High School Geometry
CCSS Scope and Sequence Proposed Scope and Sequence
by Timothy D. Kanold

First Semester – 75 Instructional days

Unit 1: Geometric Foundations and Tools (10 Instructional days)

G-CO Congruence

Experiment with transformations in the plane (*Supporting Cluster*)

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc

Make geometric constructions (*Supporting Cluster*)

G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 1 Notes:
This first Unit connects to the grades 6-8 Geometry standards and the definitions established there, as outlined in G.CO.1, and provide an opportunity to learn to use tools that will be pervasive throughout the course.

During this course, students explore geometric ideas using key tools of geometry including: Constructions, Transformations and Coordinate Properties (Algebraic) in the plane. This first unit should focus on compass and straightedge constructions, both by hand and in dynamic geometry environments —such as Cabri Jr. on a TI-84+ or similar Calculator.

The goal is to both create constructions and ask to students to explore *why* the constructions work. This unit also considers other tools, such as paper folding, etc. As with the compass and straightedge constructions, it is important to both mechanically use the tools and to expect students to work toward mathematical explanations and justifications using figures from G-CO. 13.

Unit 2: Rigid Transformations (15 Instructional days)

G-CO Congruence

Experiment with transformations in the plane (*Supporting Cluster*)

G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions (*Major Cluster*)

G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Unit 2 Notes:
Building upon the informal experiences with basic geometric objects and relationships, the goal in this Unit is to increase the precision and the use of the definitions established in Unit 1.

The emphasis in this unit should be on the role of definitions and communicating mathematical explanations and arguments that cause and preserve congruence, rather than on developing a deductive axiomatic system. In Grade 8 rigid motions were explored, but in this course they must be more precisely defined, and their properties explored.

This unit should provide the detail necessary to use transformations as a “Proving” tool throughout future Chapters. G-CO.6 and takes the time needed to develop understanding and fluency about congruence of different shapes based on the rigid motions of rotation, reflection, and translation.

Unit 3: Proving Geometric Theorems (18 Instructional days)

G-CO Congruence

Prove geometric theorems (*Major Cluster*)

G-CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.

G-GPE Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically (*Major Cluster*)

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, [prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;] prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2)*.

Unit 3 Notes:
This unit establishes Coordinate Geometry proof using both slope and distance formula. Much of the *Geometric System* at this stage is verified using three methods: Constructions (or some such tool – including *dynamic* software), Rigid Motions from Unit 2, or Coordinate Geometry (A major cluster area). This allows for all three methods then, to be used throughout the rest of the course.

This unit brings together many of the usual theorems of geometry. The emphasis should be on the many roles of proof and a focus on the mathematical practice of making viable arguments and critiquing the reasoning of others (#3). Multiple representations (coordinates, synthetic, and algebraic proofs of properties can be analyzed and compared) and the lines and angles theorem are explored… thus leading to the opportunity for multiple solution pathways.

Note too, the special properties to be developed for understanding as revealed in the Standards for this unit—including work with parallel lines and transversals, essential to the Triangle Sum Theorem in the next Unit.

At this stage in the course students are armed with three different methods for proving true and for applying various properties of the Geometry discipline. The real work of the course can begin.

Unit 4: Triangle Congruence and Triangle Properties (20 Instructional days)

G-CO Congruence

Prove geometric theorems (*Major Cluster*)

G-CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; [the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;] the medians of a triangle meet at a point.*

Understand congruence in terms of rigid motions (*Major Cluster)*

G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G-C Circles

Understand and apply theorems about circles (*Additional Cluster*)

G-C.3 Construct the inscribed and circumscribed circles of a triangle, [and prove properties of angles for a quadrilateral inscribed in a circle.]

Unit 4 Notes:
This is a typical unit on Triangle congruence, but from the lens of transformations and the definition of congruence using rigid motions.

Note that G-CO.6 would be an extension from the limitations presented on this topic in Unit 2

This unit also connects back to *G-CO.13: Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle* from Unit 1. Given time, one should pursue the location and the properties of the Centroid, the Incenter, the Circumcenter and the Orthocenter.

Unit 5: Quadrilateral Properties (12 instructional days)

G-CO Congruence

Experiment with transformations in the plane (*Supporting Cluster and Major*)

G-CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

G-C Circles

Understand and apply theorems about circles (*Additional Cluster*)

G-C.3 Construct [the inscribed and circumscribed circles of a triangle,] and prove properties of angles for a quadrilateral inscribed in a circle.

G-GPE Expressing Geometric Properties with Equations

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; [prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).]*

Unit 5 Notes:
Making connections between geometric relationships and properties and algebraic expressions is an important mathematical process. This final unit of first semester provides opportunities to go back and forth between algebraic and geometric representations of concepts as students should use both Transformational and coordinate (Expressing Geometric Properties with Equations G-GPE) approaches to the properties of parallelograms.

Note that in this unit and in Unit 4, standard G-C.3 from the Circle domain are introduced as appropriate.

The unit should also build upon the previous unit making connections between similarity and linearity, for example in $f(x) = mx$ the slope m can be seen as a " Shape" factor for a right triangle and the x can viewed as the scale factor of a dilation centered at the origin (This can be thought of as a lead in for the next unit on similarity).

This unit could use two more instructional days, if time allows. And the unit should take advantage of technology such as Cabri Jr. for allowing a robust discovery of many of these properties.

This Unit signifies the end of First Semester.

Second Semester – 80 Instructional days

Unit 6: Similarity Transformations (20 Instructional days)

G-SRT Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations (*Major Cluster*)

G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Understand similarity in terms of similarity transformations

G-SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*.

G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-GPE Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically (*Major Cluster*)

G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-CO Congruence

Prove geometric theorems (*Major Cluster*)

G-CO.10 Prove theorems about triangles. *Theorems include: [measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent;] the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; [the medians of a triangle meet at a point.]*

***Unit 6 notes**:*
This opening of 2nd Semester Unit is packed. From defining similarity transformation as the composition of a dilation followed by a congruence, to proving that the meaning of similarity for triangles is the

equality of all corresponding pairs of angles and the *proportionality* of all corresponding pairs of sides, to proving certain theorems based on similarity or congruence, to using the coordinate plane; the work of this chapter is extensive. This Chapter also includes proof of the Pythagorean theorem – but does not include any right triangle trigonometry (next unit).

The is also a connection to be made between similarity and linearity (why is the graph of a linear function a straight line, and understanding slope as a constant rate of change).

This Unit should end by formalizing and connecting to the Grade 8 standard 8.EE 6 *Understand the connections between proportional relationships, lines, and linear equations.*

Unit 7: Trigonometric Ratios (15 Instructional days)

G-SRT Similarity, Right Triangles, and Trigonometry

Define trigonometric ratios and solve problems involving right triangles (*Major Cluster and Additional Cluster*)

Prove theorems involving similarity (*Major Cluster*)

G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-SRT Similarity, Right Triangles, and Trigonometry (optional "+" Standards)

G-SRT.9 (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 7 Notes:
In this unit, the trig ratios are established using "within" similarity comparisons for right triangles i.e. the ratio of the lengths of two edges in a right triangle is an important "shape invariant". These ratios have been given names: sine = opposite/hypotenuse, cosine = adjacent/hypotenuse and tangent = opposite/adjacent.

All of the properties described in G-SRT5-8 are explored resulting in the student being able to "solve" any right triangle. Important in this unit is the definition of the trig ratios and the connections made to triangle similarity issues – that the side ratios in the right triangles are related to the acute angles of the triangle.

This unit can and should be extended to include G-SRT. 9-11 — unless these standards are listed as part of a year 4 course in your school. And, as with all of the units in 2nd Semester, presentation of this unit with a graphing calculator is essential.

Unit 8: Properties of Circles (15 Instructional days)

G-C Circle

Understand and apply theorems about circles (*Additional Cluster*)

G-C.1 Prove that all circles are similar.

G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Find arc lengths and areas of sectors of circles

G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

G-GPE Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section (*Additional Cluster)*

G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Unit 8 Notes:
This unit establishes the definitions of congruent and similar circles, and includes all of the key properties of circles, treated both synthetically, with attention to constructions, and analytically using coordinate proof. Note that G-C.3 would be treated here only if you chose NOT to do so in Units 4 and 5.

Arc length is important to this unit, and its connection to the radian measure of an angle as the constant of proportionality. Establish via similarity the relationship between the arc length of a sector of a circle and the radius of that circle.

Note too that G-GPE.2 — *Derive the equation of a parabola given a focus and directrix* is an Algebra 2 Standard. Also note that this is an additional cluster here. However, there is probably space to treat this in some fashion if desired.

Unit 9: Geometric Modeling, Measurement and Dimension (15 Instructional days)

G-GMD Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems (*Additional Cluster*)

G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*.

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects (*Additional Cluster*)

G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

G-MG Modeling with Geometry

Apply geometric concepts in modeling situations (*Major Cluster*)

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or human torso as a cylinder).★

G-MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Unit 9 Notes:
This unit should be driven by the modeling aspect – using the measurement – 2 and 3-D, to facilitate the modeling. The unit could be split in 2, but it is preferred to integrate the lessons. They might go in the order of G – GMD.1, G-GMD.4 (This will be very new), then G-GMD.3 while integrating G-MG 1-3.

This unit should treat geometric measurement more comprehensively and make connections and explain the various volume formulas with attention to the structure of the algebraic expressions – how are they generated? Moving between 2 and 3 dimensions via slicing and rotation, and establishing area phenomena as fundamental contexts for things quadratic and volume phenomena as fundamental contexts for cubic relationships is helpful.

This Unit builds on and connects to the 7th grade geometry standard: *7G-G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume*

Unit 10: Understanding Probability (15 Instructional days)

S-CP Conditional Probability and the Rules of Probability★

Understand independence and conditional probability and use them to interpret data (*Additional Cluster*)

S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

S-CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S-CP.3 Understand the conditional probability of A given B as P(A and B) / P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Use the rules of probability to compute probabilities of compound events in a uniform probability model (*Additional Cluster*)

S-CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.★

S-CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★

Unit 10 Notes:
This Unit is clearly an Algebra 2 Unit for PARCC. For SBAC it won't matter as much. However, it should be taught at the end of the Geometry year. *After the PARCC Exam*. It relieves the pressure of a PACKED Algebra 2 course. And it can have a "connection" to Geometry by using geometric probability contexts where applicable.

Note that S – CP.6 and 7 could be saved until Algebra 2 if so desired. This unit builds upon knowledge of probability first learned in seventh grade that includes uses of random sampling to draw inferences, compares inferences between populations, investigates chance processes, and develops, uses and evaluates probability models.

The unit introduces foundational properties and rules of probabilities, starting with events as subsets of sample space and introducing independence and conditional probability. The unit includes treatment of rules of probability to determine likelihood of compound events and introduces counting principals to calculate probability..