

Resource Title: Secondary One Mathematics Student Edition

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Core Subject Area: Secondary II Mathematics

Mathematics, Secondary II

Standard	Designated Sections
Unit 1: Extending the Number System	
Extend the properties of exponents to rational exponents.	
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.</i>	Module 3 Task 1 Experimenting with Exponents Module 3 Task 2 Half Interested Module 3 Task 3 More Interesting Module 3 Task 3 More Interesting Module 3 Task 4 Radical Ideas
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Module 3 Task 3 More Interesting Module 3 Task 4 Radical Ideas
Use properties of rational and irrational numbers.	

<i>Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.</i>	
N.RN.3 Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.	Module 3 Task 9 My Irrational and Imaginary Friends Module 3 Task 10 iNumbers
Perform arithmetic operations with complex numbers. <i>Limit to multiplications that involve i^2 as the highest power of i.</i>	
N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	Module 3 Task 9 My Irrational and Imaginary Friends Module 3 Task 10 iNumbers
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Module 3 Task 9 My Irrational and Imaginary Friends Module 3 Task 10 iNumbers
Perform arithmetic operations on polynomials. Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x .	
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Module 3 Task 10 iNumbers
Unit 2: Quadratic Functions and Modeling	
Interpret functions that arise in applications in terms of a context. <i>Focus on quadratic functions; compare with linear and exponential functions studied in Secondary Mathematics I.</i>	
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>	Module 4 Task 7 More Features, More Functions *This standard shows up as a related standard throughout many tasks in Modules 1, 2, 3, and 4.
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of</i>	Module 4 Task 1 Some of This, Some of That Module 4 Task 2 Bike Lovers

<p><i>person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★</i></p>	<p>Module 4 Task 2 Bike Lovers Module 4 Task 3 More Functions with Features Module 4 Task 4 Reflections of a Bike Lover</p>
<p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★</p>	<p>Module 1 Task 5 Look Out Below Module 1 Task 6 Tortoise and the Hare Module 3 Task 1 Experimenting With Exponents</p>
<p>Analyze functions using different representations. <i>For F.IF.7b, compare and contrast absolute value, step and piecewise- de- fined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and useful- ness when examining piecewise- defined functions. Note that this unit, and in particular in F.IF.8b, ex- tends the work begun in Secondary Mathematics I on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.</i></p>	
<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p>	<p>Module 2 Task 1 Shifty y’s Module 2 Task 2 Transformer’s: More Than Meets the y’s Module 4 Task 1 Some of This, Some of That Module 4 Task 2 Bike Lovers Module 4 Task 3 More Functions with Features Module 4 Task 4 Reflections of a Bike Lover</p>
<p>F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</i></p>	<p>Module 2 Task 3 Building the Perfect Square Module 2 Task 4 Factor Fixin’ Module 2 Task 5 Lining Up Quadratics Module 2 Task 6 I’ve Got a Fill-in Module 3 Task 3 More Interesting</p>
<p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p>	<p>Module 1 Task 2 I Rule Module 1 Task 3 Scott’s Macho March Module 1 Task 6 Tortoise and the Hare</p> <p>*This standard shows up as a related standard throughout many tasks in Modules 1, 2, 3, and 4.</p>

<p>Build a function that models a relationship between two quantities. <i>Focus on situations that exhibit a quadratic or exponential relationship.</i></p>	
<p>F.BF.1 Write a function that describes a relationship between two quantities.*</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p>	<p>Module 1 Task 1 Something to Talk About Module 1 Task 2 I Rule Module 1 Task 3 Scott's Macho March Module 1 Task 4 Rabbit Run Module 1 Task 5 Look Out Below Module 1 Task 6 Tortoise and Hare Module 2 Task 4 Factor Fixin' Module 2 Task 5 Lining Up Quadratics Module 2 Task 6 I've Got a Fill-in</p>
<p>Build new functions from existing functions. <i>For F.BF.3, focus on quadratic functions and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2, x > 0$.</i></p>	
<p>F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>Module 2 Task 1 Shifty y's Module 2 Task 2 Transformer's: More Than Meets the y's</p>
<p>F.BF.4 Find inverse functions.</p> <p>a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. <i>For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.</i></p> <p>c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting the domain.</p>	<p>Module 4 Task 5 What's Your Pace? Module 4 Task 6 Bernie's Bikes</p>
<p>Construct and compare linear, quadratic, and exponential models and solve problems. <i>Compare linear and exponential growth studied in Secondary Mathematics I to quadratic growth.</i></p>	
<p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>	<p>Module 1 Task 3 Scott's Macho March Module 1 Task 6 Tortoise and Hare Module 1 Task 7 How does it Grow?</p>

<p>Unit 3: Expressions and Equations</p> <p>Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.</p>	
<p>Interpret the structure of expressions.</p> <p><i>Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Secondary Mathematics I to rational exponents focusing on those that represent square or cube roots.</i></p>	
<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>b Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</i></p>	<p>Module 1 Task 1 Something to Talk About</p> <p>Module 1 Task 2 I Rule</p> <p>Module 1 Task 4 Rabbit Run</p> <p>Module 1 Task 5 Look Out Below</p> <p>Module 1 Task 6 Tortoise and Hare</p>
<p>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</p>	<p>Module 2 Task 3 Building The Perfect Square</p> <p>Module 2 Task 4 Factor Fixin'</p> <p>Module 2 Task 5 Lining Up Quadratics</p>
<p>Write expressions in equivalent forms to solve problems.</p> <p>It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.</p>	
<p>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.«</p> <p>a. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>c. Use the properties of exponents to transform expressions for exponential functions.</p> <p><i>For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></p>	<p>Module 2 Task 4 Factor Fixin'</p> <p>Module 2 Task 5 Lining Up Quadratics</p> <p>Module 2 Task 6 I've Got a Fill-in</p> <p>Module 3 Task 3 More Interesting</p>
<p>Create equations that describe numbers or relationships.</p> <p><i>Extend work on linear and exponential equations in Secondary Mathematics I to quadratic equations. Extend A.CED.4 to formulas involving squared variables</i></p>	
<p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems.</p>	<p>Module 3 Task 6 Curbside Rivalry</p>

<p><i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p>	<p>Module 3 Task 7 Perfecting My Quads Module 3 Task 11 Quadratic Quandaries</p>
<p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p>	<p>Module 1 Task 1 Something to Talk About Module 1 Task 2 I Rule Module 1 Task 4 Rabbit Run Module 1 Task 5 Look Out Below Module 1 Task 6 Tortoise and Hare</p>
<p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law $V = IR$ to highlight resistance R.</i></p>	<p>Module 3 Task 5 Throwing an Interception Module 3 Task 6 Curbside Rivalry Module 3 Task 7 Perfecting My Quads</p>
<p>Solve equations and inequalities in one variable. <i>Extend to solving any quadratic equation with real coefficients, including those with complex solutions.</i></p>	
<p>A.REI.4 Solve quadratic equations in one variable.</p> <p>a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</p> <p>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</p>	<p>Module 3 Task 5 Throwing an Interception Module 3 Task 6 Curbside Rivalry Module 3 Task 7 Perfecting My Quads Module 3 Task 8 To Be Determined</p>
<p>Use complex numbers in polynomial identities and equations. <i>Limit to quadratics with real coefficients.</i></p>	
<p>N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.</p>	<p>Module 3 Task 8 To Be Determined Module 3 Task 9 My Irrational and Imaginary Friends</p>
<p>N.CN.8 Extend polynomial identities to the complex numbers. <i>For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.</i></p>	<p>Module 3 Task 8 To Be Determined Module 3 Task 9 My Irrational and Imaginary Friends</p>
<p>N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>	<p>Module 3 Task 8 To Be Determined Module 3 Task 9 My Irrational and Imaginary Friends</p>
<p>Solve systems of equations.</p>	

<p>Include systems consisting of one linear and one quadratic equation.</p> <p>Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point $(3/5, 4/5)$ on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.</p>	
<p>A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</p>	<p>Module 3 Task 6 Curbside Rivalry Module 3 Task 7 Perfecting My Quads</p>
<p>Unit 4: Descriptive Statistics</p> <p>Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.</p>	
<p>Understand independence and conditional probability and use them to interpret data.</p> <p>Build on work with two-way tables from Secondary Mathematics I Unit 4 (S.ID.5) to develop understanding of conditional probability and independence.</p>	
<p>S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p>	<p>Module 9 Task 3 Fried Freddy’s</p> <p>*S.CP.1 is a related standard in several tasks throughout Module 9</p>
<p>S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</p>	<p>Module 9 Task 3 Fried Freddy’s Module 9 Task 5 Freddy Revisited Module 9 Task 6 Striving for Independence</p>
<p>S.CP.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</p>	<p>Module 9 Task 5 Freddy Revisited Module 9 Task 6 Striving for Independence</p>
<p>S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</p>	<p>Module 9 Task 2 Chocolate vs Vanilla Module 9 Task 5 Freddy Revisited Module 9 Task 6 Striving for Independence</p>

<p>S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i></p>	<p>Module 9 Task 5 Freddy Revisited Module 9 Task 6 Striving for Independence</p>
<p>Use the rules of probability to compute probabilities of compound events in a uniform probability model.</p>	
<p>S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.</p>	<p>Module 9 Task 1 TB or Not TB Module 9 Task 2 Chocolate vs Vanilla Module 9 Task 3 Fried Freddy's Module 9 Task 4 Visualizing with Venn Module 9 Task 6 Striving for Independence</p>
<p>S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.</p>	<p>Module 9 Task 3 Fried Freddy's Module 9 Task 4 Visualizing with Venn</p>
<p>S.CP.8 Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model.</p>	<p>Module 9 Task 6 Striving for Independence</p>
<p>Use probability to evaluate outcomes of decisions. <i>This unit sets the stage for work in Secondary Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.</i></p>	
<p>S.MD.1 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p>	<p>*S.MD.1 is found in several Ready, Set, Go's in Module 9</p>
<p>S.MD.2 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>	<p>Module 9 Task 1 TB or Not TB</p>
<p>Unit 5: Similarity, Right Triangle Trigonometry, and Proof</p> <p><i>Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem.</i></p> <p><i>It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.</i></p>	

Understand similarity in terms of similarity transformations.	
<p>G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.</p> <p>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</p> <p>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</p>	Module 6 Task 1 Photocopy Faux Pas
<p>G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p>	Module 6 Task 2 Triangle Dilations Module 6 Task 3 Similar Triangles and Other Figures
<p>G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>	Module 6 Task 3 Similar Triangles and Other Figures
<p>Prove geometric theorems.</p> <p><i>Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 6.</i></p>	
<p>G.CO.9 Prove theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i></p>	Module 5 Task 2 Do You See What I See? Module 5 Task 3 It's All in Your Head Module 5 Task 4 Parallelism Preserved Module 5 Task 5 Conjectures and Proof Module 6 Task 5 Measured Reasoning
<p>G.CO.10 Prove theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i></p>	Module 5 Task 1 How Do You Know That? Module 5 Task 2 Do You See What I See? Module 5 Task 3 It's All in Your Head Module 5 Task 5 Conjectures and Proof Module 5 Task 8 Centers of a Triangle Module 6 Task 5 Measured Reasoning
<p>G.CO.11 Prove theorems about parallelograms. <i>Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each</i></p>	Module 5 Task 2 Do You See What I See? Module 5 Task 6 Parallelogram Conjectures and Proof

<i>other, and conversely, rectangles are parallelograms with congruent diagonals.</i>	Module 5 Task 7 Guess My Parallelogram
Prove theorems involving similarity.	
G.SRT.4 Prove theorems about triangles. <i>Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.</i>	Module 6 Task 4 Cut By A Transversal Module 6 Task 5 Measured Reasoning Module 6 Task 7 Pythagoras By Proportions
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figure	Module 6 Task 2 Triangle Dilations Module 6 Task 5 Measured Reasoning Module 6 Task 7 Pythagoras By Proportions
Use coordinates to prove simple geometric theorems algebraically.	
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	Module 6 Task 6 Yard Work in Segments
Define trigonometric ratios and solve problems involving right triangles.	
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Module 6 Task 8 Are Relationships Predictable? Module 6 Task 9 Relationships with Meaning Module 6 Task 11 Solving Right Triangles Using Trigonometric Relationships
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	Module 6 Task 9 Relationships with Meaning Module 6 Task 10 Finding the Value of a Relationship Module 6 Task 11 Solving Right Triangles Using Trigonometric Relationships
G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	Module 6 Task 8 Are Relationships Predictable? Module 6 Task 10 Finding the Value of a Relationship
Prove and apply trigonometric identities. In this course, limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. A course with a greater focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could continue to be limited to acute angles in Mathematics II. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.	
F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$,	Module 6 Task 9 Relationships with Meaning

or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle.	Module 6 Task 11 Solving Right Triangles Using Trigonometric Relationships
<p>Unit 6: Circles with and without Coordinates</p> <p>In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.</p>	
Understand and apply theorems about circles.	
G.C.1 Prove that all circles are similar.	Module 7 Task 2 Circle Dilations
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i>	Module 7 Task 1 Centered Module 7 Task 3 Cyclic Polygons Module 7 Task 6 Circular Reasoning
G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Module 7 Task 3 Cyclic Polygons
G.C.4 Construct a tangent line from a point outside a given circle to the circle.	Module 7 Task 3 Cyclic Polygons
<p>Find arc lengths and areas of sectors of circles.</p> <p><i>Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.</i></p>	
G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Module 7 Task 7 Pied Module 7 Task 8 Madison's Round Garden Module 7 Task 9 Rays and Radians
<p>Translate between the geometric description and the equation for a conic section.</p> <p><i>Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.</i></p>	
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an	Module 8 Task 1 Circling Triangles Module 8 Task 2 Getting Centered

equation.	Module 8 Task 3 Circe Challenges
G.GPE.2 Derive the equation of a parabola given a focus and directrix.	Module 8 Task 4 Directing Our Focus Module 8 Task 5 Functioning with Parabolas Module 8 Task 6 Turn It Around
Use coordinates to prove simple geometric theorems algebraically. Include simple proofs involving circles.	
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.</i>	Module 8 Task 1 Circling Triangles (Or Triangulating Circles) Module 8 Task 2 Getting Centered Module 8 Task 3 Circle Challenges
Explain volume formulas and use them to solve problems. <i>Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.</i>	
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i>	Module 7 Task 4 Planning the Gazebo Module 7 Task 5 From Polygons to Circles Module 7 Task 10 Sand Castles
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	Module 7 Task 10 Sand Castles