## Secondary Two Mathematics: An Integrated Approach Module 1 Quadratic Functions

#### By

#### The Mathematics Vision Project:

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In partnership with the Utah State Office of Education



#### Module 1 – Quadratic Functions

Classroom Task: 1.1 Something to Talk About – A Develop Understanding Task

An introduction to quadratic functions, designed to elicit representations and surface a new type of

pattern and change (F.BF.1, A.SSE.1, A.CED.2)

Ready, Set, Go Homework: Quadratic Functions 1.1

Classroom Task: 1.2 | Rule – A Solidify Understanding Task

Solidification of quadratic functions begins as quadratic patters are examined in multiple representations

and contrasted with linear relationships (F.BF.1, A.SSE.1, A.CED.2)

Ready, Set, Go Homework: Quadratic Functions 1.2

Classroom Task: 1.3 Scott's Macho March – A Solidify Understanding Task

Focus specifically on the nature of change between values in a quadratic being linear. (F-BF, F-LE)

Ready, Set, Go Homework: Quadratic Functions 1.3

Classroom Task: 1.4 Rabbit Run – A Solidify Understanding Task

Focus on maximum/minimum point as well as domain and range for quadratics (F.BF.1, A.SSE.1,

A.CED.2)

Ready, Set, Go Homework: Quadratic Functions 1.4

Classroom Task: 1.5 Look out Below – A Solidify Understanding Task

Examining quadratic functions on various sized intervals to determine average rates of change (F.BF.1,

**A.SSE.1, A.CED.2**)

Ready, Set, Go Homework: Quadratic Functions 1.5

Classroom Task: 1.6 Tortoise and Hare – A Solidify Understanding Task

Comparing quadratic and exponential functions to clarify and distinguish between type of growth in each as well as how that growth appears in each of their representations (F.BF.1, A.SSE.1, A.CED.2, F.LE.3)

Ready, Set, Go Homework: Quadratic Functions 1.6

Classroom Task: 1.7 How Does it Grow – A Practice Understanding Task

Incorporating quadratics with the understandings of linear and exponential functions (F.LE.1, F.LE.2,

F.LE.3)

Ready, Set, Go Homework: Quadratic Functions 1.7



#### 1.1 Something to Talk About

A Develop Understanding Task

Cell phones often indicate the strength of the phone's signal with a series of bars. The logo below shows how this might look for various levels of service.



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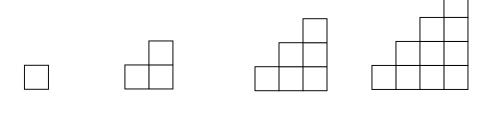


Figure 1 Figure 2 Figure 3 Figure 4

1. Assuming the pattern continues, draw the next figure in the sequence.

2. How many blocks will be in the size 10 logo?

3. Examine the sequence of figures and find a rule or formula for the number of tiles in any figure number.



#### Ready, Set, Go!



Ready

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Topic: Distributive Property

#### Simplify the following expressions

1. 3(2x + 7)

2. -12(5x-4)

3. 5a(-3a+13)

4. 9x(6x-2)

- 5.  $\frac{2x}{3}(12x+18)$
- 6.  $\frac{4a}{5}(10a-25b)$
- 7.  $\frac{-4x}{11}(121x + 22)$

Set Topic: Recognizing linear, exponential, and quadratic equations.

In each set of 3 functions, one will be linear and one will be exponential. One of the three will be a new category of function. List the characteristics in each table that helped you to identify the linear and the exponential functions. What are some characteristics of the new function? Find an explicit and recursive equation for each.

8. Linear, exponential, or a new kind of function.

$\boldsymbol{x}$	f(x)	
6	64	
7	128	
8	256	
9	512	
10	1024	

f(x)6 36 49 8 64 9 81 10

x	f(x)
6	11
7	13
8	15
9	17
10	19

Type and characteristics?

Type and characteristics?

100

Type and characteristics?

Explicit equation:

Explicit equation:

Explicit equation:

Recursive equation:

Recursive equation:

Recursive equation:

9. Linear, exponential, or a new kind of function?

d.

x	f(x)
-2	-17
-1	-12
0	-7
1	-2
2	3

x	f(x)
-2	1/25
-1	1/5
0	1
1	5
2	25

x	f(x)	
-2	9	
-1	6	
0	5	
1	6	
2	9	

Type and characteristics?

Type and characteristics?

Type and characteristics?

Explicit equation:

Explicit equation:

Explicit equation:

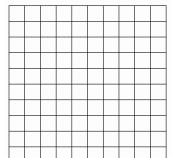
Recursive equation:

Recursive equation:

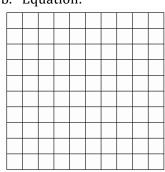
Recursive equation:

10. Graph the functions from the tables in #8. Add any additional characteristics you notice from the graph. Place your axes so that you can show all 5 points. Identify your scale. Write your explicit equation above the graph.

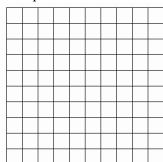
a. Equation:



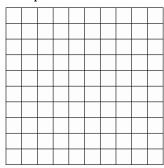
b. Equation:



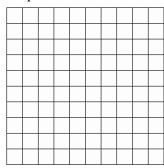
c. Equation:



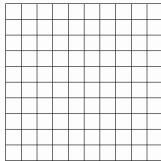
d. Equation:



e. Equation:



f. Equation:



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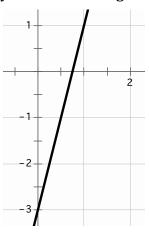


Go

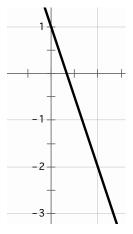
Topic: rates of change

Identify the rate of change in each of the representations below.

11.



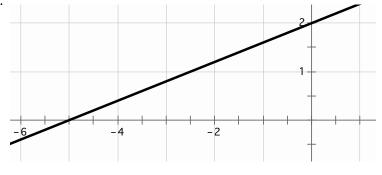
12.



13.				
f(x)				
65				
68				
71				
74				

$$f(0) = 7; f(n+1) = f(n) + 5$$

15.



16.

Slope of  $\overrightarrow{AB}$ A(-3, 12) B(-11, -16)

17. George is loading freight into an elevator. He notices that the weight limit for the elevator is 1000 lbs. He knows that he weighs 210 lbs. He has loaded 15 boxes into the elevator. Each box weighs 50 lbs. Identify the rate of change for this situation.

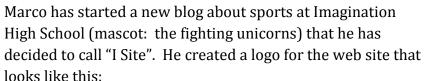
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10.					
Independent variable	4	5	6	7	8
Dependent variable	5	5.5	6	6.5	7

19. 
$$f(-4) = 24$$
 and  $f(6) = -36$ 

#### 1.2 I Rule!

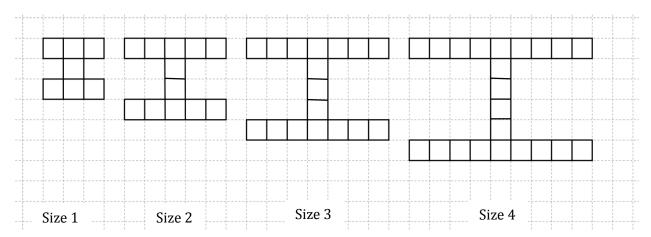
#### A Solidify Understanding Task





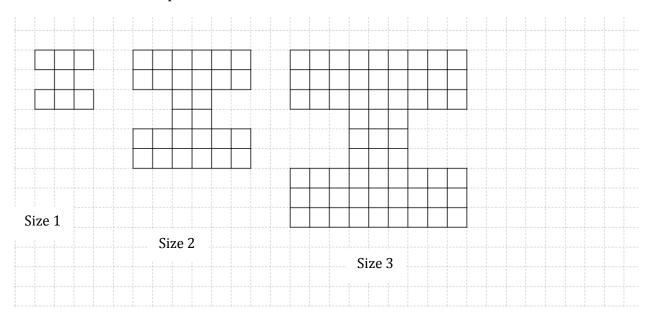
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He is working on creating the logo in various sizes to be placed on different pages on the website. Marco developed the following designs:

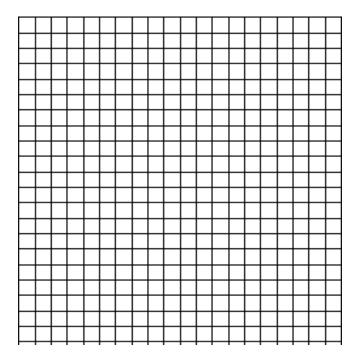


- 1. How many squares will be needed to create the size 100 logo?
- 2. Develop a mathematical model for the number of squares in the logo for size n.

Marco decides to experiment with making his logo "blockier" so that it looks stronger. Here's what he came up with:



3. Assuming that Marco continues with the pattern as it has begun, draw the next figure, size 4, and find the number of blocks in the figure.



4. Develop a mathematical model for the number of blocks in a logo of size n.

5. Compare the models that you developed for the first set of logos to the second set of logos. In what ways are they similar? In what ways are they different?

#### Ready, Set, Go!



#### Ready

Topic: Adding and multiplying binomials

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#### Simplify the following expressions

1a. 
$$(2x + 7) + (5x + 3)$$

2a. 
$$(6x - 1) + (x - 10)$$

3a. 
$$(8x + 3) + (3x - 4)$$

4a. 
$$(-5x + 2) + (7x - 13)$$

5a. 
$$(12x + 3) + (-4x + 3)$$

6. 
$$(x+5)(x-5)$$

b. 
$$(2x + 7)(5x + 3)$$

b. 
$$(6x - 1)(x - 10)$$

b. 
$$(8x + 3)(3x - 4)$$

b. 
$$(-5x + 2)(7x - 13)$$

b. 
$$(12x + 3)(-4x + 3)$$

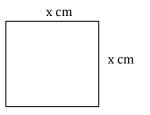
- 7. Compare your answers in 1 5 part a to your answers in #1 #5 part b respectively. Look for a pattern in the answers. How are they different?
- 8. The answer to #6 is a different "shape" than the other *part b* answers, even though you were still multiplying. Explain how it is different from the other products. Try to explain *why* it is different. Think of 2 more examples of multiplication of two binomials that would do the same thing as #6.
- 9. Try adding the two binomials in #6. (x + 5) + (x 5) =\_\_\_\_\_\_ Is this answer a different "shape" than the other *part a* answers? Explain.

#### Set

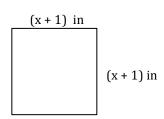
Topic: Comparing area and perimeter

Calculate the *perimeter* and the *area* of the figures below. (You answers will contain a variable.)

10.



11.



a. Perimeter: \_\_\_\_\_

b. Area: \_\_\_\_\_

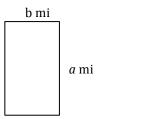
a. Perimeter: \_\_\_\_\_

b. Area: \_\_\_\_\_

12.

 (a + 5) ft	
	(b + 3) ft

13.



a. Perimeter: \_\_\_\_\_

b. Area: \_\_\_\_\_

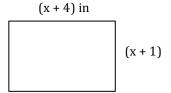
a. Perimeter: \_\_\_\_\_

b. Area: \_\_\_\_\_

14.

ŀ.	(x + 3) m	
		(x - 2) m

15.



a. Perimeter: \_\_\_\_\_

b. Area: \_\_\_\_\_

a. Perimeter: \_\_\_\_\_

b. Area:

- 16. Compare the perimeter to the area in each of problems (10-15).
- a. What do the perimeters and areas have in common?
- b. In what way are the numbers and units in the perimeters and areas different?

Go

Topic: Greatest Common Factor (GCF)

#### Find the GCF of the given numbers.

- 17. 15abc<sup>2</sup> and 25a<sup>3</sup>bc
- 18. 12x<sup>5</sup>y and 32x<sup>6</sup>y
- 19. 17pqr and 51pqr<sup>3</sup>

20.  $7x^2$  and 21x

- 21.  $6x^2$ , 18x, and -12
- 22.  $4x^2$  and 9x

- 23.  $11x^2y^2$ ,  $33x^2y$ , and  $3xy^2$
- 24. 16a<sup>2</sup>b, 24ab, and 16b
- 25. 49s<sup>2</sup>t<sup>2</sup> and 36s<sup>2</sup>t<sup>2</sup>

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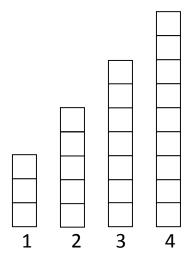
#### 1.3 Scott's Macho March

A Solidify Understanding Task

After looking in the mirror and feeling flabby, Scott decided that he really needs to get in shape. He joined a gym and added push-ups to his daily exercise routine. He started keeping track of the number of push-ups he completed each day in the bar graph below, with day one



showing he completed three push-ups. After four days, Scott was certain he can continue this pattern of increasing the number of push-ups for at least a few months.



1. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.

Scott's gym is sponsoring a "Macho March" promotion. The goal of "Macho March" is to raise money for charity by doing push-ups. Scott has decided to participate and has sponsors that will donate money to the charity if he can do a total of at least 500 push-ups, and they will donate an additional \$10 for every 100 push-ups he can do beyond that.

2. Estimate the total number of push-ups that Scott will do in a month if he continues to increase the number of push-ups he does each day in the pattern shown above.

3.	How many push-ups will Scott have done after a week?
4.	Model the total number of push-ups that Scott has completed on any given day during "Macho March". Include both recursive and explicit equations.
5.	Will Scott meet his goal and earn the donation for the charity? Will he get a bonus? If so, how much? Explain.

#### Name:

#### Ready, Set, Go!



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#### Ready

Topic: Fundamental Theorem of Arithmetic

The prime factorization of a number is given. Multiply each number to find the whole number that each factorization represents.

1. 
$$2^4 \times 3^1 \times 5^2$$

2. 
$$3^4 \times 5^2 \times 7^2$$

$$3.5^2 \times 11^2 \times 13^1$$

The following problems are factorizations of numerical expressions called quadratics. Given the factors, multiply to find the quadratic expression. Add the like terms. Write the x<sup>2</sup> term first, the x-term second, and the constant term last. (Example:  $ax^2 + bx + c$ .)

4. 
$$(x+5)(x-7)$$

5. 
$$(x + 8)(x + 3)$$

6. 
$$2(x-9)(x-4)$$

7. 
$$3(x+1)(x-4)$$

8. 
$$2(3x-5)(x-1)$$

7. 
$$3(x+1)(x-4)$$
 8.  $2(3x-5)(x-1)$  9.  $2(5x-7)(3x+1)$ 

#### Set

Use first and second differences to identify the pattern in the tables as linear, quadratic, or neither. Write the recursive equation for the patterns that are linear or quadratic.

10.

x	у
-3	-23
-2	-17
-1	-11
0	-5
1	1
2	7
3	13

- a. Pattern:
- b. Recursive equation:

11.

X	У
-3	4
-2	0
-1	-2
0	-2
1	0
2	4
3	10

- a. Pattern:
- b. Recursive equation:

x	у
-3	-15
-2	-10
-1	-5
0	0
1	5
2	10
3	15

- a. Pattern:
- b. Recursive equation:

#### Quadratic Functions 1.3

13.

$\boldsymbol{x}$	у
-3	24
-2	22
-1	20
0	18
1	16
2	14
3	12

14.

x	y
-3	48
-2	22
-1	6
0	0
1	4
2	18
3	42
	-

15.

x	у
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

a. Pattern:

b. Recursive equation:

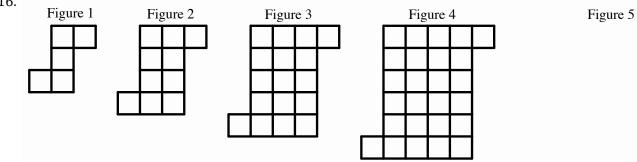
a. Pattern:

b. Recursive equation:

a. Pattern:

b. Recursive equation:

16.



- a. Draw figure 5.
- b. Predict the number of squares in figure 30. Show what you did to get your prediction.

#### Go

Topic: Interpreting recursive equations to write a sequence

Write the first five terms of the sequence.

17. 
$$f(0) = -5$$
;  $f(n+1) = f(n) + 8$  18.  $f(0) = 24$ ;  $f(n+1) = f(n) - 5$ 

18. 
$$f(0) = 24$$
;  $f(n + 1) = f(n) - 5$ 

19. 
$$f(0) = 25$$
;  $f(n + 1) = 3f(n)$ 

20. 
$$f(0) = 6$$
;  $f(n + 1) = 2f(n)$ 

#### 1.4 Rabbit Run

#### A Solidify Understanding Task

Misha has a new rabbit that she named "Wascal". She wants to build Wascal a pen so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.



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1. If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?

2. If Misha wants a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.

3. Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.

4.	What	kind	οf	function	ic	this?	Why?
т.	vvIIat	MIIIU	UΙ	Iuncuon	13	umsi	V V I I V :

5. How does this function compare to the second type of block I logos in *I Rule*?

#### Name: Quad

Quadratic Functions | 1.4

#### Ready, Set, Go!



#### Ready

Topic: applying the slope formula

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Calculate the slope of the line between the given points. Use your answer to indicate which line is the steepest.

1. A (-3, 7) B (-5, 17)

2. H (12, -37) K (4, -3)

3. P(-11, -24) Q(21, 40)

4. R (55, -75) W(-15, -40)

#### **Set** Topic: Investigating perimeters and areas

Adam and his brother are responsible for feeding their horses. In the spring and summer the horses graze in an unfenced pasture. The brothers have erected a portable fence to corral the horses in a grazing area. Each day the horses eat all of the grass inside the fence. Then the boys move it to a new area where the grass is long and green. The porta-fence consists of 16 separate pieces of fencing each 10 feet long. The brothers have always arranged the fence in a long rectangle with one length of fence on each end and 7 pieces on each side making the grazing area 700 sq. ft. Adam has learned in his math class that a rectangle can have the same perimeter but different areas. He is beginning to wonder if he can make his daily job easier by rearranging the fence so that the horses have a bigger grazing area. He begins by making a table of values. He lists all of the possible areas of a rectangle with a perimeter of 160 ft., while keeping in mind that he is restricted by the lengths of his fencing units. He realizes that a rectangle that is oriented horizontally in the pasture will cover a different section of grass than one that is oriented vertically. So he is considering the two rectangles as different in his table. Use this information to answer questions 5 – 9 on the next page.

Horizontal

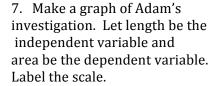
Vertical



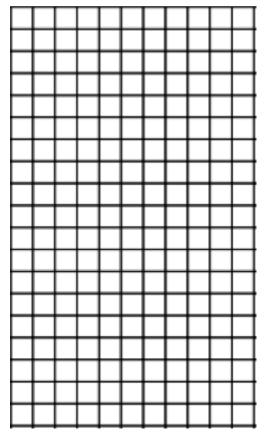
5. Fill in Adam's table with all of the arrangements for the fence. (The first one is done for you.)

	Length in "fencing" units	Width in "fencing" units	Length in ft.	Width in ft.	Perimeter (ft)	Area (ft) <sup>2</sup>
	1 unit	7 units	10 ft	70 ft	160 ft	700 ft <sup>2</sup>
a.	2 units				160 ft	
b.	3 units				160 ft	
c.	4 units				160 ft	
d.	5 units				160 ft	
e.	6 units				160 ft	
f.	7 units				160 ft	

6. Discuss Adam's findings. Explain how you would rearrange the sections of the porta-fence so that Adam will be able to do less work.



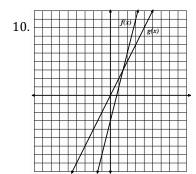
- 8. What is the shape of your graph?
- 9. Explain what makes this function be a quadratic.

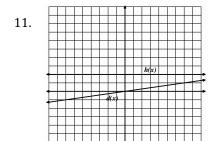


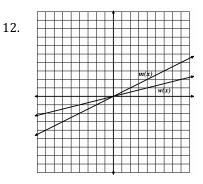
#### Go

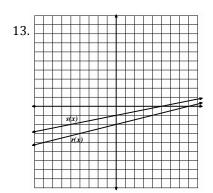
Topic: Comparing linear and exponential rates of change.

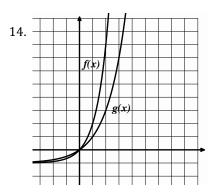
#### Indicate which function is changing faster.

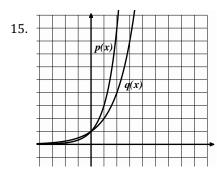


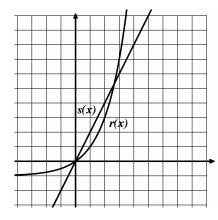












- 16a. Examine the graph at the left from 0 to 1. Which graph do you think is growing faster?
- b. Now look at the graph from 2 to 3. Which graph is growing faster in this interval?

#### 1.5 Look Out Below!

#### A Solidify Understanding Task

What happens when you drop a ball? It falls to the ground.

That question sounds as silly as "Why did the chicken cross the road?" (To get to the other side.) Seriously, it took scientists until the sixteenth and seventeenth centuries to fully understand the physics and mathematics of falling bodies. We now know that gravity acts on the object that is falling in a way that causes it to



accelerate as it falls. That means that if there is no air resistance, it falls faster and faster, covering more distance in each second as it falls. If you could slow the process down so that you could see the position of the object as it falls, it would look something like the picture below.



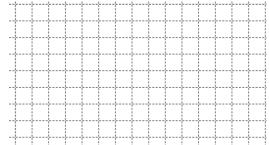
To be more precise, objects fall at a constant rate of acceleration on earth of about 32 feet per second per second. The simplest case occurs when the object starts from rest, that is, when its speed is zero when t=0. In this case, the object's instantaneous speed after 1 second is 32 feet per second; after 2 seconds, its instantaneous speed is 2(32) = 64 feet per second; and so on. Other planets and moons each have a different rate of acceleration, but the basic principal remains the same. If the acceleration on a particular planet is g, then the object's instantaneous speed after 1 second is g units per second; after 2 seconds, its instantaneous speed is 2g units per second; and so on.

In this task, we will explore the mathematics of falling objects, but before we start thinking about falling objects we need to begin with a little work on the relationship between speed, time, and distance.

#### Part 1: Average speed and distance travelled

Consider a car that is traveling at a steady rate of 30 feet per second. At time t = 0, the driver of the car starts to increase his speed (accelerate) in order to pass a slow moving vehicle. The speed increases at a constant rate so that 20 seconds later, the car is traveling at a rate of 40 feet per second.

a. Graph the car's speed as a function of time for this 20-second time interval.



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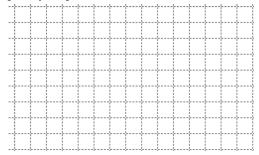


- b. Calculate the average speed of the car for this 20-second time interval.
- c. Find the total distance the car travels during this 20-second time interval.
- d. Explain how to use area to find the total distance the car travels during this 20-second interval.

This problem illustrates an important principle: *If an object is traveling with constant acceleration, then its average speed over any time interval is the average of its beginning speed and its final speed during that time interval.* 

Let's apply this idea to a penny that is dropped (initial speed is 0 when t = 0) from the top of the Empire State Building.

- 1. What will its speed be after 1 second?
- 2. Graph the penny's speed as a function of time in the 1 second interval.



- 3. What is the average speed of the penny in the 1-second interval?
- 4. What is the total distance that the penny fell in the 1-second interval?

#### Part 2: Falling, Falling, Falling

When the astronauts went to the moon, they performed Galileo's experiment to test the idea that any two objects, no matter their mass, will fall at the same rate if there is no air resistance (like on the moon). Because the moon doesn't have air resistance, we are going to pretend like we're the astronauts dropping moon rocks and thinking about what happens. On the surface of the moon the constant acceleration increases the speed of a falling object by 6 feet per second each second. That is, if an object is dropped near the surface of the moon (e.g., its initial speed is zero when t=0), then the object's instantaneous speed after 1 second is 6 feet per second, after 2 seconds, its instantaneous speed is 12 feet per second, and so on.

1. Using this information, create a table for the speed of an object that is dropped from a height of 200 feet above the surface of the moon as a function of the elapsed time (in seconds) since it was dropped.

- 2. Add another column to your table to keep track of the distance the object has fallen as a function of elapsed time. Explain how you are finding these distances.
- 3. Approximately how long will it take for the object to hit the surface of the moon?
- 4. Write an equation for the distance the object has fallen as a function of elapsed time *t*.

5.	Write an equation for the height of the object above the surface of the moon as a function of elapsed time $t$ .
6.	Suppose the object was not dropped, but was thrown downward from a height of 250 feet above the surface of the moon with an initial speed of 10 feet per second. Rewrite your equation for the height of the object above the surface of the moon as a function of elapsed time $t$ to take into account this initial speed.
7.	How is your work on these <i>falling objects problems</i> related to your work with the <i>rabbit runs?</i>
8.	Why are the "distance fallen" and "height above the ground" functions quadratic?

#### Ready, Set, Go!



Ready

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Topic: Evaluating exponential functions

Find the indicated value of the function for each value of x.  $x = \{-2, -1, 0, 1, 2, 3\}$ 

1. 
$$f(x) = 3^x$$

2. 
$$g(x) = 5^x$$

3. 
$$h(x) = 10^x$$

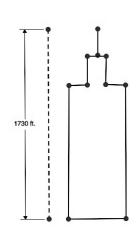
4. 
$$k(x) = \left(\frac{1}{2}\right)^x$$

$$5. \quad m(x) = \left(\frac{1}{3}\right)^x$$

#### Set

The Sears Tower in Chicago is 1730 feet tall. If a penny were let go from the top of the tower, the position above the ground s(t) of the penny at any given time t would be  $s(t) = -16t^2 + 1730$ .

6. Fill in the missing positions in the chart. Then add to get the distance fallen.



Distance from ground

a.	 1 sec
b.	2 sec
c.	 3 sec
d.	 4 sec
e.	5 sec
f.	6 sec
g.	7 sec
h.	 8 sec
i.	 9 sec
j.	10 sec

- 7. How far above the ground is the penny when 7 seconds have passed?
- 8. How far has it fallen when 7 seconds have passed?
- 9. Has the penny hit the ground at 10 seconds? Justify your answer.

The average rate of change of an object is given by the formula  $r = \frac{d}{t}$ , where r is the rate of change, d is the distance traveled, and t is the time it took to travel the given distance. We often use some form of this formula when we are trying to calculate how long a trip may take.

10. If our destination is 225 miles away and we can average 75 mph, then we should arrive in 3

hours.  $\left| \frac{225 \text{ mile}}{75 \text{ mph}} \right| = 3 \text{ hours}$  In this case you would be rearranging the formula so that  $t = \frac{d}{r}$ .

However, if your mother finds out that the trip only took 2 ½ hours, she will be upset. Use the rate formula to explain why.

11. How is the slope formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$  like the formula for rate?

For the following questions, refer back to the penny spoken about in questions 6 - 9.

- 12. Find the average rate of change for the penny on the interval [0, 1] seconds.
- 13. Find the average rate of change for the penny on the interval [6, 7] seconds.
- 14. Explain why the penny's average speed is different from 0 to 1 second than between the 6th and 7th seconds.
- 15. What is the average speed of the penny from [0,10] seconds?
- 16. What is the average speed of the penny from [9,10] seconds?
- 17. Find the first differences on the table where you recorded the position of the penny at each second. What do these differences tell you?
- 18. Take the difference of the first differences. (This would be called the 2<sup>nd</sup> difference.) Did your answer surprise you? What do you think this means?

#### Go

**Topic:** Evaluating functions

- 19. Find f(9) given that  $f(x) = x^2 + 10$ .
- 20. Find g(-3) given that  $g(x) = x^2 + 2x + 4$ .
- 21. Find h(-11) given that  $h(x) = 2x^2 + 9x 43$ .
- 22. Find r(-1) given that  $r(x) = -5x^2 3x + 9$ .
- 23. Find  $s(\frac{1}{2})$  given that  $s(x) = x^2 + \frac{5}{4}x \frac{1}{2}$ .
- 24. Find p(3) given that  $p(x) = 5^x + 2x$ .
- Find q(2) given that  $q(x) = 7^x + 11x$

### 1.6 The Tortoise and the Hare

A Solidify Understanding Task



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In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

 $d = t^2$  (d in meters and t in seconds)

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

 $d = 2^t$  (d in meters and t in seconds)

1. At what time does the hare catch up to the tortoise?

2. If the race course is very long, who wins: the tortoise or the hare? Why?

3. At what time(s) are they tied?



4.	If the race course wer	e 15 meters	long who wins.	the tortoise	or the hare?	Why?
1.	ii tiit latt toulst wel	c 15 mctcrs	iong wife wills,	, the tortorse	or the marc.	vviiy.

5. Use the properties  $d = 2^t$  and  $d = t^2$  to explain the **speeds** of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$
[0, 2)		
[O 4]		
[2, 4)		
[4,∞)		
1		

#### Ready, Set, Go!



Ready

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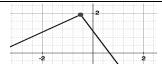
Topic: Recognizing functions

Identify which of the following representations are functions. If it is NOT a function state how you would fix it so it was.

1.  $D = \{(4,-1)(3,-6)(2,-1)(1,2)(0,4)(2,5)\}$ 

2. The number of calories you have burned since midnight at any time during the day.

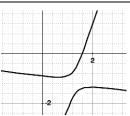
3.



4.

X	-12	-8	-6	-4
f(x)	25	25	25	25

5.



6.



#### Set

Topic: Comparing rates of change in linear, quadratic, and exponential functions

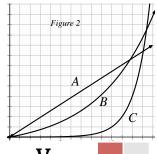
The graph at the right shows a time vs. distance graph of two cars traveling in the same direction along the freeway.

Figure 1

A

B

- 7. Which car has the cruise control on? How do you know?
- 8. Which car is accelerating? How do you know?
- 9. Identify the interval in *figure 1* where car A seems to be going faster than car B.
- 10. Identify the interval in *figure 1* where car B seems to be going faster than car A.
- 11. What in the graph indicates the speed of the cars?
- 12. A third car *C* is now shown in the graph (*see figure 2*). All 3 cars have the same destination. If the destination is a distance of 12 units from the origin, which car do you predict will arrive first? Justify your answer.



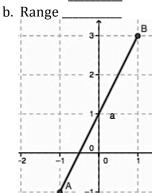
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#### Go

Topic: Identifying domain and range from a graph.

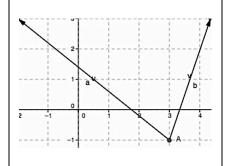
State the domain and range of each graph. Use interval notation where appropriate.

13a. Domain \_\_



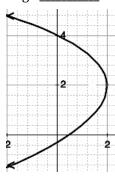
14a. Domain \_\_\_\_\_

b. Range \_\_\_\_\_

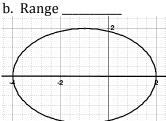


15a. Domain \_\_\_

b. Range \_\_\_\_

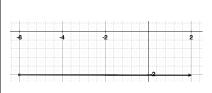


16a. Domain



17a. Domain \_\_\_

b. Range \_\_\_\_\_



18a. Domain \_\_\_\_\_

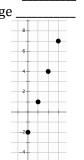
b. Range \_\_\_\_

19a. Domain

b. Range

20a. Domain\_

b. Range



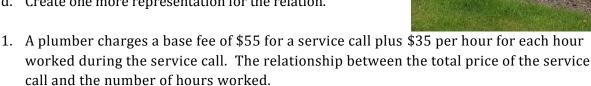
21. Are the domains of #19 and #20 the same? Explain.

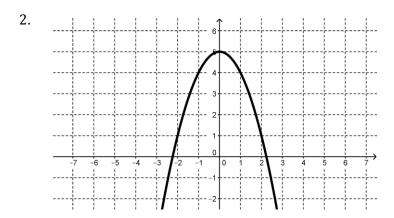
#### 1.7 How Does It Grow?

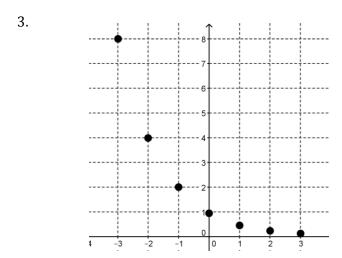
#### A Practice Understanding Task

#### For each relation given:

- a. Identify whether or not the relation is a function;
- b. Determine if the function is linear, exponential, quadratic or neither;
- c. Describe the type of growth
- d. Create one more representation for the relation.



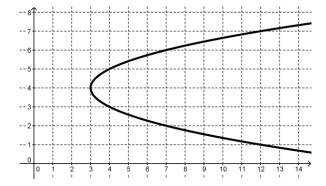






4. 
$$y = \frac{1}{3}(x-2)^2 + 4$$



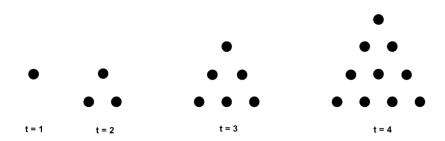


6. 
$$y = \frac{1}{3}(x-2) + 4$$

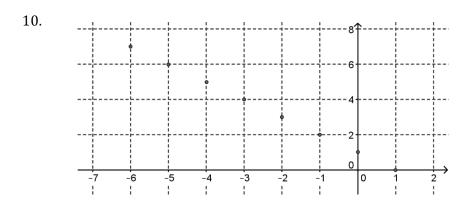
7. The relationship between the speed of a car and the distance it takes to stop when travelling at that speed.

Speed (mph)	Stopping
	Distance (ft)
10	12.5
20	36.0
30	69.5
40	114.0
50	169.5
60	249.0
70	325.5

8. The relationship between the number of dots in the figure and the time, *t*.



9. The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. The relationship between the amount of caffeine in the bloodstream and the number of hours from the time the adult drinks the caffeinated beverage.



11. 
$$y = (4x + 3)(x - 6)$$

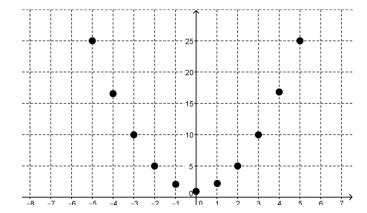
- 12. Mary Contrary wants to build a rectangular flower garden surrounded by a walkway 4 meters wide. The flower garden will be 6 meters longer than it is wide.
  - a. The relationship between the width of the garden and the perimeter of the walkway.
  - b. The relationship between the width of the garden and area of the walkway.

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13. 
$$y = \left(\frac{1}{3}\right)^{x-2} + 4$$

14.



#### Ready, Set, Go!



Ready

Topic: transforming lines

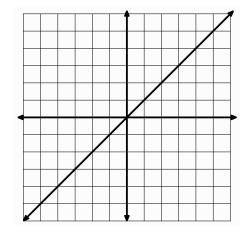
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1. Graph the following linear equations on the grid. The equation y = x has been graphed for you. For each new equation explain what the number 3 does to the graph of y = x. Pay attention to the y-intercept, the x-intercept, and the slope. Identify what changes in the graph and what stays the same.

a. 
$$y = x + 3$$

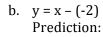
b. 
$$y = x - 3$$

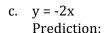
c. 
$$y = 3x$$

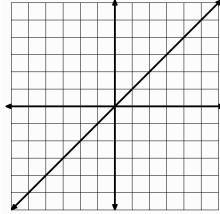


2. The graph of y = x is given. (*See figure 2*.) For each equation predict what you think the number -2 will do to the graph. Then graph the equation.

a. 
$$y = x + (-2)$$
  
Prediction:





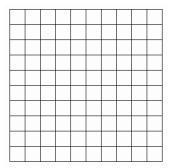


#### Set

Topic: Distinguishing between linear, exponential, and quadratic functions

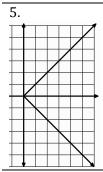
#### For each relation given:

- a. Identify whether or not the relation is a function. (If it's not a function, skip b d.)
- b. Determine if the function is Linear, Exponential, Quadratic or Neither.
- c. Describe the type of growth.
- d. Express the relation in the indicated form.
- 3. I had 81 freckles on my nose before I began using vanishing cream. After the first week I had 27, the next week 9, then 3...
- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Make a graph. Label your axes and the scale Show all 4 points.

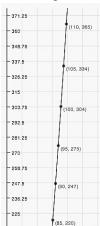


4.	X	у
т.	0	80
	1	$80\frac{2}{3}$
	2	$80\frac{1}{3}$
	3	80
	4	79 <sup>2</sup>

- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Write the explicit equation.



- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Create a table
- 6. Speed in mph of a baseball vs. distance in ft.



- a. Function?
- b. Linear, Exponential, Quadratic or Neither
- c. How does it grow?
- d. Predict the distance the baseball flies if it leaves the bat at a speed of 115 mph.

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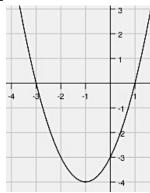
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#### Go

Match the function on the left with the equivalent function on the right.

$$_{---} 7. \quad f(x) = -2x + 5$$

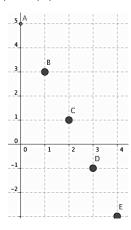
8.



a. 
$$f(x) = 5(2)^x$$

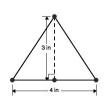
b.

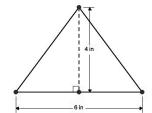
d.



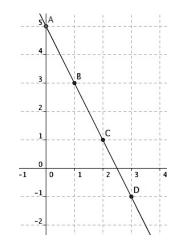
\_10. The area of the triangles below.







c. 
$$f(1) = 2$$
;  $f(n+1) = f(n) + 2n$ 



\_\_\_\_11. 
$$f(0) = 5$$
;  $f(n) = 2 * f(n-1)$ 

\_\_\_\_12. 
$$f(0) = 5$$
;  $f(n) = f(n-1)-2$ 

e. 
$$y + x = 0$$

f. 
$$y = (x - 1)(x + 3)$$

g. 
$$A = 7000(1.03)^{20}$$