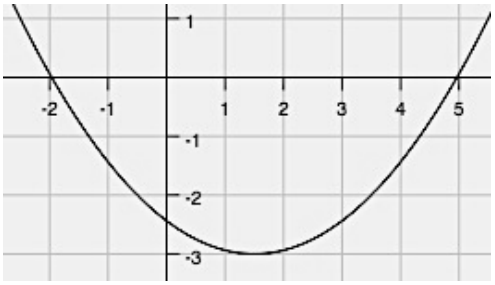
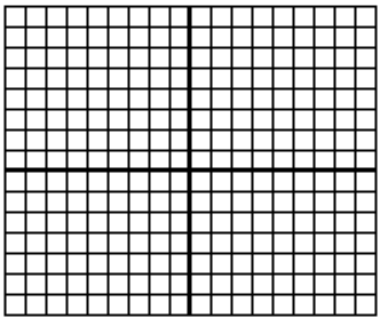


6.



7. What connection exists between the coordinates of the vertex and the equation of the axis of symmetry?

8. Look back at #6. Try to find a way to find the ***exact*** value of the coordinates of the vertex. Test your method with each vertex in 1 - 5. Explain your conjecture.



9. How many **x-intercepts** can a parabola have?

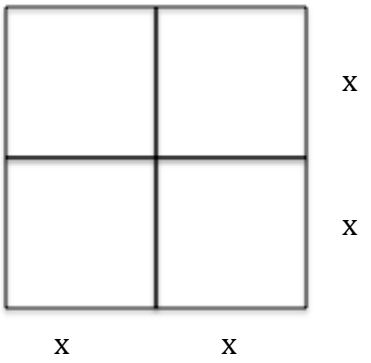
10. Sketch a parabola that has no x-intercepts, then explain what has to happen for a parabola to have no x-intercepts.

Set Topic: Transformations on quadratics

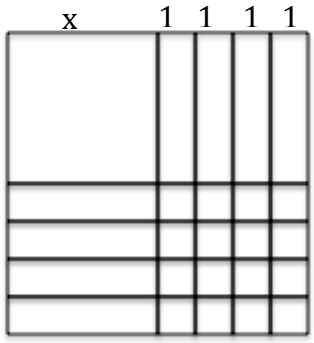
Choose the area model that is the best match for the equation.

11. $x^2 + 4$	12. $(x + 4)^2$	13. $(4x)^2$	14. $4x^2$

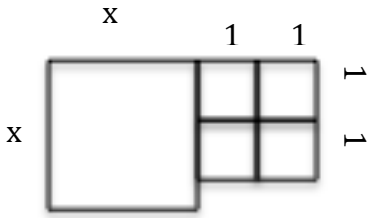
A.



B.

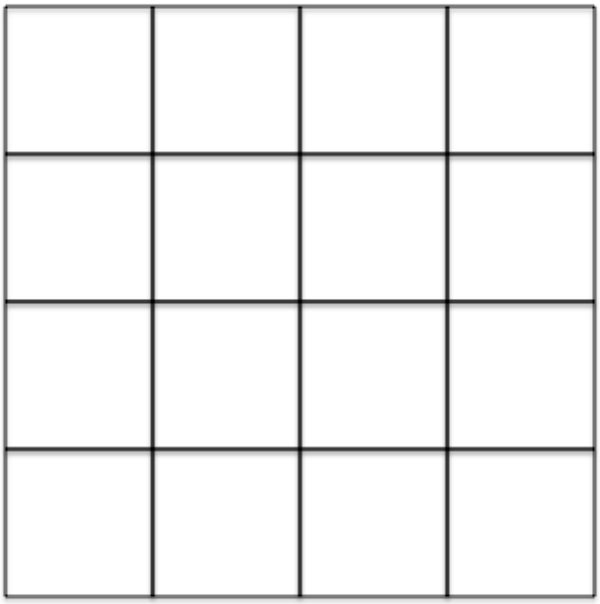


C.



D.

x

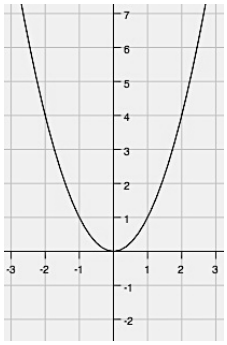


x

A table of values for $f(x) = x^2$ is given. Compare the values in the table for $g(x)$ to those for $f(x)$. Identify what stays the same and what changes. Use this information to write the vertex form of the equation of $g(x)$. Then graph $g(x)$.

Describe how the graph changed from the graph of $f(x)$.
Use words such as right, left, up, and down.

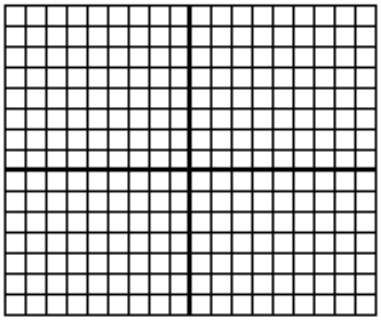
x	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9



$f(x)$

15. $g(x) =$

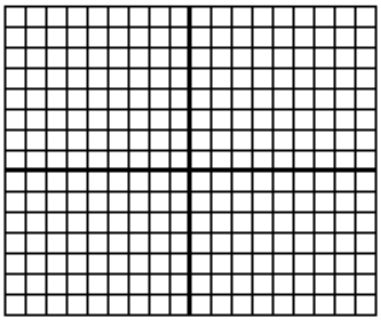
x	-3	-2	-1	0	1	2	3
$g(x)$	2	-3	-6	-7	-6	-3	2



In what way did it move?
What part of the equation shows this move?

16. $g(x)$ =

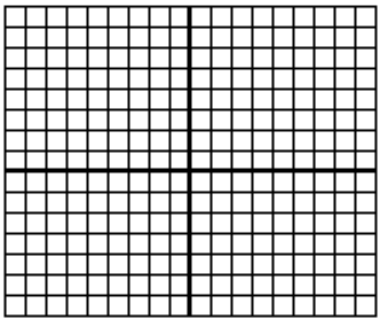
x	-3	-2	-1	0	1	2	3
$g(x)$	11	6	3	2	3	6	11



In what way did it move?
What part of the equation shows this move?

17. $g(x)$ =

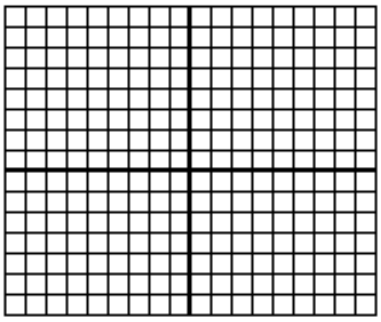
x	-4	-3	-2	-1	0	1	2
$g(x)$	9	4	1	0	1	4	9



In what way did it move?
What part of the equation shows this move?

18. $g(x)$ =

x	0	1	2	3	4	5	6
$g(x)$	9	4	1	0	1	4	9



In what way did it move?
What part of the equation shows this move?

Go Topic: Finding square roots

Simplify.

19. $\sqrt{49a^2b^6}$

20. $\sqrt{(x+13)^2}$

21. $\sqrt{(x-16)^2}$

22. $\sqrt{(36x+25)^2}$

23. $\sqrt{(11x-7)^2}$

24. $\sqrt{9m^2(2p^3-q)^2}$

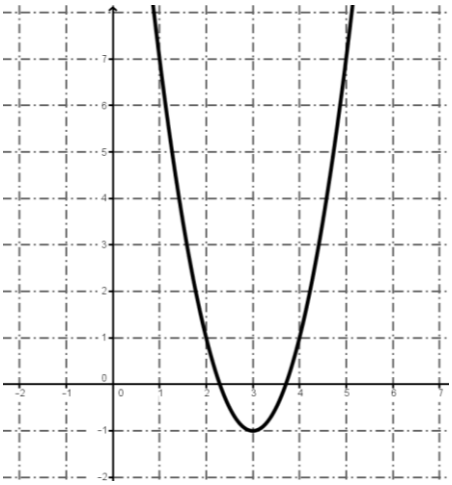
2.2 Transformers: More Than Meets the y's

A Solidify Understanding Task

Write the equation for each problem below. Use a second representation to check your equation.

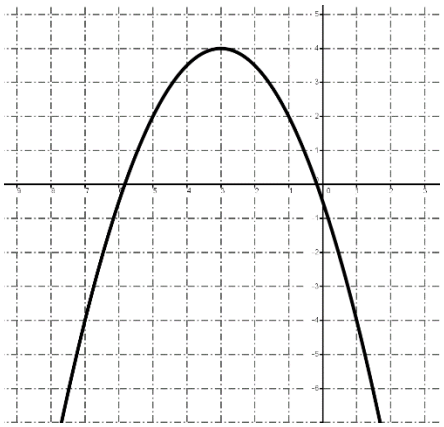
1. The area of a square with side length *x*, where the side length is decreased by 3, the area is multiplied by 2 and then 4 square units are added to the area.

2.



3.

x	*f(x)*
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14
4	23

4. 

Graph each equation without using technology. Be sure to have the exact vertex and at least two correct points on either side of the line of symmetry.

5. $f(x) = -x^2 + 3$

6. $g(x) = (x + 2)^2 - 5$

7. $h(x) = 3(x - 1)^2 + 2$

8. Given: $f(x) = a(x - h)^2 + k$
   a. What point is the vertex of the parabola?

   b. What is the equation of the line of symmetry?

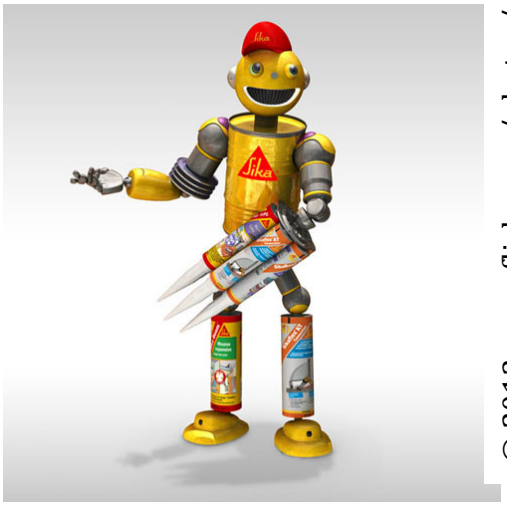
   c. How can you tell if the parabola opens up or down?

   d. How do you identify the dilation?

9. Does it matter in which order the transformations are done? Explain why or why not.

NAME Structures of Expressions | 2.2

Ready, Set, Go!



Ready

Topic: Standard form of a quadratic equation

The standard form of a quadratic equation is defined as
y = ax² + bx + c (a ≠ 0).

Identify ***a***, ***b***, and ***c*** in the following equations.

1. $y = 5x^2 + 3x + 6$

a = _____
b = _____
c = _____

2. $y = x^2 - 7x + 3$

a = _____
b = _____
c = _____

3. $y = 6x^2 - 5$

a = _____
b = _____
c = _____

4. $y = -3x^2 + 4x$

a = _____
b = _____
c = _____

5. $y = 8x^2 - 5x - 2$

a = _____
b = _____
c = _____

Multiply and write each product in the form ***y = ax² + bx + c***. Then identify ***a***, ***b***, and ***c***.

6. $y = x(x - 4)$

a = _____
b = _____
c = _____

7. $y = (x - 1)(2x - 1)$

a = _____
b = _____
c = _____

8. $y = (3x - 2)(3x + 2)$

a = _____
b = _____
c = _____

9. $y = (x + 6)(x + 6)$

a = _____
b = _____
c = _____

10. $y = (x - 3)^2$

a = _____
b = _____
c = _____

11. $y = -(x + 5)^2$

a = _____
b = _____
c = _____

NAME Structures of Expressions | 2.2

Set

Topic: Writing the equation of a parabola in vertex form.

Find a value for $\boldsymbol{\omega}$ such that the graph will have the specified number of x-intercepts.

12. $\boldsymbol{y = x^2 + \omega}$
2 x-intercepts

13. $\boldsymbol{y = x^2 + \omega}$
1 *x*-intercept

14. $\boldsymbol{y = x^2 + \omega}$
no *x*-intercepts

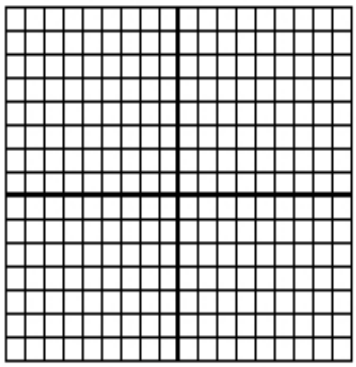
15. $\boldsymbol{y = -x^2 + \omega}$
2 x-intercepts

16. $\boldsymbol{y = -x^2 + \omega}$
1 *x*-intercept

17. $\boldsymbol{y = -x^2 + \omega}$
no *x*-intercepts

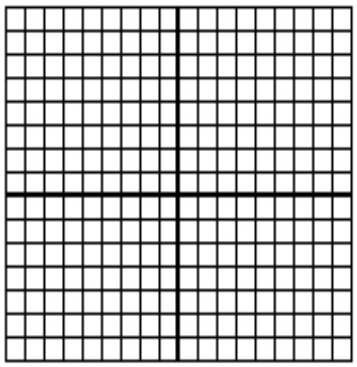
Graph the following equations. State the vertex. (Be accurate with your key points and shape!)

18. $y = (x-1)^2$



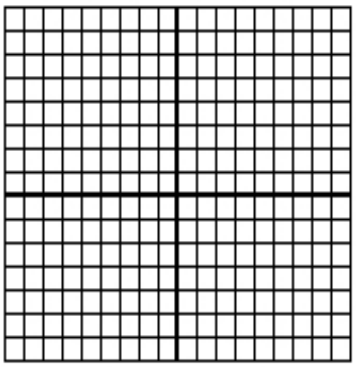
vertex? _____

19. $y = (x-1)^2 + 1$



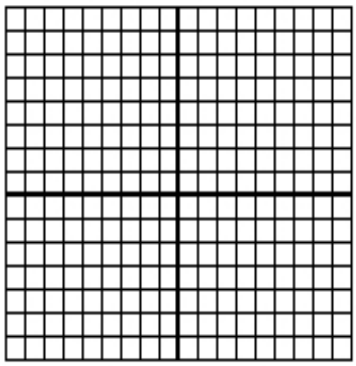
vertex? _____

20. $y = 2(x-1)^2 + 1$



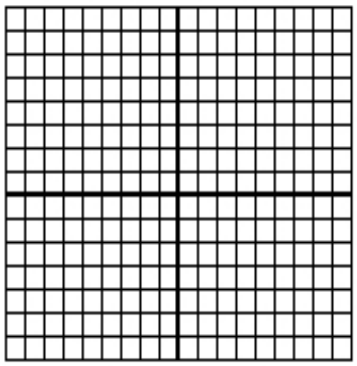
vertex? _____

21. $y = (x+3)^2$



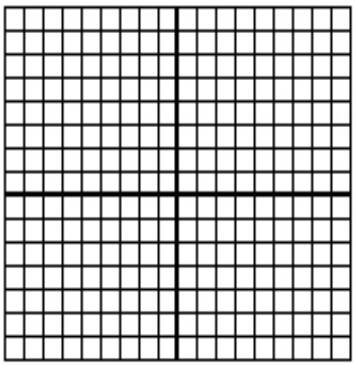
vertex? _____

22. $y = -(x+3)^2 - 4$



vertex? _____

23. $y = -0.5(x+1)^2 + 4$



vertex? _____

Go

Use the table to identify the **vertex**, the equation for the **axis of symmetry**, and state the number of **x-intercept(s)** the parabola will have, if any. Will the vertex be a **minimum** or a **maximum**?

24.

x	y
-4	10
-3	3
-2	-2
-1	-5
0	-6
1	-5
2	-2

vertex _____

A.S. _____

x-inter _____

max or min?

25.

x	y
-2	49
-1	28
0	13
1	4
2	1
3	4
4	13

vertex _____

A.S. _____

x-inter _____

max or min?

26.

x	y
-7	-9
-6	3
-5	7
-4	3
-3	-9
-2	-29
-1	-57

vertex _____

A.S. _____

x-inter _____

max or min?

27.

x	y
-8	-9
-7	-8
-6	-9
-5	-12
-4	-17
-3	-24
-2	-33

vertex _____

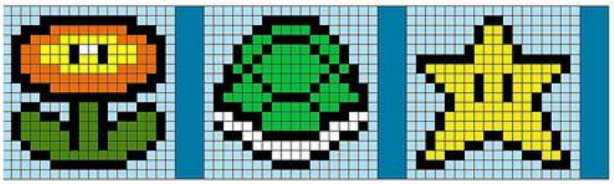
A.S. _____

x-inter _____

max or min?

2.3 Building the Perfect Square

A Solidify Understanding Task

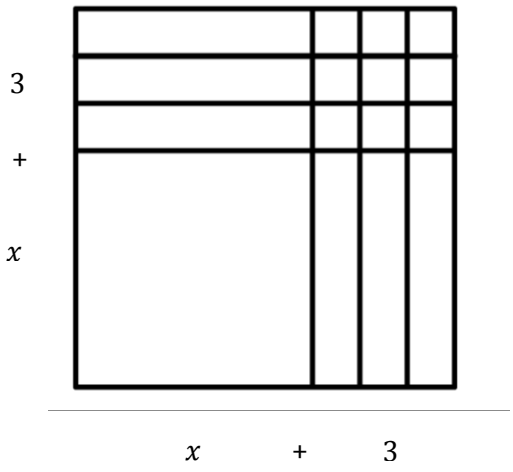


Part 1: Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that the can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to the $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

1. Block with side length: $x + 2$.

2. Block with side length: $x + 1$.

3. What patterns do you notice when you relate the diagrams to the two expressions for the area?

4. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

   a. $x^2 + 4x$

   b. $x^2 + 6x$

   c. $x^2 + 8x$

   d. $x^2 + 12x$

5. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

   Will this strategy work if b is negative? Why or why not?

   Will the strategy work if b is an odd number? What happens to c if b is odd?

Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$.

6. What would they write if the order was for 2 blocks that are 1 inch longer than the basic block? Write the area function in two equivalent forms. Verify your algebra using a diagram.

Part 2: Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x - 3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

7. Fully describe the parabola that Jenny has been assigned to graph.

8. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or any of the transformations of the parabola in this form. Now what am I supposed to do?"

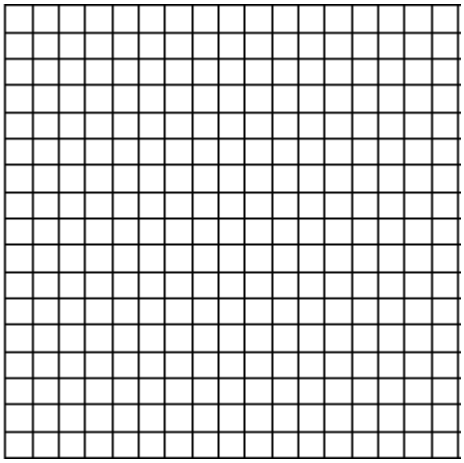
   "Wait a minute—is this the area of a perfect square?" Use your work from Part 1of this task to answer Jenny's question and justify your answer.

9. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$. Change the form of the equation, find the vertex, and graph the parabola.

   a. $y = x^2 - 6x + 9$ New form of the equation: _____

   b. Vertex of the parabola: _____

   c. Graph (with at least 3 accurate points on each side of the line of symmetry):

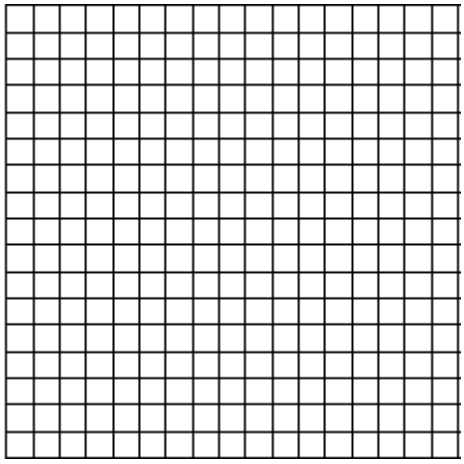


10. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x - h)^2 + k$.

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

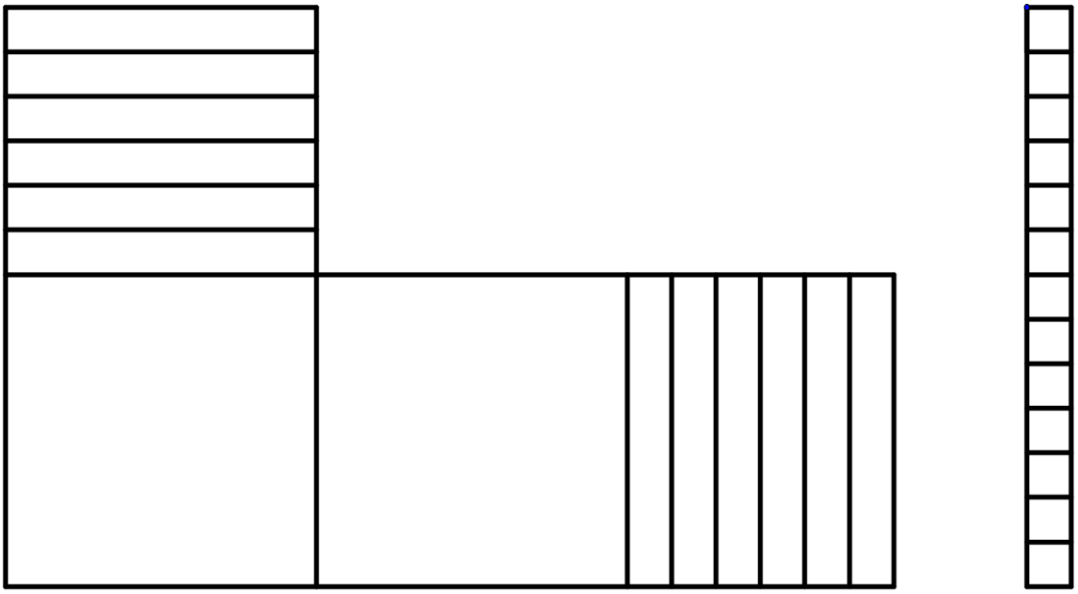
c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.



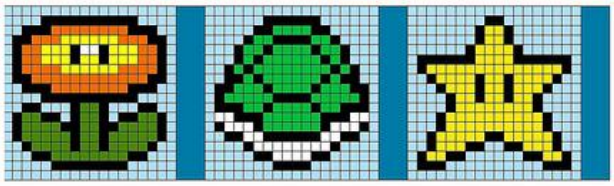
11. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$g(x) = x^2 + 7x + 10$

12. Jenny's last quadratic function to graph is $f(x) = 2x^2 + 12x + 13$. She draws the following diagram and says, "I'm not sure how this helps me. I don't see how to make this a square." Help Jenny to complete the square and find the vertex of the parabola using either the diagram or the equation.



NAME Structures of Expressions | 2.3



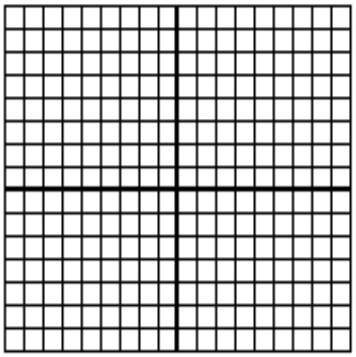
Ready, Set, Go!

Ready

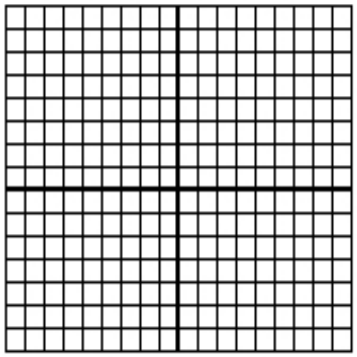
Topic: graphing lines using the intercepts

Find the x-intercept and the y-intercept.
Then graph the equation.

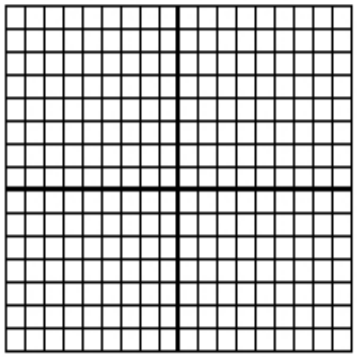
1. $3x + 2y = 12$



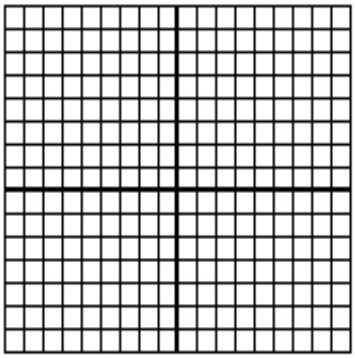
2. $8x - 12y = -24$



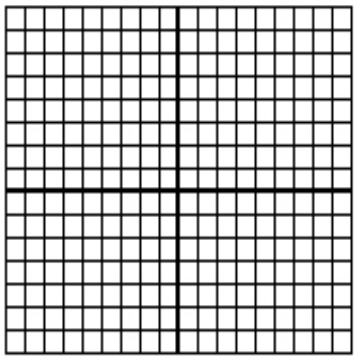
3. $3x - 7y = 21$



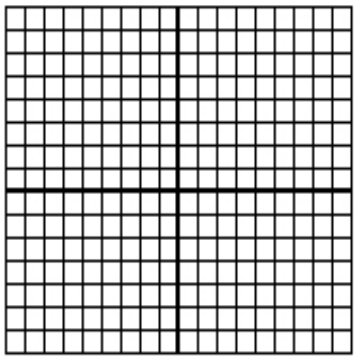
4. $5x - 10y = 20$



5. $2y = 6x - 18$



6. $y = -6x + 6$



Set

Topic: Completing the square

Multiply. Show each step. <u>Circle the pair of like terms before you simplify to a trinomial.</u>

7. $(x+5)(x+5)$ 8. $(3x-7)(3x-7)$ 9. $(9x+1)^2$ 10. $(4x-11)^2$

11. Write a rule for finding the coefficient of the x-term when multiplying and simplifying $(x+q)^2$.

Fill in the number that completes the square. Then write the trinomial in factored form.

12. $x^2+8x+\_\_\_\_$ 2. $x^2-10x+\_\_\_\_$ 3. $x^2+16x+\_\_\_\_$

4. $x^2-6x+\_\_\_\_$ 5. $x^2-22x+\_\_\_\_$ 6. $x^2+18x+\_\_\_\_$

On the next set of problems, leave the number that completes the square as a fraction. Then write the trinomial in factored form.

7. $x^2-11x+\_\_\_\_\_$ 8. $x^2+7x+\_\_\_\_\_$ 9. $x^2+15x+\_\_\_\_\_$

10. $x^2+\frac{2}{3}x+\_\_\_\_\_$ 11. $x^2-\frac{1}{5}x+\_\_\_\_\_$ 12. $x^2-\frac{3}{4}x+\_\_\_\_\_$

Find the value of "B," that will make a perfect square trinomial. Then write the trinomial in factored form.

16. $x^2+\_\_\_\_\_x+16$ 17. $x^2-\_\_\_\_\_x+121$ 18. $x^2-\_\_\_\_\_x+625$

19. $9x^2+\_\_\_\_\_x+225$ 20. $25x^2+\_\_\_\_\_x+49$ 21. $x^2+\_\_\_\_\_x+9$

22. $x^2+\_\_\_\_\_x+\frac{25}{4}$ 23. $x^2+\_\_\_\_\_x+\frac{9}{4}$ 24. $x^2+\_\_\_\_\_x+\frac{49}{4}$

Go

Find the intercepts of the graph of each equation. State whether it's an x-intercept or a y-intercept.

25. y = -4.5 26. x = 9.5 27. x = -8.2 28. y = 112

2.4 Factor Fixin'

A Solidify Understanding Task

At first, *Optima's Quilts* only made square blocks for quilters and Optima spent her time making perfect squares. Customer service representatives were trained to ask for the length of the side of the block, x, that was being ordered, and they would let the customer know the area of the block to be quilted using the formula $A(x) = x^2$.

Optima found that many customers that came into the store were making designs that required a combination of squares and rectangles. So, *Optima's Quilts* has decided to produce several new lines of rectangular quilt blocks. Each new line is described in terms of how the rectangular block has been modified from the original square block. For example, one line of quilt blocks consists of starting with a square block and extending one side length by 5 inches and the other side length by 2 inches to form a new rectangular block. The design department knows that the area of this new block can be represented by the expression: $A(x) = (x + 5)(x + 2)$, but they do not feel that this expression gives the customer a real sense of how much bigger this new block is (e.g., how much more area it has) when compared to the original square blocks.

1. Can you find a different expression to represent the area of this new rectangular block? You will need to convince your customers that your formula is correct using a diagram.

Here are some additional new lines of blocks that *Optima's Quilts* has introduced. Find two different algebraic expressions to represent each rectangle, and illustrate with a diagram why your representations are correct.

2. The original square block was extended 3 inches on one side and 4 inches on the other.

3. The original square block was extended 4 inches on only one side.

4. The original square block was extended 5 inches on each side.

5. The original square block was extended 2 inches on one side and 6 inches on the other.

6. The original square block was extended 2 inches on one side and decreased by 2 inches on the other. (One of the employees thinks that this rectangle will have the same area as the original square since one side was decreased by the same amount as the other side was increased. What do you think?)

7. Both sides of the original square block were decreased by 3 inches.

Customers start ordering custom-made blocks designs by requesting how much additional area they want beyond the original area of x^2. Once an order is taken for a certain type of block, you need to have specific instructions on how to make the new design for the manufacturing team. Your instructions need to explain how to extend the sides of a square blocks to create the new line of rectangular blocks.

The customer service department has placed the following orders on your desk. For each, describe how to make the new blocks by extending the sides of a square block with an initial side length of x. Your instructions should include diagrams, written descriptions and algebraic descriptions of the area of the rectangles in using expressions representing the lengths of the sides.

8. $x^2 + 3x + 5x + 15$

9. $x^2 + 4x + 6x + 24$

Some of the orders are written in an even more simplified algebraic code. Figure out what these entries mean by finding the sides of the rectangles that have this area. Use the sides of the rectangle to write equivalent expressions for the area.

10. $x^2 + 9x + 18$

11. $x^2 + 7x + 10$

12. $x^2 + 9x + 8$

13. $x^2 + 6x + 8$

14. What relationships or patterns do you notice when you find the sides of the rectangles for a given area of this type?

One customer service representative has received an order requesting that the length of one side of the original square block be doubled and then increased by 3 inches, and that the other side be increased by 4 inches.

15. How might you represent this order using two different algebraic expressions?

16. What are the sides of the rectangle that has the area: $2x^2 + 9x + 10$?

17. A customer called and asked for a rectangle with area given by: $x^2 + 7x + 9$. The customer service representative said that the shop couldn't make that rectangle. Do you agree or disagree? How can you tell if a rectangle can be constructed from a given area?

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Ready, Set, Go!

Ready Topic: Creating binomial quadratics

Multiply.

1. $x(4x-7)$ 2. . $5x(3x+8)$

3. Are the answers to problems 1 & 2 quadratics? Justify!

4. Write a rule for factoring a quadratic, written in standard form (**$ax^2 + bx + c$**) when **c** equals 0.

Multiply.

5. $(x+9)(x-9)$ 6. $(x+2)(x-2)$ 7. $(6x+5)(6x-5)$ 8. $(7x+1)(7x-1)$

9. The answers to problems 5, 6, 7, & 8 are quadratics. Which coefficient, **a, b,** or **c**, equals 0?

10. Multiply $(x-13)(x+13)$ (Show all of your steps.) Then multiply $(x-13)(x-13)$.

11. Multiply $(a-b)(a+b)$ (Show all of your steps.) Then multiply $(a+b)(a+b)$.

12. These problems represent two different types of special products. The first is called a ***difference of 2 squares***, while the second one is called a ***perfect square trinomial.*** If you can recognize these, you will make factoring easier for yourself. Explain how you will recognize these two special products. Include, how they are the same, how they are different, and how they factor.

difference of 2 squares	***perfect square trinomial***
Example:	*Example:*
same?	
different?	
factor?	*factor?*

Set Topic: factoring quadratic expressions

Factor the following quadratic expressions into two binomials.

13. $x^2 - 4x + 45$ 14. $x^2 - 12x + 45$ 15. $x^2 - 44x + 45$

16. $x^2 - x - 72$

17. $x^2 + 14x - 72$

18. $x^2 - 6x + 72$

19. $x^2 - 12x + 36$

20. $x^2 - 36$

21. $x^2 - 15x + 36$

22. $15x^2 - 26x + 8$

23. $15x^2 - 2x - 8$

24. $15x^2 - 37x - 8$

25. Look back at each "row" of factoring problems. Explain how it is possible for the coefficient of the middle term to be different numbers in each problem when the "outside" coefficients are basically the same.

Go

Topic: Taking the square root of perfect squares

Only some of the expressions inside the radical sign are perfect squares. Identify which ones are perfect squares and take the square root. Leave the ones that are not perfect squares under the radical sign. Do not attempt to simplify them. (*Hint: Check your answers by squaring them. You should be able to get what you started with, if you are right.)*

26. $\sqrt{(17xyz)^2}$

27. $\sqrt{(3x-7)^2}$

28. $\sqrt{121a^2b^6}$

29. $\sqrt{x^2 + 32x + 16}$

30. $\sqrt{4x^2 + 28x - 49}$

31. $\sqrt{4x^2 + 28x + 49}$

32. $\sqrt{x^2 - 16}$

33. $\sqrt{x^2 + 9}$

33. $\sqrt{x^2 + 10x + 100}$

34. $\sqrt{225x^2 + 30x + 1}$

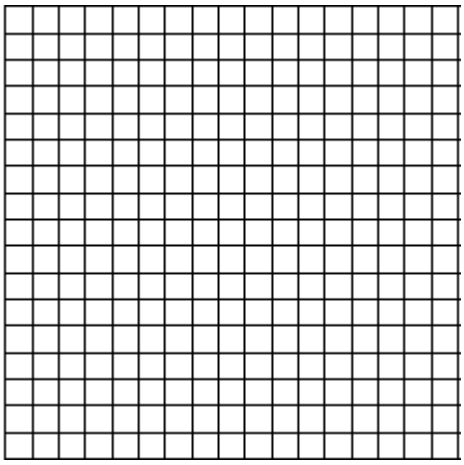
35. $\sqrt{169x^2 - 260x + 100}$

2.5 Lining Up Quadratics

A Solidify Understanding Task

Graph each function and find the vertex, the y-intercept and the x-intercepts. Be sure to properly write the intercepts as points.

1. $y = (x - 1)(x + 3)$



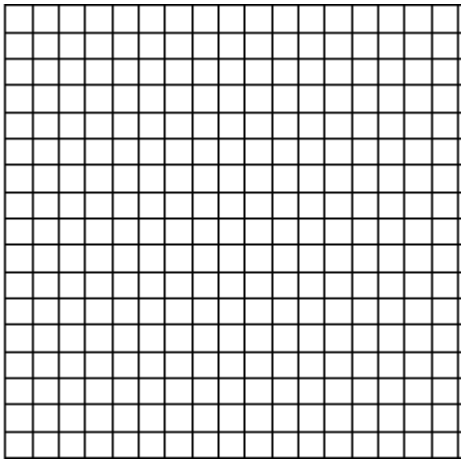
Line of Symmetry _____

Vertex _____

x-intercepts _____ _____

y-intercept _____

2. $f(x) = 2(x - 2)(x - 6)$



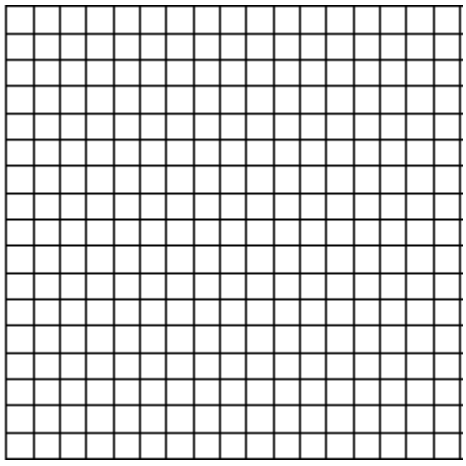
Line of Symmetry _____

Vertex _____

x-intercepts _____ _____

y-intercept _____

3. $g(x) = -x(x + 4)$



Line of Symmetry _____

Vertex _____

x-intercepts _____ _____

y-intercept _____

4. Based on these examples, how can you use a quadratic function in factored form to:
   a. Find the line of symmetry of the parabola?

   b. Find the vertex of the parabola?

   c. Find the x-intercepts of the parabola?

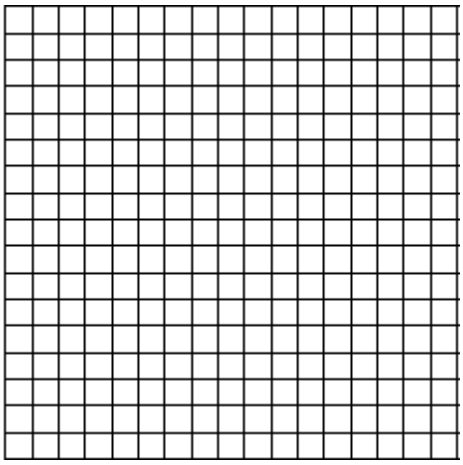
   d. Find the y-intercept of the parabola?

   e. Find the vertical stretch?

5. Choose any two **<u>linear</u>** functions and write them in the form: $f(x) = m(x - c)$, where m is the slope of the line. Graph the two functions.

   Linear function 1:

   Linear function 2:

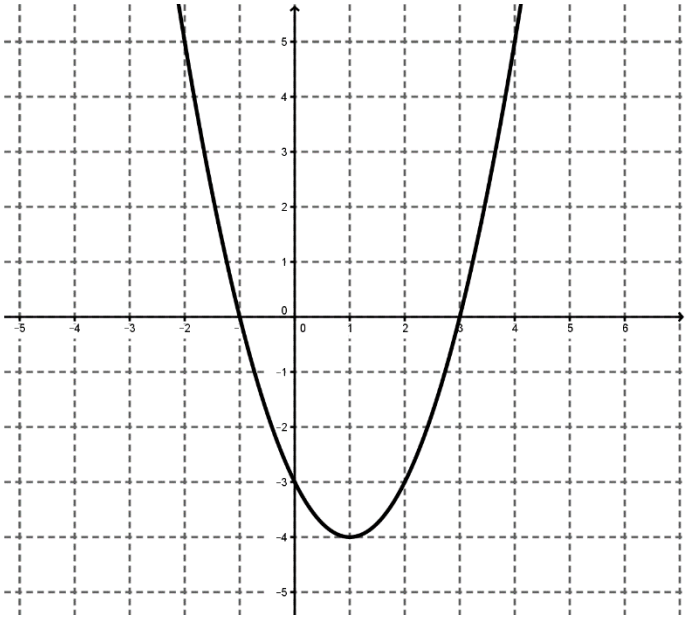


6. On the same graph as #5, graph the function *P(x)* that is the product of the two linear functions that you have chosen. What shape is created?

7. Describe the relationship between *x*-intercepts of the linear functions and the *x*-intercepts of the function *P(x).* Why does this relationship exist?

8. Describe the relationship between *y*-intercepts of the linear functions and the *y*-intercepts of the function *P(x).* Why does this relationship exist?

9. Given the parabola to the right, sketch two lines that could represent its linear factors.



10. Write an equation for each of these two lines.

11. How did you use the *x* and *y* intercepts of the parabola to select the two lines?

12. Are these the only two lines that could represent the linear factors of the parabola? If so, explain why. If not, describe the other possible lines.

13. Use your two lines to write the equation of the parabola. Is this the only possible equation of the parabola?

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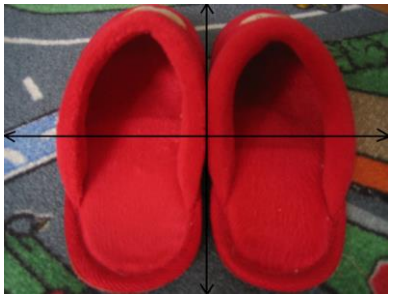

Ready, Set, Go!

Ready

Topic: Multiplying binomials using a two-way table.

Multiply the following binomials using the given two-way table to assist you.

Example:

Multiply $(2x + 3)(5x - 7)$

$(2x+3)$ \ $(5x-7)$	
$10x^2$	$-14x$
$+15x$	-21

$= 10x^2 + x - 21$

1. $(3x - 4)(7x - 5)$

2. $(9x + 2)(x + 6)$

3. $(4x - 3)(3x + 11)$

4. $(7x + 3)(7x - 3)$

5. $(3x - 10)(3x + 10)$

6. $(11x + 5)(11x - 5)$

7. $(4x + 5)^2$

8. $(x + 9)^2$

9. $(10x - 7)^2$

10. What do you notice in the "like-term" boxes in #'s 7, 8, and 9 that is different from the other problems?

Set Topic: Factored form of a quadratic function

Given the **factored form** of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

11. $y = 4(x-2)(x+6)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

12. $y = -3(x+2)(x-6)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

13. $y = (x+5)(x+7)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

14. $y = \frac{1}{2}(x-7)(x-7)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

15. $y = -\frac{1}{2}(x-8)(x+4)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

16. $y = \frac{3}{5}(x-25)(x-9)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

17. $y = \frac{3}{4}(x-3)(x+3)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

18. $y = -(x-5)(x+5)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

19. $y = \frac{2}{3}(x+10)(x+10)$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

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Go

Topic: Vertex form of a quadratic function

Given the **vertex form** of a quadratic function, identify the vertex, intercepts, and vertical stretch of the parabola.

20. $y = (x+2)^2 - 4$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

21. $y = -3(x+6)^2 + 3$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

22. $y = 2(x-1)^2 - 8$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

23. $y = 4(x+2)^2 - 64$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

24. $y = -3(x-2)^2 + 48$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

25. $y = (x+6)^2 - 1$

v:_____

x-inter(s) _____

y-inter _____

stretch _____

26. Did you notice that the parabolas in problems 11, 12, & 13 are the same as the ones in problems 23, 24, & 25 respectively? If you didn't, go back and compare the answers in problems 11, 12, & 13 and problems 23, 24, & 25.

Prove that a. $4(x-2)(x+6) = 4(x+2)^2 - 64$

b. $-3(x+2)(x-6) = -3(x-2)^2 + 48$

c. $(x+5)(x+7) = (x+6)^2 - 1$

2.6 I've Got a Fill-in

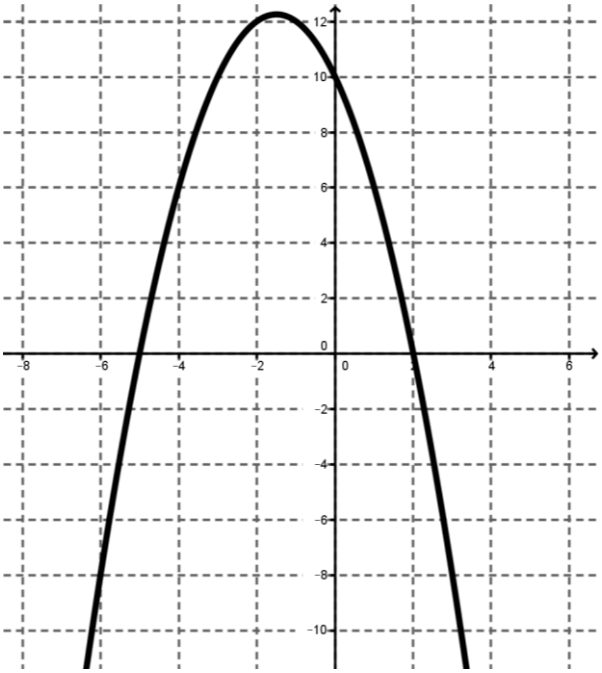
A Practice Understanding Task

For each problem below, you are given a piece of information that tells you a lot. Use what you know about that information to fill in the rest.

1.	You get this:	Fill in this:
	$y = x^2 - x - 12$	Factored form on the equation: Graph of the equation:

2.	You get this:	Fill in this:
	$y = x^2 - 6x + 3$	Vertex form of the equation:
		Graph of the equation:

3.	You get this:	Fill in this:
		Vertex form of the equation:
		Standard form of the equation:

4.	You get this:	Fill in this:
		Factored form of the equation:
		Standard form of the equation:

5.	You get this:	Fill in this:
	$y = -x^2 - 6x + 16$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and *y*-intercept

6.	You get this:	Fill in this:
	$y = 2x^2 + 12x + 13$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and *y*-intercept

7.	You get this:	Fill in this:
	$y = -2x^2 + 14x + 60$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and *y*-intercept

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Ready, Set, Go!

Ready

Topic: Let's get *READY* for the test!

A golf-pro practices his swing by driving golf balls off the edge of a cliff into a lake. The height of the ball above the lake (measured in meters) as a function of time (measured in seconds and represented by the variable t) from the instant of impact with the golf club is

$58.8 + 19.6t - 4.9t^2$.

The expressions below are equivalent:

a. $-4.9t^2 + 19.6t + 58.8$ *standard form*
b. $-4.9(t - 6)(t + 2)$ *factored form*
c. $-4.9(t - 2)^2 + 78.4$ *vertex form*

1. Which expression is the most useful for finding how many seconds it takes for the ball to hit the water? Justify your answer.

2. Which expression is the most useful for finding the maximum height of the ball? Justify your answer.

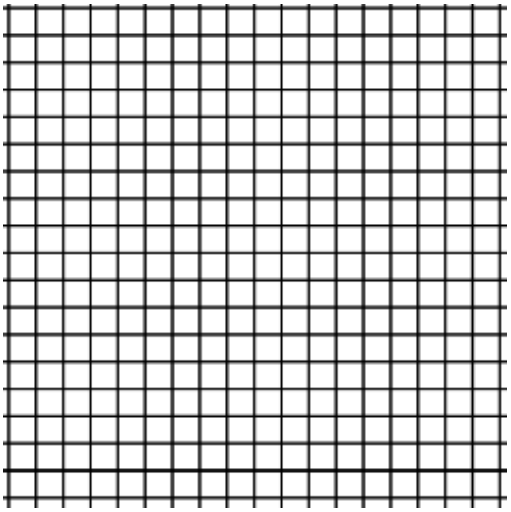
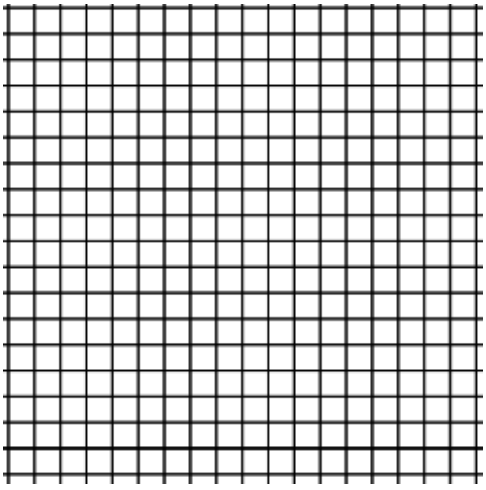
3. If you wanted to know the height of the ball at exactly 3.5 seconds, which expression would you use to find your answer? Explain why.

4. If you wanted to know the height of the cliff above the lake, which expression would you use? Explain why.

NAME Structures of Expressions | 2.6

Set

One form of a quadratic function is given. Fill-in the missing forms.

5. **Standard form** **Vertex form** **Factored form** $y = (x + 5)(x - 3)$	
Table (Show the vertex and at least 2 points on each side of the vertex.) Show the first differences and the second differences.	**Graph** 
6. **Standard form** **Vertex form** $\boldsymbol{y = -3(x - 1)^2 + 4}$ **Factored form**	
Table (Show the vertex and at least 2 points on each side of the vertex.) Show the first differences and the second differences.	**Graph** 

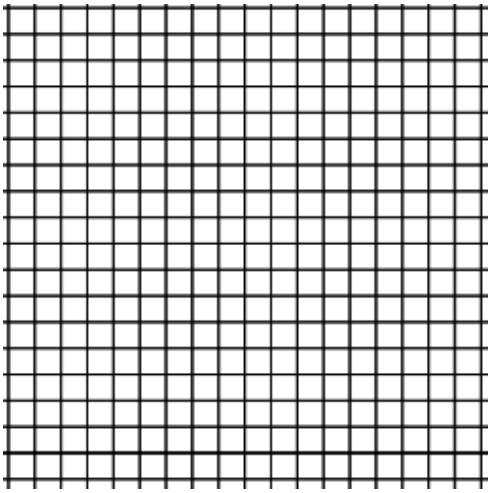
7. Standard form **Vertex form** **Factored form**

$y = -x^2 + 10x - 25$

Table (Show the vertex and at least 2 points on each side of the vertex.)

Show the first differences and the second differences.

Graph

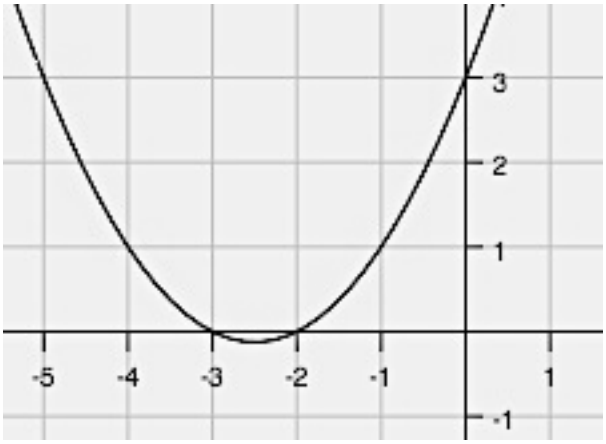


8. Standard form **Vertex form** **Factored form**

Table (Show the vertex and at least 2 points on each side of the vertex.)

Show the first differences and the second differences.

Graph



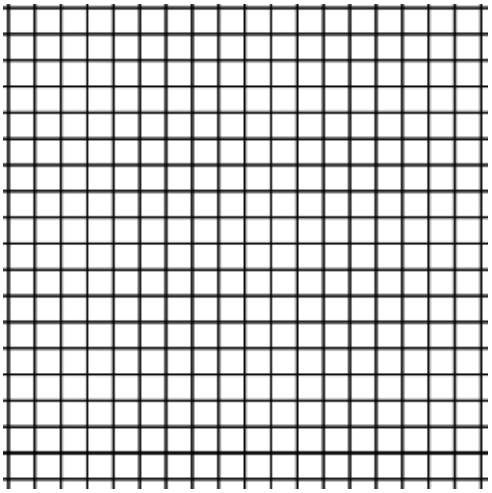
9. Standard form **Vertex form** **Factored form**

Table (Show the vertex and at least 2 points on each side of the vertex.)

x	y
0	12
1	2
2	-4
3	-6
4	-4
5	2
6	12

Show the first differences and the second differences.

Graph



Go Topic: factoring quadratics

Verify each factorization by multiplying.

10. $x^2 + 12x - 64 = (x + 16)(x - 4)$

11. $x^2 - 64 = (x + 8)(x - 8)$

12. $x^2 + 20x + 64 = (x + 16)(x + 4)$

13. $x^2 - 16x + 64 = (x - 8)(x - 8)$

Factor the following quadratic expressions. *(Hint: Some will not factor.)*

14. $x^2 - 5x + 6$

15. $x^2 - 7x + 6$

16. $x^2 - x - 6$

17. $m^2 + 16x + 63$

18. $s^2 - 3s - 1$

19. $3x^2 + 7x + 2$

20. $12n^2 - 8n + 1$

21. $3x^2 + 11x + 10$

22. $8c^2 - 11c + 3$

23. $36x^2 + 84x + 49$

24. $64x^2 - 9$

25. $25x^2 + 10x + 4$

26. Which quadratic expression above could represent the area of a square?
27. Which two in factored form could NOT be the side-lengths for a rectangle?